Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Detroit
Based on data through September 11, 2020
Outline of Slides

• Basic data from Johns Hopkins CSSE (raw and smoothed)
• Brief summary of the model
• Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
• Simulation of re-opening – possibilities for raising $R_0$
• Results with alternative parameter values:
  - Lower mortality rate, $\delta = 0.8\%$
  - Higher mortality rate, $\delta = 1.2\%$
  - Infections last longer, $\gamma = 0.15$
  - Cases resolve more quickly, $\theta = 0.2$
  - Cases resolve more slowly, $\theta = 0.07$
• Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Detroit: Daily Deaths per Million People

Detroit

Daily deaths per million people
Brief Summary of Model

• See the paper for a full exposition

• A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Detroit: Estimates of $R_0(t)$

Detroit

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Detroit: Percent Currently Infectious

Detroit
Peak I/N = 2.15%  Final I/N = 0.12%  δ = 0.010  θ = 0.10  γ = 0.20

Percent currently infectious, I/N (percent)
Detroit: Growth Rate of Daily Deaths over Past Week (percent)

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

• **Warning:** Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

• **7 days of forecasts:** Rainbow color order!
  
  ROY-G-BIV (old to new, low to high)
  
  ○ Black = current
  ○ Red = oldest, Orange = second oldest, Yellow = third oldest...
  ○ Violet (purple) = one day earlier

• For robustness graphs, same idea
  
  ○ Black = baseline (e.g. $\delta = 1.0\%$)
  ○ Red = lowest parameter value (e.g. $\delta = 0.8\%$)
  ○ Green = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:
  1. Alternatively, we fit this equation:
     \[
     \log R_0(t) = a_0 - \alpha(Daily\ Deaths)
     \]
     \[\Rightarrow \alpha \approx .05\]

     $R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline

- Robustness: Assume $R_0(t) =$ final empirical value. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With \( \alpha = .05 \) (see robustness section for \( \alpha = 0 \))
Detroit (7 days): Daily Deaths per Million People ($\alpha = 0.05$)

Detroit

$R_0=2.3/0.3/0.3$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%$Infect=16/16/16

DATA THROUGH 11 SEP 2020
Detroit (7 days): Cumulative Deaths per Million (Future, $\alpha = 0.05$)

\[
R_0 = 2.3/0.3/0.3 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect}=16/16/16
\]
Detroit (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0.05$)

Detroit

$R_0=2.3/0.3/0.3$, $\delta = 0.010$, $\alpha=0.05$, $\theta=0.1$, $\%$ Infect=16/16/16
Robustness to Mortality Rate, $\delta$
Detroit: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

Detroit

$R_0=2.3/0.3/0.3 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}=16/16/16$

DATA THROUGH 11-SEP-2020
Detroit: Daily Deaths per Million People ($\delta = 0.01/0.008/0.012$)
Detroit: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

Detroit

$R_0 = 2.3/0.3/0.3 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \text{%Infect} = 16/16/16$

DATA THROUGH 11-SEP-2020

Cumulative deaths per million people

Mar  Apr  May  Jun  Jul  Aug  Sep  Oct  Nov  Dec  Jan

2020
Reopening and Herd Immunity

– Black: assumes $R_0(today)$ remains in place forever
– Red: assumes $R_0(suppress) = 1/s(today)$
– Green: we move 25% of the way from $R_0(today)$ back to initial $R_0 = “normal”$
– Purple: we move 50% of the way from $R_0(today)$ back to initial $R_0 = “normal”$

NOTE: Lines often cover each other up
Detroit: Re-Opening ($\alpha = .05$)

Detroit

$R_0(t)=0.3$, $R_0\text{ (suppress)}=1.2$, $R_0(25/50)=0.8/1.3$, $\delta = 0.010$, $\alpha=0.05$

(Light bars = New York City, for comparison)
Detroit: Re-Opening ($\alpha = 0$)

Detroit

$R_0(t)=0.2, \ R_0(\text{suppress})=1.2, \ R_0(25/50)=0.7/1.2, \ \delta = 0.010, \ \alpha=0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Detroit (7 days): Daily Deaths per Million People ($\alpha = 0$)

Detroit

$R_0 = 2.3/0.2/0.2$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  %Infect = 16/16/16

DATA THROUGH 11 SEP 2020
Detroit (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

**Detroit**

$R_0=2.3/0.2/0.2$  $\delta = 0.010$  $\alpha=0.00$  $\theta=0.1$  %Infect=16/16/16

DATA THROUGH 11-SEP-2020
Detroit (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Detroit

$R_0=2.3/0.2/0.2 \; \delta = 0.010 \; \alpha=0.00 \; \theta=0.1 \; \%\text{Infect}=16/16/16$
Detroit: Daily Deaths per Million People ($\delta = 0.8\%$)

Detroit

$R_0 = 2.3/0.3/0.3 \quad \delta = 0.008 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 19/19/19$

SOME ERRORS IN ESTIMATION...
Detroit: Cumulative Deaths per Million (\( \delta = 0.8\% \))

\[ R_0 = 2.3/0.3/0.3 \quad \delta = 0.008 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \% \text{Infect} = 19/19/19 \]

SOME ERRORS IN ESTIMATION...
Detroit: Daily Deaths per Million People ($\delta = 1.2\%$)

Detroit

$R_0 = 2.3/0.3/0.3$  $\delta = 0.012$  $\theta = 0.1$  $\gamma = 0.2$  $\%$Infect$=13/13/13$

SOME ERRORS IN ESTIMATION...
Detroit: Cumulative Deaths per Million ($\delta = 1.2\%$)

Detroit

$R_0 = 2.3/0.3/0.3$  $\delta = 0.012$  $\theta = 0.1$  $\gamma = 0.2$  $\%Infect = 13/13/13$

SOME ERRORS IN ESTIMATION...
Detroit: Daily Deaths per Million People ($\gamma = .2/.15$)

Detroit

$R_0=2.3/0.3/0.3$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%$Infect=16/16/16

DATA THROUGH 11 SEP 2020
Detroit: Cumulative Deaths per Million $\gamma = .2/.15$

Detroit

$R_0=2.3/0.3/0.3$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect=16/16/16

DATA THROUGH 11-SEP-2020

Cumulative deaths per million people

Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan 2020
Detroit: Daily Deaths per Million People ($\theta = 0.1/0.07/0.2$)

Detroit

$R_0 = 2.3/0.3/0.3$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$Infect = 16/16/16

DATA THROUGH 11 SEP 2020
Detroit: Cumulative Deaths per Million People ($\theta = .1/.07/.2$)

Detroit

$R_0=2.3/0.3/0.3$   $\delta = 0.010$   $\alpha=0.05$   $\theta=0.1$   %Infect=16/16/16

DATA THROUGH 11-SEP-2020
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Detroit: Daily Deaths, Actual and Smoothed

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Detroit: Change in Smoothed Daily Deaths

\[
\text{Detroit: Delta } d \\
\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20
\]
Detroit: Change in (Change in Smoothed Daily Deaths)

Detroit: Delta (Delta d)
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]