Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Spain
Based on data through September 11, 2020
Outline of Slides

• Basic data from Johns Hopkins CSSE (raw and smoothed)

• Brief summary of the model

• Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)

• Simulation of re-opening – possibilities for raising $R_0$

• Results with alternative parameter values:
  
  o Lower mortality rate, $\delta = 0.8\%$
  o Higher mortality rate, $\delta = 1.2\%$
  o Infections last longer, $\gamma = 0.15$
  o Cases resolve more quickly, $\theta = 0.2$
  o Cases resolve more slowly, $\theta = 0.07$

• Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Spain: Daily Deaths per Million People (Smoothed)
Brief Summary of Model

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Spain: Estimates of $R_0(t)$

Spain

$\delta = 0.010 \quad \theta=0.10 \quad \gamma=0.20$
Spain: Percent Currently Infectious

Peak I/N = 1.02%   Final I/N = 0.03%   $\delta=0.010$   $\theta=0.10$   $\gamma=0.20$
Spain: Growth Rate of Daily Deaths over Past Week (percent)

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

- **Warning:** Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

- 7 days of forecasts: Rainbow color order!
  - ROY-G-BIV (old to new, low to high)
    - Black = current
    - Red = oldest, Orange = second oldest, Yellow = third oldest...
    - Violet (purple) = one day earlier

- For robustness graphs, same idea
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - Red = lowest parameter value (e.g. $\delta = 0.8\%$)
  - Green = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:
  1. Alternatively, we fit this equation:

     $$\log R_0(t) = a_0 - \alpha(Daily\ Deaths)$$

     \[\Rightarrow \alpha \approx .05\]

     $R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline

- Robustness: Assume $R_0(t) = \text{final empirical value. Constant in future, so no } \alpha \text{ adjustment } \rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Spain (7 days): Daily Deaths per Million People ($\alpha = .05$)

Spain

$R_0=2.1/0.2/0.2 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}= 6/6/6$

DATA THROUGH 11-SEP-2020
Spain (7 days): Cumulative Deaths per Million (Future, $\alpha = .05$)

$R_0 = 2.1/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 6/6/6$

DATA THROUGH 11-SEP-2020
Spain (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = .05$)

$$R_0 = 2.1/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 6/6/6$$

Cumulative deaths per million people

Spain

New York City

Italy

Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan 2020
Robustness to Mortality Rate, $\delta$
Spain: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

$R_0 = 2.1/0.2/0.2$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$Infect = 6/6/6

DATA THROUGH 11-SEP-2020
Spain: Daily Deaths per Million People ($\delta = .01/.008/.012$)

$R_0=2.1/0.2/0.2$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%\text{Infect}= 6/6/6$

DATA THROUGH 11-SEP-2020

Spain
Spain: Cumulative Deaths per Million ($\delta = 0.01 / 0.008 / 0.012$)

$R_0 = 2.1 / 0.2 / 0.2$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  \%Infect = 6 / 6 / 6

DATA THROUGH 11-SEP-2020
Reopening and Herd Immunity

– Black: assumes $R_0(today)$ remains in place forever
– Red: assumes $R_0(suppress) = 1/s(today)$
– Green: we move 25% of the way from $R_0(today)$ back to initial $R_0 = “normal”$
– Purple: we move 50% of the way from $R_0(today)$ back to initial $R_0 = “normal”$

NOTE: Lines often cover each other up
Spain: Re-Opening ($\alpha = .05$)

Spain

$R_0(t)=0.2$, $R_0$ (suppress)$=1.1$, $R_0(25/50)=0.7/1.2$, $\delta = 0.010$, $\alpha=0.05$

(Light bars = New York City, for comparison)
Spain: Re-Opening ($\alpha = 0$)

Spain

$R_0(t)=0.2$, $R_0$ (suppress$)=1.1$, $R_0(25/50)=0.7/1.1$, $\delta = 0.010$, $\alpha=0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Spain (7 days): Daily Deaths per Million People ($\alpha = 0$)

Spain

$R_0 = 2.1/0.2/0.2$  \( \delta = 0.010 \)  \( \alpha = 0.00 \)  \( \theta = 0.1 \)  \%Infect = 6/6/6

DATA THROUGH 11-SEP-2020
Spain (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

$R_0 = 2.1/0.2/0.2$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  $\%\text{Infect} = 6/6/6$

DATA THROUGH 11-SEP-2020
Spain (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Spain

$R_0 = 2.1/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infect} = 6/6/6$

New York City

Italy
Spain: Daily Deaths per Million People ($\delta = 0.8\%$)

Spain

$R_0=2.1/0.2/0.2$  $\delta = 0.008$  $\theta=0.1$  $\gamma=0.2$  \%Infect$= 8/8/8$

SOME ERRORS IN ESTIMATION...
Spain: Cumulative Deaths per Million ($\delta = 0.8\%$)

Spain

$R_0 = 2.1/0.2/0.2$  $\delta = 0.008$  $\theta = 0.1$  $\gamma = 0.2$  $%Infect = 8/8/8$

SOME ERRORS IN ESTIMATION...
Spain: Daily Deaths per Million People ($\delta = 1.2\%$)

Spain

$R_0 = 2.1/0.2/0.2 \quad \delta = 0.012 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 5/5/5$

SOME ERRORS IN ESTIMATION...
Spain: Cumulative Deaths per Million ($\delta = 1.2\%$)

Spain

$R_0 = 2.1/0.2/0.2 \quad \delta = 0.012 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \% \text{Infect} = 5/5/5$

SOME ERRORS IN ESTIMATION...
Spain: Daily Deaths per Million People ($\gamma = .2/.15$)

Spain

$R_0 = 2.1/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 6/6/6$

DATA THROUGH 11-SEP-2020
Spain: Cumulative Deaths per Million $\gamma = .2/.15$)

$R_0 = 2.1/0.2/0.2$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect= 6/6/6  

$\gamma = 0.25$

DATA THROUGH 11-SEP-2020
Spain: Daily Deaths per Million People ($\theta = .1 / .07 / .2$)

$R_0 = 2.1 / 0.2 / 0.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \% \text{ Infect} = 6 / 6 / 6$

DATA THROUGH 11-SEP-2020
Spain: Cumulative Deaths per Million People ($\theta = 0.1/0.07/0.2$)

$r_0 = 2.1/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \% \text{Infect} = 6/6/6$

DATA THROUGH 11-SEP-2020

Cumulative deaths per million people

$\theta \equiv 0.97$
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Spain: Daily Deaths, Actual and Smoothed

Spain: Daily deaths, $d$

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Spain: Change in Smoothed Daily Deaths

Spain: Delta d
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Spain: Change in (Change in Smoothed Daily Deaths)

Spain: Delta (Delta d)

\[
\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20
\]