Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Detroit
Based on data through October 9, 2020
Outline of Slides

• Basic data from Johns Hopkins CSSE (raw and smoothed)
• Brief summary of the model
• Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
• Simulation of re-opening – possibilities for raising $R_0$
• Results with alternative parameter values:
  ◦ Lower mortality rate, $\delta = 0.8\%$
  ◦ Higher mortality rate, $\delta = 1.2\%$
  ◦ Infections last longer, $\gamma = 0.15$
  ◦ Cases resolve more quickly, $\theta = 0.2$
  ◦ Cases resolve more slowly, $\theta = 0.07$
• Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Detroit: Daily Deaths per Million People

Detroit
Detroit: Daily Deaths per Million People (Smoothed)
**Brief Summary of Model**

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t / \gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

- Inferred from daily deaths, and
- the change in daily deaths, and
- the change in (the change in daily deaths)
(see end of slide deck for this data)
Detroit: Estimates of $R_0(t)$

Detroit

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Detroit: Percent Currently Infectious

Detroit
Peak I/N = 2.15%  Final I/N = 0.12%  δ = 0.010  θ = 0.10  γ = 0.20
Detroit: Growth Rate of Daily Deaths over Past Week (percent)

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

• Warning: Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

• 7 days of forecasts: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  ◦ Black = current
  ◦ Red = oldest, Orange = second oldest, Yellow = third oldest...
  ◦ Violet (purple) = one day earlier

• For robustness graphs, same idea
  ◦ Black = baseline (e.g. $\delta = 1.0\%$)
  ◦ Red = lowest parameter value (e.g. $\delta = 0.8\%$)
  ◦ Green = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:
  1. Alternatively, we fit this equation:

    \[
    \log R_0(t) = a_0 - \alpha(Daily Deaths)
    \]

    $\Rightarrow \alpha \approx .05$

    $R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline

- Robustness: Assume $R_0(t) = \text{final empirical value}$. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Detroit (7 days): Daily Deaths per Million People ($\alpha = .05$)

Detroit

$R_0=2.3/0.3/0.3$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect=16/16/16

DATA THROUGH 09-OCT-2020
Detroit (7 days): Cumulative Deaths per Million (Future, $\alpha = .05$)

Detroit

$R_0 = 2.3/0.3/0.3$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect=16/16/16
Detroit (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = .05$)

Detroit

$R_0 = 2.3/0.3/0.3 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect}=16/16/16$

Cumulative deaths per million people

New York City

Italy

Detroit
Robustness to Mortality Rate, $\delta$
Detroit: Cumulative Deaths per Million ($\delta = 0.01/0.008/0.012$)

Detroit

$R_0=2.3/0.3/0.3 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}=16/16/16$

DATA THROUGH 09-OCT-2020
Detroit: Daily Deaths per Million People ($\delta = .01 / .008 / .012$)

Detroit

$R_0 = 2.3 / 0.3 / 0.3 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad %\text{Infect} = 16 / 16 / 16$

DATA THROUGH 09-OCT-2020
Detroit: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

Detroit
$R_0 = 2.3/0.3/0.3 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \% \text{Infect} = 16/16/16$

DATA THROUGH 09-OCT-2020

Cumulative deaths per million people


\(\delta = 0.008\)
Reopening and Herd Immunity

- Black: assumes $R_0(\text{today})$ remains in place forever
- Red: assumes $R_0(\text{suppress}) = 1/s(\text{today})$
- Green: we move 25% of the way from $R_0(\text{today})$ back to initial $R_0 =$ “normal”
- Purple: we move 50% of the way from $R_0(\text{today})$ back to initial $R_0 =$ “normal”

NOTE: Lines often cover each other up
Detroit: Re-Opening ($\alpha = .05$)

Detroit

$R_0(t)=0.3$, $R_0(\text{suppress})=1.2$, $R_0(25/50)=0.8/1.3$, $\delta = 0.010$, $\alpha=0.05$

(Light bars = New York City, for comparison)
Detroit: Re-Opening ($\alpha = 0$)

Detroit

$R_0(t)=0.2$, $R_0(\text{suppress})=1.2$, $R_0(25/50)=0.7/1.2$, $\delta = 0.010$, $\alpha=0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Detroit (7 days): Daily Deaths per Million People ($\alpha = 0$)

Detroit

$R_0 = 2.3/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infect} = 16/16/16$

DATA THROUGH 09-OCT-2020
Detroit (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

Detroit

$R_0 = 2.3/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infect}=16/16/16$

DATA THROUGH 09-OCT-2020
Detroit (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Detroit
$R_0 = 2.3/0.2/0.2$ \hspace{1cm} $\delta = 0.010$ \hspace{1cm} $\alpha = 0.00$ \hspace{1cm} $\theta = 0.1$ \hspace{1cm} $\%\text{Infect} = 16/16/16$
Detroit: Daily Deaths per Million People ($\delta = 0.8\%$)

Detroit

$R_0 = 2.3/0.3/0.3 \quad \delta = 0.008 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect}=19/19/19$

SOME ERRORS IN ESTIMATION...
Detroit: Cumulative Deaths per Million ($\delta = 0.8\%$)

Detroit

$R_0 = 2.3/0.3/0.3 \quad \delta = 0.008 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 19/19/19$

SOME ERRORS IN ESTIMATION...
Detroit: Daily Deaths per Million People ($\delta = 1.2\%$)

Detroit

$R_0=2.3/0.3/0.3$  \(\delta = 0.012\)  \(\theta=0.1\)  \(\gamma=0.2\)  \%Infect=13/13/13

SOME ERRORS IN ESTIMATION...
Detroit: Cumulative Deaths per Million ($\delta = 1.2\%$)

Detroit

$R_0 = 2.3/0.3/0.3$  \( \delta = 0.012 \)  \( \theta = 0.1 \)  \( \gamma = 0.2 \)  \%Infect = 13/13/13

SOME ERRORS IN ESTIMATION...
Detroit: Daily Deaths per Million People ($\gamma = .2/.15$)

Detroit

$R_0 = 2.3/0.3/0.3$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$Infect = 16/16/16

DATA THROUGH 09-OCT-2020
Detroit: Cumulative Deaths per Million $\gamma = 0.2/0.15$)

Detroit

$R_0 = 2.3/0.3/0.3 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect}=16/16/16$

DATA THROUGH 09-OCT-2020

Cumulative deaths per million people

Detroit: Daily Deaths per Million People ($\theta = 0.1/0.07/0.2$)

Detroit

$R_0=2.3/0.3/0.3$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%\text{Infect}=16/16/16$

DATA THROUGH 09-OCT-2020
Detroit: Cumulative Deaths per Million People ($\theta = .1/.07/.2$)

$R_0 = 2.3/0.3/0.3$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infected = 16/16/16

DATA THROUGH 09-OCT-2020
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Detroit: Daily Deaths, Actual and Smoothed

Detroit: Daily deaths, $d$

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Detroit: Change in Smoothed Daily Deaths

Detroit: Delta d
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Detroit: Change in (Change in Smoothed Daily Deaths)

Detroit: Delta (Delta d)
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]