Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Luxembourg
Based on data through October 9, 2020
Outline of Slides

- Basic data from Johns Hopkins CSSE (raw and smoothed)
- Brief summary of the model
- Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
- Simulation of re-opening – possibilities for raising $R_0$
- Results with alternative parameter values:
  - Lower mortality rate, $\delta = 0.8\%$
  - Higher mortality rate, $\delta = 1.2\%$
  - Infections last longer, $\gamma = 0.15$
  - Cases resolve more quickly, $\theta = 0.2$
  - Cases resolve more slowly, $\theta = 0.07$
- Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Luxembourg: Daily Deaths per Million People

Luxembourg
Luxembourg: Daily Deaths per Million People (Smoothed)
Brief Summary of Model

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ $(\beta_t/\gamma)$</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Luxembourg: Estimates of $R_0(t)$

Luxembourg

$\delta = 0.010$  $\theta=0.10$  $\gamma=0.20$
Luxembourg: Percent Currently Infectious

Luxembourg
Peak I/N = 0.28% Final I/N = 0.03% δ = 0.010 θ=0.10 γ=0.20
Luxembourg: Growth Rate of Daily Deaths over Past Week (percent)

Luxembourg

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

- **Warning**: Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

- 7 days of forecasts: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  - Black = current
  - Red = oldest, Orange = second oldest, Yellow = third oldest...
  - Violet (purple) = one day earlier

- For robustness graphs, same idea
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - Red = lowest parameter value (e.g. $\delta = 0.8\%$)
  - Green = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

• Inferred from death data when we have it

• For future, two approaches:

  1. Alternatively, we fit this equation:

  \[
  \log R_0(t) = a_0 - \alpha(Daily\ Deaths)
  \]

  \[\Rightarrow \alpha \approx 0.05\]

  \[R_0\ declines\ by\ 5\ percent\ for\ each\ new\ daily\ death,\]
  \[or\ rises\ by\ 5\ percent\ when\ daily\ deaths\ decline\]

• Robustness: Assume $R_0(t) = \text{final empirical value}$. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Luxembourg (7 days): Daily Deaths per Million People ($\alpha = .05$)

Luxembourg

$R_0=1.6/0.2/0.2 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}= 2/2/2$

DATA THROUGH 09-OCT-2020


Daily deaths per million people
Luxembourg (7 days): Cumulative Deaths per Million (Future, $\alpha = .05$)

$R_0 = 1.6/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \% \text{Infect} = 2/2/2$

DATA THROUGH 09-OCT-2020
Luxembourg (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0.05$)

$R_0 = 1.6/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 2/2/2$

New York City

Italy
Robustness to Mortality Rate, $\delta$
Luxembourg: Cumulative Deaths per Million ($\delta = .01/0.008/0.012$)

DATA THROUGH 09-OCT-2020

Luxembourg

$R_0=1.6/0.2/0.2$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%Infect= 2/2/2$
Luxembourg: Daily Deaths per Million People ($\delta = 0.01/0.008/0.012$)

Luxembourg
$R_0 = 1.6/0.2/0.2$, $\gamma = 0.010$, $\alpha = 0.05$, $\theta = 0.1$, $\%\text{Infect} = 2/2/2$
Luxembourg: Cumulative Deaths per Million ($\delta = 0.01/0.008/0.012$)

Luxembourg
$R_0=1.6/0.2/0.2$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect= 2/2/2

DATA THROUGH 09-OCT-2020
Reopening and Herd Immunity

– **Black**: assumes $R_0(\text{today})$ remains in place forever
– **Red**: assumes $R_0(\text{suppress}) = 1/s(\text{today})$
– **Green**: we move 25% of the way from $R_0(\text{today})$ back to initial $R_0 = “normal”$
– **Purple**: we move 50% of the way from $R_0(\text{today})$ back to initial $R_0 = “normal”$

**NOTE**: Lines often cover each other up
Luxembourg: Re-Opening \( (\alpha = .05) \)

Luxembourg

\[ R_0(t) = 0.2, \quad R_0(\text{suppress}) = 1.0, \quad R_0(25/50) = 0.7/1.1, \quad \delta = 0.010, \quad \alpha = 0.05 \]

(Light bars = New York City, for comparison)
Luxembourg: Re-Opening ($\alpha = 0$)

$R_0(t)=0.2$, $R_0$ (suppress) = 1.0, $R_0(25/50)=0.7/1.1$, $\delta = 0.010$, $\alpha=0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Luxembourg (7 days): Daily Deaths per Million People ($\alpha = 0$)

Luxembourg

$R_0 = 1.6/0.2/0.2$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  $\%$Infect = 2/2/2

DATA THROUGH 09-OCT-2020
Luxembourg (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

Luxembourg

$R_0 = 1.6/0.2/0.2$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  $\%$Infect = 2/2/2

DATA THROUGH 09-OCT-2020
Luxembourg (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Luxembourg

$R_0 = 1.6/0.2/0.2$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  %Infect = 2/2/2

New York City

Italy
Luxembourg: Daily Deaths per Million People ($\delta = 0.8\%$)

Luxembourg

$R_0 = 1.6/0.2/0.2$ $\delta = 0.008$ $\theta = 0.1$ $\gamma = 0.2$ $%\text{Infect} = 2/2/2$

SOME ERRORS IN ESTIMATION...
Luxembourg: Cumulative Deaths per Million ($\delta = 0.8\%$)

Luxembourg

$R_0=1.6/0.2/0.2 \quad \delta = 0.008 \quad \theta=0.1 \quad \gamma=0.2 \quad \%\text{Infect}= 2/ 2/ 2$

SOME ERRORS IN ESTIMATION...
Luxembourg: Daily Deaths per Million People ($\delta = 1.2\%$)

Luxembourg

$R_0=1.6/0.2/0.2$  $\delta = 0.012$  $\theta=0.1$  $\gamma=0.2$  $\%\text{Infect}=1/1/1$

SOME ERRORS IN ESTIMATION...
Luxembourg: Cumulative Deaths per Million ($\delta = 1.2\%$)

Luxembourg

$R_0=1.6/0.2/0.2 \quad \delta = 0.012 \quad \theta=0.1 \quad \gamma=0.2 \quad \%Infect=1/1/1$

SOME ERRORS IN ESTIMATION...
Luxembourg: Daily Deaths per Million People ($\gamma = .2/.15$)

Luxembourg

$R_0 = 1.6/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 2/2/2$

DATA THROUGH 09-OCT-2020
Luxembourg: Cumulative Deaths per Million $\gamma = .2/.15$)

Luxembourg

$R_0 = 1.6/0.2/0.2$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  \%Infect = $2/2/2$

DATA THROUGH 09-OCT-2020
Luxembourg: Daily Deaths per Million People ($\theta = 1/0.07/0.2$)

Luxembourg

$R_0 = 1.6/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect}= 2/2/2$

DATA THROUGH 09-OCT-2020
Luxembourg: Cumulative Deaths per Million People ($\theta = .1 / .07 / .2$)

Data through 09-Oct-2020

$R_0 = 1.6/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 2/2/2$
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Luxembourg: Daily Deaths, Actual and Smoothed

Luxembourg: Daily deaths, d
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Luxembourg: Change in Smoothed Daily Deaths

Luxembourg: Delta $d$

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Luxembourg: Change in (Change in Smoothed Daily Deaths)

Luxembourg: Delta (Delta d)
\[\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20\]