Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Indiana Based on data through August 24, 2020
Outline of Slides

• Basic data from Johns Hopkins CSSE (raw and smoothed)
• Brief summary of the model
• Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
• Simulation of re-opening – possibilities for raising $R_0$
• Results with alternative parameter values:
  o Lower mortality rate, $\delta = 0.8\%$
  o Higher mortality rate, $\delta = 1.2\%$
  o Infections last longer, $\gamma = 0.15$
  o Cases resolve more quickly, $\theta = 0.2$
  o Cases resolve more slowly, $\theta = 0.07$
• Data underlying estimates of $R_0(t)$
Underlying data from
Johns Hopkins CSSE

– Raw data
– Smoothed = 7 day centered moving average
– No “excess deaths” correction (change as of Aug 6 run)
Indiana: Daily Deaths per Million People

![Graph showing daily deaths per million people in Indiana from April to August 2020. The graph indicates fluctuations in daily deaths with a peak in May.]
Indiana: Daily Deaths per Million People (Smoothed)

```
<table>
<thead>
<tr>
<th>Month</th>
<th>Daily Deaths per Million People</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apr</td>
<td>4</td>
</tr>
<tr>
<td>May</td>
<td>12</td>
</tr>
<tr>
<td>Jun</td>
<td>8</td>
</tr>
<tr>
<td>Jul</td>
<td>6</td>
</tr>
<tr>
<td>Aug</td>
<td>2</td>
</tr>
</tbody>
</table>
```
Brief Summary of Model

- See the paper for a full exposition

- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Indiana: Estimates of $R_0(t)$

Indiana

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$

Graph showing the trend of $R_0(t)$ from March to September 2020.
Indiana: Percent Currently Infectious

Peak I/N = 0.37%  Final I/N = 0.10%  δ = 0.010  θ = 0.10  γ = 0.20
Indiana: Growth Rate of Daily Deaths over Past Week (percent)

Indiana

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

• **Warning**: Results are often very uncertain; this can be seen by comparing across multiple graphs. See the [original paper](#).

• 7 days of forecasts: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  - Black = current
  - **Red** = oldest, **Orange** = second oldest, **Yellow** = third oldest...
  - **Violet** (purple) = one day earlier

• For robustness graphs, same idea
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - **Red** = lowest parameter value (e.g. $\delta = 0.8\%$)
  - **Green** = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:
  1. Alternatively, we fit this equation:

$$\log R_0(t) = a_0 - \alpha(Daily Deaths)$$

$$\Rightarrow \alpha \approx 0.05$$

$R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline

- Robustness: Assume $R_0(t) = \text{final empirical value}$. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Indiana (7 days): Daily Deaths per Million People ($\alpha = .05$)

Indiana

$R_0 = 1.7/1.0/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \% \text{Infect} = 5/5/7$

DATA THROUGH 24-AUG-2020
Indiana (7 days): Cumulative Deaths per Million (Future, $\alpha = .05$)

Indiana

$R_0 = 1.7/1.0/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 5/5/7

DATA THROUGH 24-AUG-2020
Indiana (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0.05$)

$R_0 = 1.7 / 1.0 / 1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%\text{Infect} = 5 / 5 / 7$
Robustness to Mortality Rate, $\delta$
Indiana: Cumulative Deaths per Million ($\delta = .01/0.008/0.012$)

Indiana

$R_0=1.7/1.0/1.1 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}=5/5/7$

DATA THROUGH 24-AUG-2020
Indiana: Daily Deaths per Million People ($\delta = .01 / .008 / .012$)

Indiana

$R_0 = 1.7 / 1.0 / 1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 5 / 5 / 7

DATA THROUGH 24-AUG-2020
Indiana: Cumulative Deaths per Million ($\delta = 0.01/0.008/0.012$)

Indiana

$R_0 = 1.7/1.0/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 5/5/7$

DATA THROUGH 24-AUG-2020
Reopening and Herd Immunity

– Black: assumes $R_0(today)$ remains in place forever
– Red: assumes $R_0(suppress) = 1/s(today)$
– Green: we move 25% of the way from $R_0(today)$ back to initial $R_0 = “normal”$
– Purple: we move 50% of the way from $R_0(today)$ back to initial $R_0 = “normal”$

NOTE: Lines often cover each other up
Indiana: Re-Opening ($\alpha = 0.05$)

Indiana

$R_0(t)=1.0$, $R_0(\text{suppress})=1.1$, $R_0(25/50)=1.3/1.5$, $\delta = 0.010$, $\alpha=0.05$
Indiana: Re-Opening ($\alpha = 0$)

Indiana

$R_0(t)=1.1, \ R_0(\text{suppress})=1.1, \ R_0(25/50)=1.3/1.5, \ \delta = 0.010, \ \alpha=0.00$
Results for alternative parameter values
Indiana (7 days): Daily Deaths per Million People ($\alpha = 0$)

Indiana

$R_0 = 1.7/1.1/1.1$, $\delta = 0.010$, $\alpha = 0.00$, $\theta = 0.1$, %Infect = 5/5/7

DATA THROUGH 24-AUG-2020
Indiana (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

Indiana

$R_0 = 1.7/1.1/1.1 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infect} = 5/5/7$

DATA THROUGH 24-AUG-2020

Cumulative deaths per million people
Indiana (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Indiana

$R_0 = 1.7/1.1/1.1$  \(\delta = 0.010\)  \(\alpha = 0.00\)  \(\theta = 0.1\)  %Infect= 5/5/7

New York City

Italy
Indiana: Daily Deaths per Million People ($\delta = 0.8\%$)

Indiana

$R_0 = 1.7/1.1/1.1$  $\delta = 0.008$  $\theta = 0.1$  $\gamma = 0.2$  $\%$Infect = 6/7/8
Indiana: Cumulative Deaths per Million ($\delta = 0.8\%$)

$$\text{Indiana}
\begin{align*}
R_0 &= 1.7/1.1/1.1 \\
\delta &= 0.008 \\
\theta &= 0.1 \\
\gamma &= 0.2 \\
\%\text{Infect} &= 6/7/8
\end{align*}$$
Indiana: Daily Deaths per Million People ($\delta = 1.2\%$)

R$_0$ = 1.7/1.0/1.0  $\delta = 0.012$  $\theta = 0.1$  $\gamma = 0.2$  %Infect = 4/5/6
Indiana: Cumulative Deaths per Million ($\delta = 1.2\%$)

Indiana

$R_0 = 1.7/1.0/1.0 \quad \delta = 0.012 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 4/5/6$
Indiana: Daily Deaths per Million People ($\gamma = .2/.15$)

Indiana

$R_0 = 1.7/1.0/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 5/5/7

DATA THROUGH 24-AUG-2020
Indiana: Cumulative Deaths per Million $\gamma = .2/.15$)

Indiana

$R_0 = 1.7/1.0/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 5/5/7$

DATA THROUGH 24-AUG-2020
Indiana: Daily Deaths per Million People ($\theta = .1/.07/.2$)

Indiana

$R_0 = 1.7/1.0/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 5/5/7$

DATA THROUGH 24-AUG-2020
Indiana: Cumulative Deaths per Million People ($\theta = .1/.07/.2$)

$R_0 = 1.7/1.0/1.1 \hspace{1em} \delta = 0.010 \hspace{1em} \alpha = 0.05 \hspace{1em} \theta = 0.1 \hspace{1em} \%Infect = 5/5/7$

DATA THROUGH 24-AUG-2020
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Indiana: Daily Deaths, Actual and Smoothed

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Indiana: Change in Smoothed Daily Deaths

Indiana: Delta d

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Indiana: Change in (Change in Smoothed Daily Deaths)

Indiana: Delta (Delta d)
\[ \delta = 0.010 \quad \theta=0.10 \quad \gamma=0.20 \]