Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Ireland
Based on data through September 11, 2020
Outline of Slides

- Basic data from Johns Hopkins CSSE (raw and smoothed)
- Brief summary of the model
- Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
- Simulation of re-opening – possibilities for raising $R_0$
- Results with alternative parameter values:
  - Lower mortality rate, $\delta = 0.8\%$
  - Higher mortality rate, $\delta = 1.2\%$
  - Infections last longer, $\gamma = 0.15$
  - Cases resolve more quickly, $\theta = 0.2$
  - Cases resolve more slowly, $\theta = 0.07$
- Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Ireland: Daily Deaths per Million People

Ireland
Ireland: Daily Deaths per Million People (Smoothed)
**Brief Summary of Model**

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Ireland: Estimates of $R_0(t)$

Ireland

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Ireland: Percent Currently Infectious

Peak I/N = 0.62%   Final I/N = 0.00%   δ = 0.010   θ=0.10   γ=0.20
Ireland: Growth Rate of Daily Deaths over Past Week (percent)

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Intepreting Results
Guide to Graphs

• **Warning**: Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

• 7 days of forecasts: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  - Black = current
  - Red = oldest, Orange = second oldest, Yellow = third oldest...
  - Violet (purple) = one day earlier

• For robustness graphs, same idea
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - Red = lowest parameter value (e.g. $\delta = 0.8\%$)
  - Green = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it

- For future, two approaches:
  
  1. Alternatively, we fit this equation:

     \[
     \log R_0(t) = a_0 - \alpha(Daily\ Deaths) 
     \]

     \[\Rightarrow \alpha \approx 0.05\]

     $R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline

- Robustness: Assume $R_0(t) = \text{final empirical value}$. Constant in future, so no $\alpha$ adjustment $\Rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Ireland (7 days): Daily Deaths per Million People ($\alpha = .05$)

$R_0 = 1.8/0.2/0.2 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%Infect= 4/ 4/ 4$

DATA THROUGH 11-SEP-2020
Ireland (7 days): Cumulative Deaths per Million (Future, $\alpha = .05$)

Ireland

$R_0 = 1.8/0.2/0.2$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $%\text{Infect} = 4/4/4$

DATA THROUGH 11-SEP-2020
Ireland (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0.05$)

$R_0=1.8/0.2/0.2$ $\delta = 0.010$ $\alpha=0.05$ $\theta=0.1$ $\%$Infect= 4/ 4/ 4

Cumulative deaths per million people

Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan 2020
Robustness to Mortality Rate, $\delta$
Ireland: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

Ireland

$R_0=1.8/0.2/0.2$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%\text{Infected}=4/4/4$

DATA THROUGH 11-SEP-2020
Ireland: Daily Deaths per Million People ($\delta = .01/.008/.012$)

Ireland

$R_0 = 1.8/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 4/4/4$

DATA THROUGH 11-SEP-2020

Daily deaths per million people

Apr May Jun Jul Aug Sep Oct Nov Dec Jan 2020
Ireland: Cumulative Deaths per Million ($\delta = 0.01/0.008/0.012$)

Ireland

$R_0 = 1.8/0.2/0.2 \space \delta = 0.010 \space \alpha = 0.05 \space \theta = 0.1 \space \%\text{Infect} = 4/4/4$

DATA THROUGH 11-SEP-2020

Cumulative deaths per million people

Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan 2020

$\delta = 0.008$
Reopening and Herd Immunity

– **Black**: assumes $R_0(\text{today})$ remains in place forever
– **Red**: assumes $R_0(\text{suppress}) = \frac{1}{s(\text{today})}$
– **Green**: we move 25% of the way from $R_0(\text{today})$ back to initial $R_0 = \text{“normal”}$
– **Purple**: we move 50% of the way from $R_0(\text{today})$ back to initial $R_0 = \text{“normal”}$

**NOTE**: Lines often cover each other up
Ireland: Re-Opening ($\alpha = 0.05$)

\[
R_0(t) = 0.2, \quad R_0(\text{suppress}) = 1.0, \quad R_0(25/50) = 0.7/1.1, \quad \delta = 0.010, \quad \alpha = 0.05
\]

(Light bars = New York City, for comparison)
Ireland: Re-Opening ($\alpha = 0$)

Ireland

$R_0(t)=0.2, \ R_0\text{(suppress)}=1.0, \ R_0(25/50)=0.7/1.1, \ \delta = 0.010, \ \alpha=0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Ireland (7 days): Daily Deaths per Million People ($\alpha = 0$)

\[ R_0 = 1.8/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infect} = 4/4/4 \]

DATA THROUGH 11-SEP-2020
Ireland (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

$R_0 = 1.8/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infect} = 4/4/4$

DATA THROUGH 11-SEP-2020
Ireland (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Ireland

$R_0 = 1.8/0.2/0.2$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  $%Infect = 4/4/4$

New York City

Italy
Ireland: Daily Deaths per Million People ($\delta = 0.8\%$)

Ireland

$R_0 = 1.8/0.2/0.2$  $\delta = 0.008$  $\theta = 0.1$  $\gamma = 0.2$  $\%$Infect $= 4/4/4$

SOME ERRORS IN ESTIMATION...
Ireland: Cumulative Deaths per Million ($\delta = 0.8\%$)

Ireland

$R_0 = 1.8/0.2/0.2$  $\delta = 0.008$  $\theta = 0.1$  $\gamma = 0.2$  $\%$Infect = 4/4/4

SOME ERRORS IN ESTIMATION...
Ireland: Daily Deaths per Million People ($\delta = 1.2\%$)

Ireland

$R_0 = 1.8/0.2/0.2$  $\delta = 0.012$  $\theta = 0.1$  $\gamma = 0.2$  $\%$Infect = 3/3/3

SOME ERRORS IN ESTIMATION...
Ireland: Cumulative Deaths per Million ($\delta = 1.2\%$)

Ireland

$R_0=1.8/0.2/0.2 \quad \delta = 0.012 \quad \theta=0.1 \quad \gamma=0.2 \quad \%\text{Infect}=3/3/3$

SOME ERRORS IN ESTIMATION...
Ireland: Daily Deaths per Million People ($\gamma = 0.2/0.15$)

Ireland

$R_0 = 1.8/0.2/0.2$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%\text{Infect} = 4/4/4$

DATA THROUGH 11-SEP-2020
Ireland: Cumulative Deaths per Million $\gamma = .2 / .15$

R₀ = 1.8/0.2/0.2  δ = 0.010  α = 0.05  θ = 0.1  % Infect = 4/4/4

DATA THROUGH 11-SEP-2020
Ireland: Daily Deaths per Million People ($\theta = .1/.07/.2$)

\[ R_0 = 1.8/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 4/4/4 \]

DATA THROUGH 11-SEP-2020
Ireland: Cumulative Deaths per Million People ($\theta = .1/0.07/0.2$)

$Ireland$

$R_0=1.8/0.2/0.2$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%Infect=4/4/4$

DATA THROUGH 11-SEP-2020
Data Underlying Estimates
of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Ireland: Daily Deaths, Actual and Smoothed

Ireland: Daily deaths, d
\( \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \)
Ireland: Change in Smoothed Daily Deaths

Ireland: Delta $d$

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Ireland: Change in (Change in Smoothed Daily Deaths)

Ireland: Delta (Delta d)

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$