Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Germany
Based on data through August 24, 2020
Outline of Slides

- Basic data from Johns Hopkins CSSE (raw and smoothed)
- Brief summary of the model
- Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
- Simulation of re-opening – possibilities for raising $R_0$
- Results with alternative parameter values:
  - Lower mortality rate, $\delta = 0.8\%$
  - Higher mortality rate, $\delta = 1.2\%$
  - Infections last longer, $\gamma = 0.15$
  - Cases resolve more quickly, $\theta = 0.2$
  - Cases resolve more slowly, $\theta = 0.07$
- Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Germany: Daily Deaths per Million People

-1 0 1 2 3 4 5 6 7

Daily deaths per million people

2020
Brief Summary of Model

• See the paper for a full exposition

• A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Germany: Estimates of $R_0(t)$

Germany

$\delta = 0.010 \quad \theta=0.10 \quad \gamma=0.20$
Germany: Percent Currently Infectious

Peak I/N = 0.15%   Final I/N = 0.00%   δ = 0.010   θ=0.10   γ=0.20
Germany: Growth Rate of Daily Deaths over Past Week (percent)

Germany
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

- **Warning**: Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

- 7 days of forecasts: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  - Black = current
  - Red = oldest, Orange = second oldest, Yellow = third oldest...
  - Violet (purple) = one day earlier

- For robustness graphs, same idea
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - Red = lowest parameter value (e.g. $\delta = 0.8\%$)
  - Green = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:
  
  1. Alternatively, we fit this equation:

$$\log R_0(t) = a_0 - \alpha \text{(Daily Deaths)}$$

$$\Rightarrow \alpha \approx .05$$

$R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline.

- Robustness: Assume $R_0(t) =$ final empirical value. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

- After peak, forecasts settle down.
- Before that, very noisy!
- If the region has not peaked, do not trust
- With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Germany (7 days): Daily Deaths per Million People \((\alpha = .05)\)

Germany

\[ R_0 = 1.5/1.0/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 1/1/1 \]

DATA THROUGH 24-AUG-2020
Germany (7 days): Cumulative Deaths per Million (Future, $\alpha = 0.05$)

$R_0 = 1.5 / 1.0 / 1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 1 / 1 / 1$

DATA THROUGH 24-AUG-2020
Germany (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = .05$)

Germany
$R_0 = 1.5/1.0/1.0$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $%Infect = 1/1/1$

New York City

Italy
Robustness to Mortality Rate, $\delta$
Germany: Cumulative Deaths per Million \((\delta = 0.01/0.008/0.012)\)

\[
\text{R}_0 = 1.5/1.0/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 1/1/1
\]

DATA THROUGH 24-AUG-2020
Germany: Daily Deaths per Million People ($\delta = .01/.008/.012$)

\[ R_0 = 1.5/1.0/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 1/1/1 \]

Data through 24-Aug-2020
Germany: Cumulative Deaths per Million ($\delta = .01/ .008/ .012$)

DATA THROUGH 24-AUG-2020

Germany

$R_0 = 1.5/1.0/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \% \text{Infect} = 1/1/1 \quad \delta = 0.008$
Reopening and Herd Immunity

– **Black**: assumes $R_0(\text{today})$ remains in place forever
– **Red**: assumes $R_0(\text{suppress}) = 1/s(\text{today})$
– **Green**: we move 25% of the way from $R_0(\text{today})$ back to initial $R_0 = \text{“normal”}$
– **Purple**: we move 50% of the way from $R_0(\text{today})$ back to initial $R_0 = \text{“normal”}$

**NOTE**: Lines often cover each other up
Germany: Re-Opening ($\alpha = 0.05$)

Germany
$R_0(t)=1.0$, $R_0\text{ (suppress)}=1.0$, $R_0\text{ (25/50)}=1.3/1.5$, $\delta = 0.010$, $\alpha=0.05$
Germany: Re-Opening ($\alpha = 0$)

Germany

$R_0(t) = 1.0$, $R_0(\text{suppress}) = 1.0$, $R_0(25/50) = 1.3/1.5$, $\delta = 0.010$, $\alpha = 0.00$
Results for alternative parameter values
Germany (7 days): Daily Deaths per Million People ($\alpha = 0$)

DATA THROUGH 24-AUG-2020

Germany

$R_0=1.5/1.0/1.0 \quad \delta = 0.010 \quad \alpha=0.00 \quad \theta=0.1 \quad \%\text{Infect}=1/1/1$

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Germany (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

\[ R_0 = 1.5/1.0/1.0 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \% \text{ Infect} = 1 \quad 1 \quad 1 \]

DATA THROUGH 24-AUG-2020

Cumulative deaths per million people

Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan 2020
Germany (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Germany

$R_0 = 1.5/1.0/1.0 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infect} = 1/1/1$

Cumulative deaths per million people

New York City

Italy
Germany: Daily Deaths per Million People ($\delta = 0.8\%$)

$R_0 = 1.5/1.0/1.0$  $\delta = 0.008$  $\theta = 0.1$  $\gamma = 0.2$  $\%$Infect$ = 1/1/1$
Germany: Cumulative Deaths per Million ($\delta = 0.8\%$)

R₀ = 1.5/1.0/1.0  $\delta = 0.008$  $\theta = 0.1$  $\gamma = 0.2$  %Infect = 1/1/1
Germany: Daily Deaths per Million People ($\delta = 1.2\%$)

Germany

$R_0=1.5/1.0/1.0$  \[\delta = 0.012 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 1/1/1\]
Germany: Cumulative Deaths per Million ($\delta = 1.2\%$)

R₀ = 1.5/1.0/1.0  $\delta = 0.012$  $\theta = 0.1$  $\gamma = 0.2$  %Infect = 1/1/1

Cumulative deaths per million people

Mar  Apr  May  Jun  Jul  Aug  Sep  Oct  Nov  Dec  2020  Jan
Germany: Daily Deaths per Million People ($\gamma = 0.2/0.15$)

\[
R_0 = 1.5/1.0/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 1/1/1
\]

DATA THROUGH 24-AUG-2020
Germany: Cumulative Deaths per Million $\gamma = 0.2/0.15$
Germany: Daily Deaths per Million People ($\theta = .1/.07/.2$)

\[ R_0 = 1.5/1.0/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 1/1/1 \]

Data through 24-Aug-2020
Germany: Cumulative Deaths per Million People ($\theta = .1/.07/.2$)

\[ R_0 = 1.5/1.0/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 1/1/1 \]

DATA THROUGH 24-AUG-2020
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Germany: Daily Deaths, Actual and Smoothed

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Germany: Change in Smoothed Daily Deaths

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Germany: Change in (Change in Smoothed Daily Deaths)

German: Delta (Delta d)
\[ \delta = 0.010 \quad \theta=0.10 \quad \gamma=0.20 \]