Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Madrid, Spain
Based on data through September 11, 2020
Outline of Slides

- Basic data from Johns Hopkins CSSE (raw and smoothed)
- Brief summary of the model
- Baseline results ($\delta = 1.0\%$, $\gamma = 0.2$, $\theta = 0.1$)
- Simulation of re-opening – possibilities for raising $R_0$
- Results with alternative parameter values:
  - Lower mortality rate, $\delta = 0.8\%$
  - Higher mortality rate, $\delta = 1.2\%$
  - Infections last longer, $\gamma = 0.15$
  - Cases resolve more quickly, $\theta = 0.2$
  - Cases resolve more slowly, $\theta = 0.07$
- Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Madrid, Spain: Daily Deaths per Million People

Madrid, Spain

Daily deaths per million people

Apr May Jun Jul Aug Sep
2020

-10
0
10
20
30
40
50

0
10
20
30
40
50
60
70
80
90
100

3
Madrid, Spain: Daily Deaths per Million People (Smoothed)
Brief Summary of Model

• See the paper for a full exposition

• A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Madrid, Spain: Estimates of $R_0(t)$

Madrid, Spain

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Madrid, Spain: Percent Currently Infectious

Madrid, Spain
Peak I/N = 2.37% Final I/N = 0.04% δ = 0.010 θ = 0.10 γ = 0.20
Madrid, Spain: Growth Rate of Daily Deaths over Past Week (percent)

Madrid, Spain

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
**Guide to Graphs**

- **Warning:** Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

- **7 days of forecasts:** Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  - Black = current
  - Red = oldest, Orange = second oldest, Yellow = third oldest...
  - Violet (purple) = one day earlier

- For robustness graphs, same idea
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - Red = lowest parameter value (e.g. $\delta = 0.8\%$)
  - Green = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:
  
  1. Alternatively, we fit this equation:

     \[
     \log R_0(t) = a_0 - \alpha (\text{Daily Deaths})
     \]

     \[\Rightarrow \alpha \approx 0.05\]

     $R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline

- Robustness: Assume $R_0(t) =$ final empirical value. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Madrid, Spain (7 days): Daily Deaths per Million People ($\alpha = .05$)

Data through 11-Sep-2020

$R_0 = 2.2/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 13/13/13$
Madrid, Spain (7 days): Cumulative Deaths per Million (Future, $\alpha = .05$)

Madrid, Spain

$R_0=2.2/0.2/0.2$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect=13/13/13

DATA THROUGH 11-SEP-2020
Madrid, Spain (7 days): Cumulative Deaths per Million, Log Scale ($\alpha =$)

Madrid, Spain
$R_0 = 2.2/0.2/0.2$, $\delta = 0.010$, $\alpha = 0.05$, $\theta = 0.1$, %Infect = 13/13/13
Robustness to Mortality Rate, $\delta$
Madrid, Spain: Cumulative Deaths per Million ($\delta = 0.01/0.008/0.012$)

Madrid, Spain

$R_0=2.2/0.2/0.2$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect=13/13/13

DATA THROUGH 11-SEP-2020
Madrid, Spain: Daily Deaths per Million People ($\delta = .01/0.008/0.012$)

Madrid, Spain

$R_0 = 2.2/0.2/0.2$  \( \delta = 0.010 \)  \( \alpha = 0.05 \)  \( \theta = 0.1 \)  \%Infect = 13/13/13

DATA THROUGH 11-SEP-2020
Madrid, Spain: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

Madrid, Spain

$R_0=2.2/0.2/0.2$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect=13/13/13

DATA THROUGH 11-SEP-2020

$\delta = 0.008$
Reopening and Herd Immunity

– Black: assumes $R_0(today)$ remains in place forever
– Red: assumes $R_0(suppress) = 1/s(today)$
– Green: we move 25% of the way from $R_0(today)$ back to initial $R_0 = “normal”$
– Purple: we move 50% of the way from $R_0(today)$ back to initial $R_0 = “normal”$

NOTE: Lines often cover each other up
Madrid, Spain: Re-Opening ($\alpha = 0.05$)

Madrid, Spain

$R_0(t)=0.2$, $R_0\text{(suppress)}=1.1$, $R_0(25/50)=0.7/1.2$, $\delta = 0.010$, $\alpha=0.05$

(Light bars = New York City, for comparison)
Madrid, Spain: Re-Opening ($\alpha = 0$)

Madrid, Spain
$R_0(t) = 0.2$, $R_0\text{(suppress)} = 1.1$, $R_0(25/50) = 0.7/1.2$, $\delta = 0.010$, $\alpha = 0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Madrid, Spain (7 days): Daily Deaths per Million People ($\alpha = 0$)

Madrid, Spain

$R_0 = 2.2/0.2/0.2$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  $\%Infect = 13/13/13$

DATA THROUGH 11-SEP-2020
Madrid, Spain (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

Madrid, Spain

$R_0 = 2.2 / 0.2 / 0.2$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  $\%$Infect$=13/13/13$

DATA THROUGH 11-SEP-2020
Madrid, Spain (7 days): Cumulative Deaths per Million, Log Scale (\(\alpha = \))

Madrid, Spain

\(R_0=2.2/0.2/0.2\)  \(\delta = 0.010\)  \(\alpha=0.00\)  \(\theta=0.1\)  \%Infect=13/13/13
Madrid, Spain: Daily Deaths per Million People ($\delta = 0.8\%$)

Madrid, Spain

$R_0 = 2.2/0.2/0.2$ $\delta = 0.008$ $\theta = 0.1$ $\gamma = 0.2$ $\%\text{Infect} = 16/16/16$

SOME ERRORS IN ESTIMATION...
Madrid, Spain: Cumulative Deaths per Million ($\delta = 0.8\%$)

Madrid, Spain

$R_0=2.2/0.2/0.2 \delta = 0.008 \ \theta=0.1 \ \gamma=0.2 \ \%\text{Infect}=16/16/16$

SOME ERRORS IN ESTIMATION...
Madrid, Spain: Daily Deaths per Million People ($\delta = 1.2\%$)

Madrid, Spain

$R_0 = 2.2/0.2/0.2$  $\delta = 0.012$  $\theta = 0.1$  $\gamma = 0.2$  $\%\text{Infect} = 11/11/11$

SOME ERRORS IN ESTIMATION...
Madrid, Spain: Cumulative Deaths per Million ($\delta = 1.2\%$)

Madrid, Spain

$R_0=2.2/0.2/0.2$  $\delta = 0.012$  $\theta=0.1$  $\gamma=0.2$  $\%\text{Infect}=11/11/11$

SOME ERRORS IN ESTIMATION...
Madrid, Spain: Daily Deaths per Million People ($\gamma = .2 / .15$)

Madrid, Spain

$R_0 = 2.2 / 0.2 / 0.2$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%\text{Infect} = 13/13/13$

DATA THROUGH 11-SEP-2020
Madrid, Spain: Cumulative Deaths per Million $\gamma = .2/1.15$}

Madrid, Spain

$R_0 = 2.2/0.2/0.2 \delta = 0.010 \alpha = 0.05 \theta = 0.1$ %Infect = 13/13/13

$\gamma = 0.15$
Madrid, Spain: Daily Deaths per Million People ($\theta = .1/.07/.2$)

Madrid, Spain

$R_0=2.2/0.2/0.2$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%\text{Infect}=13/13/13$

DATA THROUGH 11-SEP-2020
Madrid, Spain: Cumulative Deaths per Million People ($\theta = .1/.07/.2$)

Madrid, Spain

$R_0 = 2.2/0.2/0.2$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 13/13/13

DATA THROUGH 11-SEP-2020

Cumulative deaths per million people

Mar  Apr  May  Jun  Jul  Aug  Sep  Oct  Nov  Dec  Jan  2020
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Madrid, Spain: Daily Deaths, Actual and Smoothed

Madrid, Spain: Daily deaths, d
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Madrid, Spain: Change in Smoothed Daily Deaths

\[ \Delta d = 0.010 \quad \Theta = 0.10 \quad \Gamma = 0.20 \]
Madrid, Spain: Change in (Change in Smoothed Daily Deaths)

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]