Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Massachusetts
Based on data through August 24, 2020
Outline of Slides

- Basic data from Johns Hopkins CSSE (raw and smoothed)
- Brief summary of the model
- Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
- Simulation of re-opening – possibilities for raising $R_0$
- Results with alternative parameter values:
  - Lower mortality rate, $\delta = 0.8\%$
  - Higher mortality rate, $\delta = 1.2\%$
  - Infections last longer, $\gamma = 0.15$
  - Cases resolve more quickly, $\theta = 0.2$
  - Cases resolve more slowly, $\theta = 0.07$
- Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Massachusetts: Daily Deaths per Million People

Massachusetts
Massachusetts: Daily Deaths per Million People (Smoothed)

Massachusetts
Brief Summary of Model

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Massachusetts: Estimates of $R_0(t)$

Massachusetts

$\delta = 0.010$  $\theta = 0.10$  $\gamma = 0.20$
Massachusetts: Percent Currently Infectious

Peak I/N = 1.48%  Final I/N = 0.14%  δ = 0.010  ϑ = 0.10  γ = 0.20
Massachusetts: Growth Rate of Daily Deaths over Past Week (percent)

\[\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20\]
Notes on Interpreting Results
Guide to Graphs

• **Warning:** Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

• 7 days of forecasts: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  - Black = current
  - Red = oldest, Orange = second oldest, Yellow = third oldest...
  - Violet (purple) = one day earlier

• For robustness graphs, same idea
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - Red = lowest parameter value (e.g. $\delta = 0.8\%$)
  - Green = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:
  1. Alternatively, we fit this equation:

        \[ \log R_0(t) = a_0 - \alpha(Daily\ Deaths) \]

        \[ \Rightarrow \alpha \approx 0.05 \]

        $R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline

- Robustness: Assume $R_0(t) =$ final empirical value. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Massachusetts (7 days): Daily Deaths per Million People ($\alpha = .05$)

Massachusetts

$R_0 = 2.0/1.8/1.3 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 14/18/25$

DATA THROUGH 24-AUG-2020
Massachusetts (7 days): Cumulative Deaths per Million (Future, $\alpha = 0.05$)

Massachusetts

$R_0 = 2.0/1.8/1.3$  \( \delta = 0.010 \)  \( \alpha = 0.05 \)  \( \theta = 0.1 \)  \%Infect = 14/18/25

DATA THROUGH 24-AUG-2020
Massachusetts (7 days): Cumulative Deaths per Million, Log Scale ($\alpha =$)

Massachusetts

$R_0=2.0/1.8/1.3 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}=14/18/25$

Cumulative deaths per million people
Robustness to Mortality Rate, $\delta$
Massachusetts: Cumulative Deaths per Million ($\delta = .01 / .008 / .012$)

Massachusetts

$R_0 = 2.0/1.8/1.3 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect}=14/18/25$

DATA THROUGH 24-AUG-2020
Massachusetts: Daily Deaths per Million People \((\delta = 0.01/0.008/0.012)\)

\[
\begin{align*}
R_0 &= 2.0/1.8/1.3 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 14/18/25
\end{align*}
\]
Massachusetts: Cumulative Deaths per Million ($\delta = 0.01/0.008/0.012$)

Massachusetts

$R_0 = 2.0/1.8/1.3 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect}=14/18/25$

DATA THROUGH 24-AUG-2020
Reopening and Herd Immunity

– Black: assumes $R_0(today)$ remains in place forever
– Red: assumes $R_0(suppress) = 1/s(today)$
– Green: we move 25% of the way from $R_0(today)$ back to initial $R_0 = “normal”$
– Purple: we move 50% of the way from $R_0(today)$ back to initial $R_0 = “normal”$

NOTE: Lines often cover each other up
Massachusetts: Re-Opening ($\alpha = 0.05$)

Massachusetts

$R_0(t)=1.8$, $R_0^{(\text{suppress})}=1.2$, $R_0^{(25/50)}=1.8/1.9$, $\delta = 0.010$, $\alpha=0.05$
Massachusetts: Re-Opening ($\alpha = 0$)

Massachusetts

$R_0(t)=1.9, \ R_0(\text{suppress})=1.5, \ R_0(25/50)=1.9/1.9, \ \delta = 0.010, \ \alpha=0.00$
Results for alternative parameter values
Massachusetts (7 days): Daily Deaths per Million People ($\alpha = 0$)

Massachusetts

$R_0 = 2.0/1.9/1.9$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  %Infect $= 14/35/72$

DATA THROUGH 24-AUG-2020
Massachusetts (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

Massachusetts

$R_0 = 2.0/1.9/1.9$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  $\%$Infect$=14/35/72$

DATA THROUGH 24-AUG-2020
Massachusetts (7 days): Cumulative Deaths per Million, Log Scale ($\alpha =$ 1)

Massachusetts
$R_0 = 2.0/1.9/1.9$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  $\%$ Infect $= 14/35/72$

New York City
Italy
Massachusetts: Daily Deaths per Million People ($\delta = 0.8\%$)

Massachusetts

$R_0=2.0/1.8/1.4$  $\delta = 0.008$  $\theta=0.1$  $\gamma=0.2$  $\%Infect=17/22/31$
Massachusetts: Cumulative Deaths per Million ($\delta = 0.8\%$)

\[ R_0 = 2.0/1.8/1.4 \quad \delta = 0.008 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 17/22/31 \]
Massachusetts: Daily Deaths per Million People ($\delta = 1.2\%$)

Massachusetts

$R_0 = 2.0/1.7/1.2 \quad \delta = 0.012 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \% \text{Infect} = 11/15/21$
Massachusetts: Cumulative Deaths per Million ($\delta = 1.2\%$)

Massachusetts

$R_0=2.0/1.7/1.2 \quad \delta = 0.012 \quad \theta=0.1 \quad \gamma=0.2 \quad \%\text{Infect}=11/15/21$
Massachusetts: Daily Deaths per Million People ($\gamma = .2/.15$)

Massachusetts

$R_0 = 2.0/1.8/1.3 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 14/18/25$

DATA THROUGH 24-AUG-2020
Massachusetts: Cumulative Deaths per Million $\gamma = .2/.15$)

Massachusetts
$R_0=2.0/1.8/1.3 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}=14/18/25$

DATA THROUGH 24-AUG-2020

Cumulative deaths per million people

Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan 2020
Massachusetts: Daily Deaths per Million People ($\theta = .1/.07/.2$)

**Massachusetts**

$R_0=2.0/1.8/1.3 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}=14/18/25$

DATA THROUGH 24-AUG-2020
Massachusetts: Cumulative Deaths per Million People ($\theta = 0.1 / 0.07 / 0.2$)

Massachusetts

$R_0=2.0/1.8/1.3 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1$  
%Infect=14/18/25  

DATA THROUGH 24-AUG-2020
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Massachusetts: Daily Deaths, Actual and Smoothed

Massachusetts: Daily deaths, $d$

$$
\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20
$$
Massachusetts: Change in (Change in Smoothed Daily Deaths)

\[
\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20
\]