Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Madrid, Spain
Based on data through October 9, 2020
Outline of Slides

- Basic data from Johns Hopkins CSSE (raw and smoothed)
- Brief summary of the model
- Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
- Simulation of re-opening – possibilities for raising $R_0$
- Results with alternative parameter values:
  - Lower mortality rate, $\delta = 0.8\%$
  - Higher mortality rate, $\delta = 1.2\%$
  - Infections last longer, $\gamma = 0.15$
  - Cases resolve more quickly, $\theta = 0.2$
  - Cases resolve more slowly, $\theta = 0.07$
- Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Madrid, Spain: Daily Deaths per Million People
Madrid, Spain: Daily Deaths per Million People (Smoothed)
Brief Summary of Model

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ $(\beta_t/\gamma)$</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Madrid, Spain: Estimates of $R_0(t)$

Madrid, Spain
$\delta = 0.010 \ \theta = 0.10 \ \gamma = 0.20$
Madrid, Spain: Percent Currently Infectious

Peak I/N = 2.37%  Final I/N = 0.04%  δ = 0.010  θ = 0.10  γ = 0.20
Madrid, Spain: Growth Rate of Daily Deaths over Past Week (percent)

Madrid, Spain
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

- **Warning:** Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

- **7 days of forecasts:** Rainbow color order!
  - ROY-G-BIV (old to new, low to high)
    - Black = current
    - Red = oldest, Orange = second oldest, Yellow = third oldest...
    - Violet (purple) = one day earlier

- **For robustness graphs, same idea**
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - Red = lowest parameter value (e.g. $\delta = 0.8\%$)
  - Green = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:

1. Alternatively, we fit this equation:

$$\log R_0(t) = a_0 - \alpha(Daily\ Deaths)$$

$$\Rightarrow \alpha \approx .05$$

$R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline.

- Robustness: Assume $R_0(t) =$ final empirical value. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Madrid, Spain (7 days): Daily Deaths per Million People ($\alpha = 0.05$)

Madrid, Spain

$R_0 = 2.2/0.2/0.2$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%\text{Infect} = 13/13/13$

DATA THROUGH 09-OCT-2020
Madrid, Spain (7 days): Cumulative Deaths per Million (Future, $\alpha = 0.05$)

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**Madrid, Spain**

$R_0 = 2.2/0.2/0.2$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 13/13/13

Data through 09-Oct-2020
Madrid, Spain (7 days): Cumulative Deaths per Million, Log Scale

Madrid, Spain

$R_0 = 2.2/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infec}=13/13/13$
Robustness to Mortality Rate, $\delta$
Madrid, Spain: Cumulative Deaths per Million ($\delta = 0.01/0.008/0.012$)
Madrid, Spain: Daily Deaths per Million People \( (\delta = .01/.008/.012) \)

Madrid, Spain

\[ R_0 = 2.2/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \% \text{Infect} = 13/13/13 \]

DATA THROUGH 09-DEC-2020
Madrid, Spain: Cumulative Deaths per Million ($\delta = 0.01/0.008/0.012$)

DATA THROUGH 09-OCT-2020

Madrid, Spain
$R_0 = 2.2/0.2/0.2$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 13/13/13

Cumulative deaths per million people
Reopening and Herd Immunity

– Black: assumes $R_0(today)$ remains in place forever
– Red: assumes $R_0(suppress) = 1/s(today)$
– Green: we move 25% of the way from $R_0(today)$ back to initial $R_0 = “normal”$
– Purple: we move 50% of the way from $R_0(today)$ back to initial $R_0 = “normal”$

NOTE: Lines often cover each other up
Madrid, Spain: Re-Opening ($\alpha = 0.05$)

Madrid, Spain

$R_0(t) = 0.2$, $R_0(\text{suppress}) = 1.1$, $R_0(25/50) = 0.7/1.2$, $\delta = 0.010$, $\alpha = 0.05$

(Light bars = New York City, for comparison)
Madrid, Spain: Re-Opening ($\alpha = 0$)

Madrid, Spain

$R_0(t)=0.2$, $R_0(\text{suppress})=1.1$, $R_0(25/50)=0.7/1.2$, $\delta = 0.010$, $\alpha=0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Madrid, Spain (7 days): Daily Deaths per Million People ($\alpha = 0$)

Madrid, Spain

$R_0 = 2.2/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infected}=13/13/13$

DATA THROUGH 09-OCT-2020
Madrid, Spain (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

Madrid, Spain
$R_0 = 2.2/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infect}=13/13/13$

DATA THROUGH 09-OCT-2020
Madrid, Spain (7 days): Cumulative Deaths per Million, Log Scale ($\alpha =$

Madrid, Spain

$R_0=2.2/0.2/0.2 \quad \delta = 0.010 \quad \alpha=0.00 \quad \theta=0.1 \quad \%\text{Infect}=13/13/13$
Madrid, Spain: Daily Deaths per Million People ($\delta = 0.8\%$)

$R_0=2.2/0.2/0.2$  $\delta = 0.008$  $\theta=0.1$  $\gamma=0.2$  $%\text{Infect}=16/16/16$

SOME ERRORS IN ESTIMATION...
Madrid, Spain: Cumulative Deaths per Million ($\delta = 0.8\%$)

Madrid, Spain
$R_0 = 2.2/0.2/0.2$  $\delta = 0.008$  $\theta = 0.1$  $\gamma = 0.2$  $\%$Infect = 16/16/16

SOME ERRORS IN ESTIMATION...
Madrid, Spain: Daily Deaths per Million People ($\delta = 1.2\%$)

Madrid, Spain

$R_0=2.2/0.2/0.2$  $\delta = 0.012$  $\theta=0.1$  $\gamma=0.2$  %Infect=11/11/11

SOME ERRORS IN ESTIMATION...
Madrid, Spain: Cumulative Deaths per Million ($\delta = 1.2\%$)

Madrid, Spain

$R_0 = 2.2/0.2/0.2 \quad \delta = 0.012 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \% \text{Infect} = 11/11/11$

SOME ERRORS IN ESTIMATION...
Madrid, Spain: Daily Deaths per Million People ($\gamma = .2/.15$)

Madrid, Spain
$R_0 = 2.2/0.2/0.2$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 13/13/13

DATA THROUGH 09-OCT-2020
Madrid, Spain: Cumulative Deaths per Million $\gamma = .2/.15$)
Madrid, Spain: Daily Deaths per Million People ($\theta = .1/0.07/0.2$)

Madrid, Spain

$R_0 = 2.2/0.2/0.2$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%\text{Infect} = 13/13/13$

DATA THROUGH 09-OCT-2020
Madrid, Spain: Cumulative Deaths per Million People ($\theta = .1/.07/.2$)

Madrid, Spain

$R_0=2.2/0.2/0.2$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%$Infect=13/13/13

DATA THROUGH 09-OCT-2020
Data Underlying Estimates of Time-Varying $R_0$

- Inferred from daily deaths, and
- the change in daily deaths, and
- the change in (the change in daily deaths)
Madrid, Spain: Daily Deaths, Actual and Smoothed

Madrid, Spain: Daily deaths, \( d \)
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Madrid, Spain: Change in Smoothed Daily Deaths

Madrid, Spain: Delta $d$

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Madrid, Spain: Change in (Change in Smoothed Daily Deaths)

Madrid, Spain: Delta (Δd)

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]