Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Costa Rica
Based on data through September 11, 2020
Outline of Slides

• Basic data from Johns Hopkins CSSE (raw and smoothed)
• Brief summary of the model
• Baseline results \((\delta = 1.0\%, \gamma = 0.2, \theta = 0.1)\)
• Simulation of re-opening – possibilities for raising \(R_0\)
• Results with alternative parameter values:
  o Lower mortality rate, \(\delta = 0.8\%\)
  o Higher mortality rate, \(\delta = 1.2\%\)
  o Infections last longer, \(\gamma = 0.15\)
  o Cases resolve more quickly, \(\theta = 0.2\)
  o Cases resolve more slowly, \(\theta = 0.07\)
• Data underlying estimates of \(R_0(t)\)
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Costa Rica: Daily Deaths per Million People

Costa Rica
Costa Rica: Daily Deaths per Million People (Smoothed)
Brief Summary of Model

• See the paper for a full exposition

• A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Costa Rica: Estimates of $R_0(t)$

Costa Rica

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Costa Rica: Percent Currently Infectious

Costa Rica
Peak I/N = 0.13%  Final I/N = 0.13%  δ = 0.010  θ = 0.10  γ = 0.20
Notes on Interpreting Results
Guide to Graphs

- **Warning:** Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

- 7 days of forecasts: Rainbow color order!
  
  ROY-G-BIV (old to new, low to high)
  
  - Black = current
  - **Red** = oldest, **Orange** = second oldest, **Yellow** = third oldest...
  - **Violet (purple)** = one day earlier

- For robustness graphs, same idea
  
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - **Red** = lowest parameter value (e.g. $\delta = 0.8\%$)
  - **Green** = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:
  1. Alternatively, we fit this equation:

\[
\log R_0(t) = a_0 - \alpha(Daily\ Deaths)
\]

\[\Rightarrow \alpha \approx .05\]

$R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline

- Robustness: Assume $R_0(t) = \text{final empirical value}$. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Costa Rica (7 days): Daily Deaths per Million People ($\alpha = .05$)

Costa Rica

$R_0=1.4/1.2/1.1$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%$Infect= 1/3/13
Costa Rica (7 days): Cumulative Deaths per Million (Future, $\alpha = .05$)

$R_0 = 1.4/1.2/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$ Infect = $1/3/13$

DATA THROUGH 11-SEP-2020
Costa Rica (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0.05$)

Costa Rica

$R_0 = 1.4/1.2/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 1/3/13$
Robustness to Mortality Rate, $\delta$
Costa Rica: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

Costa Rica

$R_0 = 1.4/1.2/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%\text{Infect} = 1/3/13$

DATA THROUGH 11-SEP-2020

Cumulative deaths per million people

Jul 12  Jul 26  Aug 09  Aug 23  Sep 06  2020
Costa Rica: Daily Deaths per Million People \( (\delta = .01/.008/.012) \)

\[
\begin{align*}
R_0 &= 1.4/1.2/1.1 \\
\delta &= 0.010 \\
\alpha &= 0.05 \\
\theta &= 0.1 \\
\%\text{Infect} &= 1/3/13
\end{align*}
\]

DATA THROUGH 11-SEP-2020
Costa Rica: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

Costa Rica

$R_0 = 1.4/1.2/1.1 \hspace{.5cm} \delta = 0.010 \hspace{.5cm} \alpha = 0.05 \hspace{.5cm} \theta = 0.1 \hspace{.5cm} \%\text{Infect} = 1/3/13$

DATA THROUGH 11-SEP-2020

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Costa Rica

Cumulative deaths per million people


0 200 400 600 800 1000 1200 1400

Cumulative deaths per million people

Costa Rica
Reopening and Herd Immunity

– Black: assumes $R_0$(today) remains in place forever

– Red: assumes $R_0$(suppress) = $1/s(today)$

– Green: we move 25% of the way from $R_0$(today) back to initial $R_0$ = “normal”

– Purple: we move 50% of the way from $R_0$(today) back to initial $R_0$ = “normal”

NOTE: Lines often cover each other up
Costa Rica: Re-Opening ($\alpha = .05$)

Costa Rica

$R_0(t)=1.2$, $R_0\text{(suppress)}=1.0$, $R_0(25/50)=1.4/1.6$, $\delta = 0.010$, $\alpha=0.05$

(Light bars = New York City, for comparison)
Costa Rica: Re-Opening ($\alpha = 0$)

Costa Rica

$R_0(t) = 1.3$, $R_0\text{(suppress)} = 1.0$, $R_0(25/50) = 1.4/1.6$, $\delta = 0.010$, $\alpha = 0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Costa Rica (7 days): Daily Deaths per Million People ($\alpha = 0$)

Costa Rica

$R_0=1.4/1.3/1.3$ \hspace{1em} $\delta = 0.010$ \hspace{1em} $\alpha=0.00$ \hspace{1em} $\theta=0.1$ \hspace{1em} $\%\text{Infect}=1/5/39$

DATA THROUGH 11-SEP-2020
Costa Rica (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

Costa Rica

$R_0 = 1.4/1.3/1.3$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  $\%\text{Infect} = 1/5/39$

DATA THROUGH 11-SEP-2020
Costa Rica (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Costa Rica

$R_0 = 1.4/1.3/1.3$  $\delta = 0.010$  $\alpha=0.00$  $\theta=0.1$  $\%$Infect$=1/5/39$
Costa Rica: Daily Deaths per Million People ($\delta = 0.8\%$)

Costa Rica

$R_0=1.4/1.2/1.2$  $\delta = 0.008$  $\theta=0.1$  $\gamma=0.2$  $%\text{Infect}=2/4/15$
Costa Rica: Cumulative Deaths per Million ($\delta = 0.8\%$)

Costa Rica

$R_0 = 1.4/1.2/1.2 \quad \delta = 0.008 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 2/4/15$
Costa Rica: Daily Deaths per Million People ($\delta = 1.2\%$)

Costa Rica

$R_0 = 1.4/1.2/1.1$  $\delta = 0.012$  $\theta = 0.1$  $\gamma = 0.2$  $\%$Infect = 1/3/11
Costa Rica: Cumulative Deaths per Million ($\delta = 1.2\%$)

Costa Rica

$R_0 = 1.4/1.2/1.1$  $\delta = 0.012$  $\theta = 0.1$  $\gamma = 0.2$  $%\text{Infect} = 1/3/11$
Costa Rica: Daily Deaths per Million People ($\gamma = .2/.15$)

Data through 11-Sep-2020

$R_0 = 1.4/1.2/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$Infect = 1/3/13
Costa Rica: Cumulative Deaths per Million $\gamma = .2/1.15$)

Costa Rica

$R_0=1.4/1.2/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 1/3/13$

DATA THROUGH 11-SEP-2020

Cumulative deaths per million people


$\gamma = 0.15$

$\gamma = 0.2$
Costa Rica: Daily Deaths per Million People ($\theta = .1/.07/.2$)

\[ R_0 = 1.4/1.2/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \% \text{Infect} = 1/3/13 \]
Costa Rica: Cumulative Deaths per Million People ($\theta = .1/1.07/1.2$)

Costa Rica

$R_0 = 1.4/1.2/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%Infect = 1/3/13$

DATA THROUGH 11-SEP-2020

Cumulative deaths per million people

Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Costa Rica: Daily Deaths, Actual and Smoothed

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Costa Rica: Change in Smoothed Daily Deaths

Costa Rica: Delta $d$

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Costa Rica: Change in (Change in Smoothed Daily Deaths)

Costa Rica: Delta (Delta d)
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]