Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Denmark
Based on data through October 9, 2020
Outline of Slides

- Basic data from Johns Hopkins CSSE (raw and smoothed)
- Brief summary of the model
- Baseline results \( (\delta = 1.0\%, \gamma = 0.2, \theta = 0.1) \)
- Simulation of re-opening – possibilities for raising \( R_0 \)
- Results with alternative parameter values:
  - Lower mortality rate, \( \delta = 0.8\% \)
  - Higher mortality rate, \( \delta = 1.2\% \)
  - Infections last longer, \( \gamma = 0.15 \)
  - Cases resolve more quickly, \( \theta = 0.2 \)
  - Cases resolve more slowly, \( \theta = 0.07 \)
- Data underlying estimates of \( R_0(t) \)
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Denmark: Daily Deaths per Million People

![Graph showing daily deaths per million people in Denmark from April to October 2020. The graph indicates fluctuations in daily deaths throughout the months, with peaks in April and May, and generally lower numbers in subsequent months.]

Daily deaths per million people
Brief Summary of Model

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Denmark: Estimates of $R_0(t)$

Denmark

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$

![Graph showing estimates of $R_0(t)$ for Denmark with months from March to November 2020 and corresponding $R_0(t)$ values].
Denmark: Percent Currently Infectious

Peak I/N = 0.15%  Final I/N = 0.02%  δ = 0.010  θ=0.10  γ=0.20
Denmark: Growth Rate of Daily Deaths over Past Week (percent)

Denmark
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

- **Warning**: Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

- **7 days of forecasts**: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  - Black = current
  - Red = oldest, Orange = second oldest, Yellow = third oldest...
  - Violet (purple) = one day earlier

- For robustness graphs, same idea
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - Red = lowest parameter value (e.g. $\delta = 0.8\%$)
  - Green = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it.
- For future, two approaches:
  - Alternatively, we fit this equation:
    \[ \log R_0(t) = a_0 - \alpha(Daily\ Deaths) \]
    \[ \Rightarrow \alpha \approx 0.05 \]
    - $R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline.

- Robustness: Assume $R_0(t) = \text{final empirical value}$. Constant in future, so no $\alpha$ adjustment $\Rightarrow \alpha = 0$.
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Denmark (7 days): Daily Deaths per Million People ($\alpha = .05$)

DATA THROUGH 09-OCT-2020

Denmark

$R_0 = 1.5/1.3/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 1/2/4$
Denmark (7 days): Cumulative Deaths per Million (Future, $\alpha = .05$)

Denmark

$R_0= 1.5/1.3/1.0 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}= 1/2/4$

DATA THROUGH 09-OCT-2020
Denmark (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = .05$)

Denmark

$R_0 = 1.5/1.3/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 1/2/4$

New York City

Italy
Robustness to Mortality Rate, $\delta$
Denmark: Cumulative Deaths per Million ($\delta = 0.01/0.008/0.012$)

DATA THROUGH 09-OCT-2020

Denmark

$R_0=1.5/1.3/1.0$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect= 1/ 2/ 4
Denmark: Daily Deaths per Million People ($\delta = 0.01/0.008/0.012$)

Denmark

$R_0 = 1.5/1.3/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \% \text{Infect} = 1/2/4$

DATA THROUGH 09-OCT-2020
Denmark: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

Denmark

$R_0 = 1.5/1.3/1.0 \ \delta = 0.010 \ \alpha = 0.05 \ \theta = 0.1 \ %Infect = 1/2/4$

DATA THROUGH 09-OCT-2020
Reopening and Herd Immunity

– Black: assumes $R_0(todays)$ remains in place forever
– Red: assumes $R_0(suppress) = 1/s(today)$
– Green: we move 25% of the way from $R_0(today)$
  back to initial $R_0 = “normal”$
– Purple: we move 50% of the way from $R_0(today)$
  back to initial $R_0 = “normal”$

NOTE: Lines often cover each other up
Denmark: Re-Opening ($\alpha = .05$)

Denmark

$R_0(t)=1.3$, $R_0($suppress$)=1.0$, $R_0(25/50)=1.4/1.6$, $\delta = 0.010$, $\alpha=0.05$

(Light bars = New York City, for comparison)
Denmark: Re-Opening ($\alpha = 0$)

Denmark

$R_0(t)=1.3$, $R_0(\text{suppress})=1.0$, $R_0(25/50)=1.4/1.6$, $\delta = 0.010$, $\alpha=0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Denmark (7 days): Daily Deaths per Million People ($\alpha = 0$)

Denmark

$R_0=1.5/1.3/1.3$  $\delta = 0.010$  $\alpha=0.00$  $\theta=0.1$  %Infect= 1/ 2/12

DATA THROUGH 09-OCT-2020
Denmark (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

Denmark

$R_0 = 1.5/1.3/1.3 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infect} = 1/2/12$

DATA THROUGH 09-OCT-2020
Denmark (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Denmark

$R_0 = 1.5/1.3/1.3 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infect} = 1/2/12$

New York City

Italy
Denmark: Daily Deaths per Million People ($\delta = 0.8\%$)

Denmark

$R_0=1.5/1.3/1.0 \quad \delta = 0.008 \quad \theta=0.1 \quad \gamma=0.2 \quad \%\text{Infect}=1/2/5$
Denmark: Cumulative Deaths per Million ($\delta = 0.8\%$)

Denmark

$R_0 = 1.5/1.3/1.0$  $\delta = 0.008$  $\theta = 0.1$  $\gamma = 0.2$  $\%\text{Infect} = 1/2/5$
Denmark: Daily Deaths per Million People ($\delta = 1.2\%$)

$R_0 = 1.5/1.2/1.0 \quad \delta = 0.012 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 1/1/4$
Denmark: Cumulative Deaths per Million ($\delta = 1.2\%$)

Denmark

$R_0=1.5/1.2/1.0$  $\delta = 0.012$  $\theta=0.1$  $\gamma=0.2$  $\%\text{Infect}=1/1/4$
Denmark: Daily Deaths per Million People \((\gamma = .2/.15)\)

Denmark

\(R_0 = 1.5/1.3/1.0\)
\(\delta = 0.010\)
\(\alpha = 0.05\)
\(\theta = 0.1\)
\(\%\text{Infect} = 1/2/4\)

Data through 09-Oct-2020
Denmark: Cumulative Deaths per Million $\gamma = .2/.15$)

Denmark

$R_0=1.5/1.3/1.0 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}=1/2/4$

DATA THROUGH 09-OCT-2020
Denmark: Daily Deaths per Million People ($\theta = 0.1/0.07/0.2$)

Denmark

$R_0 = 1.5/1.3/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 1/2/4$

DATA THROUGH 09-OCT-2020
Denmark: Cumulative Deaths per Million People ($\theta = .1/.07/.2$)

Denmark

$R_0=1.5/1.3/1.0$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 1/2/4

DATA THROUGH 09-OCT-2020
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Denmark: Daily Deaths, Actual and Smoothed

Denmark: Daily deaths, \( d \)
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Denmark: Change in Smoothed Daily Deaths

Denmark: Delta d
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Denmark: Change in (Change in Smoothed Daily Deaths)

\[
\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20
\]