Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Colorado
Based on data through August 24, 2020
Outline of Slides

- Basic data from Johns Hopkins CSSE (raw and smoothed)
- Brief summary of the model
- Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
- Simulation of re-opening – possibilities for raising $R_0$
- Results with alternative parameter values:
  - Lower mortality rate, $\delta = 0.8\%$
  - Higher mortality rate, $\delta = 1.2\%$
  - Infections last longer, $\gamma = 0.15$
  - Cases resolve more quickly, $\theta = 0.2$
  - Cases resolve more slowly, $\theta = 0.07$
- Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Colorado: Daily Deaths per Million People

Daily deaths per million people

-5 0 5 10 15 20 25

April May June July August 2020
Colorado: Daily Deaths per Million People (Smoothed)
**Brief Summary of Model**

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Colorado: Estimates of $R_0(t)$

$R_0(t)$

Colorado

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Colorado: Percent Currently Infectious

Peak I/N = 0.29%  Final I/N = 0.03%  δ = 0.010  θ = 0.10  γ = 0.20
Colorado: Growth Rate of Daily Deaths over Past Week (percent)

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

• **Warning:** Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

• 7 days of forecasts: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  
  - **Black** = current
  - **Red** = oldest, **Orange** = second oldest, **Yellow** = third oldest...
  - **Violet** (purple) = one day earlier

• For robustness graphs, same idea
  
  - **Black** = baseline (e.g. $\delta = 1.0\%$)
  - **Red** = lowest parameter value (e.g. $\delta = 0.8\%$)
  - **Green** = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:
  1. Alternatively, we fit this equation:

$$\log R_0(t) = a_0 - \alpha(Daily\ Deaths)$$

$$\Rightarrow \alpha \approx .05$$

$R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline.

- Robustness: Assume $R_0(t) =$ final empirical value. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Colorado (7 days): Daily Deaths per Million People ($\alpha = .05$)

Colorado

$R_0 = 1.6/0.9/0.9$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 3/3/4

DATA THROUGH 24-AUG-2020
Colorado (7 days): Cumulative Deaths per Million (Future, $\alpha = .05$)

\[ R_0 = 1.6/0.9/0.9 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \% \text{Infect} = 3/3/4 \]

DATA THROUGH 24-AUG-2020
Colorado (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = .05$)

Colorado

$R_0=1.6/0.9/0.9$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect= 3/ 3/ 4

New York City

Italy
Robustness to Mortality Rate, $\delta$
Colorado: Cumulative Deaths per Million ($\delta = 0.01/0.008/0.012$)

Colorado

$R_0=1.6/0.9/0.9 \; \delta = 0.010 \; \alpha=0.05 \; \theta=0.1 \; \%Infect=3/3/4$

DATA THROUGH 24-AUG-2020
Colorado: Daily Deaths per Million People ($\delta = .01/.008/.012$)

$R_0 = 1.6/0.9/0.9$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect= 3/3/4

DATA THROUGH 24-AUG-2020
Colorado: CumulativeDeaths per Million ($\delta = .01/.008/.012$)

R$_0$=1.6/0.9/0.9 $\delta = 0.010$ $\alpha=0.05$ $\theta=0.1$ %Infect= 3/3/4

DATA THROUGH 24-AUG-2020
Reopening and Herd Immunity

– Black: assumes $R_0(\text{today})$ remains in place forever
– Red: assumes $R_0(\text{suppress}) = 1/s(\text{today})$
– Green: we move 25% of the way from $R_0(\text{today})$ back to initial $R_0 = \text{“normal”}$
– Purple: we move 50% of the way from $R_0(\text{today})$ back to initial $R_0 = \text{“normal”}$

NOTE: Lines often cover each other up
Colorado: Re-Opening ($\alpha = .05$)

Colorado

$R_0(t)=0.9, \ R_0(\text{suppress})=1.0, \ R_0(25/50)=1.1/1.4, \ \delta = 0.010, \ \alpha=0.05$
Colorado: Re-Opening ($\alpha = 0$)

Colorado

$R_0(t)=0.8$, $R_0(\text{suppress})=1.0$, $R_0(25/50)=1.1/1.4$, $\delta = 0.010$, $\alpha=0.00$
Results for alternative parameter values
Colorado (7 days): Daily Deaths per Million People (\(\alpha = 0\))

\[ R_0 = 1.6/0.8/0.8 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infect} = 3/3/4 \]

DATA THROUGH 24-AUG-2020

[Graph showing the daily deaths per million people in Colorado from April to January 2020.]
Colorado (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

$R_0 = 1.6 / 0.8 / 0.8$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  $\%$Infect $= 3 / 3 / 4$

DATA THROUGH 24-AUG-2020
Colorado (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Colorado

$R_0=1.6/0.8/0.8 \quad \delta = 0.010 \quad \alpha=0.00 \quad \theta=0.1 \quad \%\text{Infect}=3/3/4$

New York City

Italy
Colorado: Daily Deaths per Million People ($\delta = 0.8\%$)

Colorado

R$_0=1.6/0.9/0.9$  $\delta = 0.008$  $\theta=0.1$  $\gamma=0.2$  %Infect= 4/ 4/ 4
Colorado: Cumulative Deaths per Million ($\delta = 0.8\%$)

Colorado

$R_0=1.6/0.9/0.9$  $\delta = 0.008$  $\theta=0.1$  $\gamma=0.2$  %Infect= 4/4/4
Colorado: Daily Deaths per Million People ($\delta = 1.2\%$)

Colorado

$R_0=1.6/0.8/0.9 \quad \delta = 0.012 \quad \theta=0.1 \quad \gamma=0.2 \quad \%Infect=3/3/3$
Colorado: Cumulative Deaths per Million ($\delta = 1.2\%$)

R$_0$ = 1.6/0.8/0.9  \( \delta = 0.012 \)  \( \theta = 0.1 \)  \( \gamma = 0.2 \)  \%Infect = 3/3/3
Colorado: Daily Deaths per Million People ($\gamma = .2/.15$)

$R_0 = 1.6/0.9/0.9$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$Infect = 3/3/4

DATA THROUGH 24-AUG-2020
Colorado: Cumulative Deaths per Million $\gamma = .2/.15$ (DATA THROUGH 24-AUG-2020)

Colorado

$R_0 = 1.6/0.9/0.9 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%Infect = 3/3/4$

$\gamma = 0.25$
Colorado: Daily Deaths per Million People ($\theta = .1/.07/.2$)

Colorado

$R_0=1.6/0.9/0.9$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%Infect=3/3/4$

DATA THROUGH 24-AUG-2020
Colorado: Cumulative Deaths per Million People $(\theta = 0.1/0.07/0.2)$

Colorado

$R_0 = 1.6/0.9/0.9$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%\text{Infect} = 3/3/4$

DATA THROUGH 24-AUG-2020

Cumulative deaths per million people

Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan 2020
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Colorado: Daily Deaths, Actual and Smoothed

Colorado: Daily deaths, $d$

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Colorado: Change in Smoothed Daily Deaths

Colorado: Delta d

$\delta = 0.010 \quad \theta=0.10 \quad \gamma=0.20$
Colorado: Change in (Change in Smoothed Daily Deaths)

Colorado: Delta (Δd)

\[
\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20
\]