Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Michigan
Based on data through September 11, 2020
Outline of Slides

• Basic data from Johns Hopkins CSSE (raw and smoothed)
• Brief summary of the model
• Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
• Simulation of re-opening – possibilities for raising $R_0$
• Results with alternative parameter values:
  - Lower mortality rate, $\delta = 0.8\%$
  - Higher mortality rate, $\delta = 1.2\%$
  - Infections last longer, $\gamma = 0.15$
  - Cases resolve more quickly, $\theta = 0.2$
  - Cases resolve more slowly, $\theta = 0.07$
• Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

– Raw data
– Smoothed = 7 day centered moving average
– No “excess deaths” correction (change as of Aug 6 run)
Michigan: Daily Deaths per Million People

Daily deaths per million people
Michigan: Daily Deaths per Million People (Smoothed)
**Brief Summary of Model**

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Michigan: Estimates of $R_0(t)$

Michigan
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Michigan: Percent Currently Infectious

Peak I/N = 1.01%  Final I/N = 0.08%  δ = 0.010  θ = 0.10  γ = 0.20
Notes on Interpreting Results
Guide to Graphs

• **Warning:** Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

• 7 days of forecasts: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  - **Black** = current
  - **Red** = oldest, **Orange** = second oldest, **Yellow** = third oldest...
  - **Violet** (purple) = one day earlier

• For robustness graphs, same idea
  - **Black** = baseline (e.g. $\delta = 1.0\%$)
  - **Red** = lowest parameter value (e.g. $\delta = 0.8\%$)
  - **Green** = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it

- For future, two approaches:
  
  1. Alternatively, we fit this equation:

  \[
  \log R_0(t) = a_0 - \alpha(Daily\ Deaths) \]

  \[\Rightarrow \alpha \approx .05\]

  $R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline

- Robustness: Assume $R_0(t) = \text{final empirical value}$. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Michigan (7 days): Daily Deaths per Million People ($\alpha = .05$)

Michigan

$R_0=2.0/1.2/1.1 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}=9/9/11$

DATA THROUGH 11-SEP-2020
Michigan (7 days): Cumulative Deaths per Million (Future, $\alpha = .05$)

$$R_0 = 2.0/1.2/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 9/9/11$$

DATA THROUGH 11-SEP-2020
Michigan (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = .05$)

Michigan
$R_0 = 2.0/1.2/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 9/9/11$

New York City
Italy
Robustness to Mortality Rate, $\delta$
Michigan: Cumulative Deaths per Million ($\delta = 0.01/0.008/0.012$)

Michigan

$R_0=2.0/1.2/1.1 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}=9/9/11$

DATA THROUGH 11-SEP-2020
Michigan: Daily Deaths per Million People ($\delta = .01/.008/.012$)

DATA THROUGH 11-SEP-2020

Michigan

$R_0 = 2.0/1.2/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 9/9/11
Michigan: Cumulative Deaths per Million ($\delta = 0.01/0.008/0.012$)

Michigan

$R_0 = 2.0/1.2/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad %\text{Infect} = 9/9/11$

DATA THROUGH 11-SEP-2020

Cumulative deaths per million people

Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan 2020
Reopening and Herd Immunity

– Black: assumes $R_0(\text{today})$ remains in place forever
– Red: assumes $R_0(\text{suppress}) = \frac{1}{s(\text{today})}$
– Green: we move 25% of the way from $R_0(\text{today})$ back to initial $R_0 = \text{“normal”}$
– Purple: we move 50% of the way from $R_0(\text{today})$ back to initial $R_0 = \text{“normal”}$

NOTE: Lines often cover each other up
Michigan: Re-Opening ($\alpha = .05$)

Michigan

$R_0(t)=1.2, \ R_0(\text{suppress})=1.1, \ R_0(25/50)=1.4/1.6, \ \delta = 0.010, \ \alpha=0.05$

(Light bars = New York City, for comparison)
Michigan: Re-Opening ($\alpha = 0$)

Michigan

$R_0(t)=1.2$, $R_0\text{(suppress)}=1.1$, $R_0(25/50)=1.4/1.6$, $\delta = 0.010$, $\alpha=0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Michigan (7 days): Daily Deaths per Million People ($\alpha = 0$)

Michigan

$R_0=2.0/1.2/1.2$  $\delta = 0.010$  $\alpha=0.00$  $\theta=0.1$  %Infect= 9/9/12

DATA THROUGH 11-SEP-2020
Michigan (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

Michigan

$R_0 = 2.0/1.2/1.2$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  $%\text{Infect} = 9/9/12$

DATA THROUGH 11-SEP-2020
Michigan (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Michigan

$R_0 = 2.0/1.2/1.2$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  $\%$Infect = 9/9/12
Michigan: Daily Deaths per Million People ($\delta = 0.8\%$)

Michigan

$R_0 = 2.0/1.2/1.1$  $\delta = 0.008$  $\theta = 0.1$  $\gamma = 0.2$  $\%$ Infect = 11/12/14
Michigan: Cumulative Deaths per Million ($\delta = 0.8\%$)

Michigan

$R_0=2.0/1.2/1.1$  $\delta = 0.008$  $\theta=0.1$  $\gamma=0.2$  $\%Infect=11/12/14$
Michigan: Daily Deaths per Million People ($\delta = 1.2\%$)

\[ R_0 = 2.0/1.1/1.1 \quad \delta = 0.012 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 7/8/9 \]
Michigan: Cumulative Deaths per Million ($\delta = 1.2\%$)

Michigan

$R_0 = 2.0/1.1/1.1 \quad \delta = 0.012 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 7/8/9$
Michigan: Daily Deaths per Million People ($\gamma = .2/.15$)

Michigan

$R_0 = 2.0/1.2/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$ Infect = 9/9/11

DATA THROUGH 11-SEP-2020
Michigan: Cumulative Deaths per Million $\gamma = .2/1.5$

$R_0 = 2.0/1.2/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 9/9/11$

DATA THROUGH 11-SEP-2020

$\gamma = 0.15$

$\gamma = 0.2$
Michigan: Daily Deaths per Million People \( (\theta = .1/.07/.2) \)

Michigan

\[ R_0 = 2.0/1.2/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 9/9/11 \]

DATA THROUGH 11-SEP-2020
Michigan: Cumulative Deaths per Million People ($\theta = .1 / .07 / .2$)

**Michigan**

$R_0=2.0/1.2/1.1$  \( \delta = 0.010 \)  \( \alpha = 0.05 \)  \( \theta = 0.1 \)  %Infect = 9/9/11

DATA THROUGH 11-SEP-2020
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Michigan: Daily Deaths, Actual and Smoothed

Michigan: Daily deaths, d

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Michigan: Change in Smoothed Daily Deaths

Michigan: Delta $d$

$\delta = 0.010  \ \theta = 0.10  \ \gamma = 0.20$
Michigan: Change in (Change in Smoothed Daily Deaths)

Michigan: Delta (Delta d)
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]