Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Kansas
Based on data through October 9, 2020
Outline of Slides

- Basic data from Johns Hopkins CSSE (raw and smoothed)
- Brief summary of the model
- Baseline results ($\delta = 1.0\%$, $\gamma = 0.2$, $\theta = 0.1$)
- Simulation of re-opening – possibilities for raising $R_0$
- Results with alternative parameter values:
  - Lower mortality rate, $\delta = 0.8\%$
  - Higher mortality rate, $\delta = 1.2\%$
  - Infections last longer, $\gamma = 0.15$
  - Cases resolve more quickly, $\theta = 0.2$
  - Cases resolve more slowly, $\theta = 0.07$
- Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Kansas: Daily Deaths per Million People

![Graph showing daily deaths per million people in Kansas from April to October 2020. The y-axis represents daily deaths per million people, ranging from 0 to 14. The x-axis represents the months from April to October. The graph shows fluctuations in daily deaths throughout the months.](image-url)
Kansas: Daily Deaths per Million People (Smoothed)
Brief Summary of Model

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Kansas: Estimates of $R_0(t)$

Kansas

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Kansas: Percent Currently Infectious

Kansas
Peak I/N = 0.20%  Final I/N = 0.18%  δ = 0.010  θ = 0.10  γ = 0.20
Kansas: Growth Rate of Daily Deaths over Past Week (percent)

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

• **Warning:** Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

• 7 days of forecasts: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  - Black = current
  - Red = oldest, Orange = second oldest, Yellow = third oldest...
  - Violet (purple) = one day earlier

• For robustness graphs, same idea
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - Red = lowest parameter value (e.g. $\delta = 0.8\%$)
  - Green = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:
  
  1. Alternatively, we fit this equation:

  $\log R_0(t) = a_0 - \alpha \text{(Daily Deaths)}$

  $\Rightarrow \alpha \approx .05$

  $R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline

- Robustness: Assume $R_0(t) = \text{final empirical value}$. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Kansas (7 days): Daily Deaths per Million People ($\alpha = 0.05$)

$R_0 = 1.5/1.2/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 4/5/10

DATA THROUGH 09-OCT-2020
Kansas (7 days): Cumulative Deaths per Million (Future, $\alpha = .05$)

Kansas

$R_0 = 1.5/1.2/1.1$  \(\delta = 0.010\) \(\alpha = 0.05\) \(\theta = 0.1\)  \%Infect = 4/5/10

DATA THROUGH 09-OCT-2020

Cumulative deaths per million people
Kansas (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = .05$)

Kansas

$R_0 = 1.5/1.2/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$Infect$= 4/5/10$

Kansas

New York City

Italy

Cumulative deaths per million people

Robustness to Mortality Rate, $\delta$
Kansas: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

R$_0$=1.5/1.2/1.1  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect= 4/ 5/10

DATA THROUGH 09-OCT-2020
Kansas: Daily Deaths per Million People ($\delta = 0.01/0.008/0.012$)

Kansas

$R_0 = 1.5/1.2/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%\text{Infect}= 4/5/10$

DATA THROUGH 09-OCT-2020
Kansas: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

$R_0 = 1.5/1.2/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 4/5/10$

DATA THROUGH 09-OCT-2020
Reopening and Herd Immunity

– Black: assumes $R_0$ (today) remains in place forever
– Red: assumes $R_0$ (suppress) = $1/s$ (today)
– Green: we move 25% of the way from $R_0$ (today) back to initial $R_0$ = “normal”
– Purple: we move 50% of the way from $R_0$ (today) back to initial $R_0$ = “normal”

NOTE: Lines often cover each other up
Kansas: Re-Opening \( (\alpha = .05) \)

Kansas

\[ R_0(t) = 1.2, \quad R_0(\text{suppress}) = 1.1, \quad R_0(25/50) = 1.4/1.6, \quad \delta = 0.010, \quad \alpha = 0.05 \]

(Light bars = New York City, for comparison)
Kansas: Re-Opening ($\alpha = 0$)

Kansas

$R_0(t)=1.2$, $R_0$ (suppress) = 1.1, $R_0(25/50)=1.4/1.6$, $\delta = 0.010$, $\alpha = 0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Kansas (7 days): Daily Deaths per Million People ($\alpha = 0$)

Kansas

$R_0 = 1.5/1.2/1.2$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  %Infect$= 4/7/28$

DATA THROUGH 09-OCT-2020
Kansas (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

Kansas

$R_0 = 1.5/1.2/1.2 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infect} = 4/7/28$

DATA THROUGH 09-OCT-2020
Kansas (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Kansas

$R_0 = 1.5/1.2/1.2$, $\delta = 0.010$, $\alpha = 0.00$, $\theta = 0.1$ %Infect = 4/7/28

New York City

Italy
Kansas: Daily Deaths per Million People ($\delta = 0.8\%$)

Kansas

$R_0=1.5/1.2/1.1 \quad \delta = 0.008 \quad \theta=0.1 \quad \gamma=0.2 \quad \%\text{Infect}=4/7/12$
Kansas: Cumulative Deaths per Million ($\delta = 0.8\%$)

Kansas

$R_0 = 1.5/1.2/1.1 \quad \delta = 0.008 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 4/7/12$
Kansas: Daily Deaths per Million People ($\delta = 1.2\%$)

Kansas

$R_0=1.5/1.2/1.1$  $\delta = 0.012$  $\theta=0.1$  $\gamma=0.2$  %Infect= 3/ 5/ 9
Kansas: Cumulative Deaths per Million ($\delta = 1.2\%$)

Kansas

$R_0 = 1.5/1.2/1.1 \quad \delta = 0.012 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%Infect = 3/5/9$
Kansas: Daily Deaths per Million People ($\gamma = .2/.15$)

Kansas

$R_0=1.5/1.2/1.1$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%\text{Infect}=4/5/10$

DATA THROUGH 09-OCT-2020
Kansas: Cumulative Deaths per Million $\gamma = .2/.15$)

Kansas

$R_0=1.5/1.2/1.1 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}= 4/5/10$

DATA THROUGH 09-OCT-2020

Cumulative deaths per million people


$\gamma = 0.15$

$\gamma = 0.2$
Kansas: Daily Deaths per Million People ($\theta = .1 / .07 / .2$)

Kansas

$R_0 = 1.5 / 1.2 / 1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$ Infect = 4/5/10

DATA THROUGH 09-OCT-2020
Kansas: Cumulative Deaths per Million People ($\theta = .1/.07/.2$)

Kansas

$R_0 = 1.5/1.2/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$Infect$ = 4/5/10$

DATA THROUGH 09-OCT-2020
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Kansas: Daily Deaths, Actual and Smoothed

Kansas: Daily deaths, \( d \)
\[ \delta = 0.010 \quad \theta=0.10 \quad \gamma=0.20 \]
Kansas: Change in Smoothed Daily Deaths

Kansas: Delta $d$

$\delta = 0.010 \ \theta=0.10 \ \gamma=0.20$
Kansas: Change in (Change in Smoothed Daily Deaths)

Kansas: Delta (Δδ)

δ = 0.010  θ = 0.10  γ = 0.20