Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Mississippi
Based on data through October 9, 2020
Outline of Slides

• Basic data from Johns Hopkins CSSE (raw and smoothed)
• Brief summary of the model
• Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
• Simulation of re-opening – possibilities for raising $R_0$
• Results with alternative parameter values:
  o Lower mortality rate, $\delta = 0.8\%$
  o Higher mortality rate, $\delta = 1.2\%$
  o Infections last longer, $\gamma = 0.15$
  o Cases resolve more quickly, $\theta = 0.2$
  o Cases resolve more slowly, $\theta = 0.07$
• Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Mississippi: Daily Deaths per Million People

Mississippi
Brief Summary of Model

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Mississippi: Estimates of $R_0(t)$

Mississippi

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Mississippi: Percent Currently Infectious

Mississippi

Peak I/N = 0.55%  Final I/N = 0.20%  δ = 0.010  θ = 0.10  γ = 0.20

Percent currently infectious, I/N (percent)
Mississippi: Growth Rate of Daily Deaths over Past Week (percent)

Mississippi
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

• **Warning**: Results are often very uncertain; this can be seen by comparing across multiple graphs. See the [original paper](#).

• 7 days of forecasts: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  - Black = current
  - **Red** = oldest, **Orange** = second oldest, **Yellow** = third oldest...
  - **Violet** (purple) = one day earlier

• For robustness graphs, same idea
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - **Red** = lowest parameter value (e.g. $\delta = 0.8\%$)
  - **Green** = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:
  1. Alternatively, we fit this equation:

$$\log R_0(t) = a_0 - \alpha(Daily\ Deaths)$$

$$\Rightarrow \alpha \approx .05$$

$R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline

- Robustness: Assume $R_0(t) = \text{final empirical value}$. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Mississippi (7 days): Daily Deaths per Million People ($\alpha = .05$)

Mississippi

$R_0=1.5/1.0/1.1 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}=11/12/13$

DATA THROUGH 09-OCT-2020
Mississippi (7 days): Cumulative Deaths per Million (Future, $\alpha = 0.05$)

Mississippi

$R_0 = 1.5/1.0/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 11/12/13$

DATA THROUGH 09-OCT-2020

Cumulative deaths per million people
Mississippi (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0.05$)

- Cumulative deaths per million people

**Mississippi**

\[ R_0 = 1.5/1.0/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 11/12/13 

Graph showing cumulative deaths over time for Mississippi, New York City, and Italy.
Robustness to Mortality Rate, $\delta$
Mississippi: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

\[ R_0 = 1.5/1.0/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 11/12/13 \]

DATA THROUGH 09-OCT-2020
Mississippi: Daily Deaths per Million People ($\delta = .01/.008/.012$)

Mississippi

$R_0=1.5/1.0/1.1$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%Infect=11/12/13$

Data through 09-Oct-2020
Mississippi: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

DATA THROUGH 09-OCT-2020

Mississippi

$R_0 = 1.5/1.0/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 11/12/13$
Reopening and Herd Immunity

– **Black**: assumes $R_0(today)$ remains in place forever
– **Red**: assumes $R_0\text{(suppress)} = 1/s(today)$
– **Green**: we move 25% of the way from $R_0(today)$ back to initial $R_0 = \text{“normal”}$
– **Purple**: we move 50% of the way from $R_0(today)$ back to initial $R_0 = \text{“normal”}$

**NOTE**: Lines often cover each other up
Mississippi: Re-Opening ($\alpha = .05$)

Mississippi

$R_0(t)=1.0, \quad R_0(\text{suppress})=1.1, \quad R_0(25/50)=1.2/1.5, \quad \delta = 0.010, \quad \alpha=0.05$

(Light bars = New York City, for comparison)
Mississippi: Re-Opening ($\alpha = 0$)

Mississippi

$R_0(t)=0.9, \ R_0\text{ (suppress)}=1.1, \ R_0\text{ (25/50)}=1.2/1.5, \ \delta = 0.010, \ \alpha=0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Mississippi (7 days): Daily Deaths per Million People ($\alpha = 0$)

Mississippi

$R_0=1.5/0.9/0.9$  $\delta = 0.010$  $\alpha=0.00$  $\theta=0.1$  $\%\text{Infect}=11/12/12$

DATA THROUGH 09-OCT-2020
Mississippi (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

**Mississippi**

$R_0 = 1.5/0.9/0.9$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  $\%Infect = 11/12/12$

DATA THROUGH 09-OCT-2020
Mississippi (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Mississippi

$R_0 = 1.5/0.9/0.9$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  $\%$ Infect = 11/12/12
Mississippi: Daily Deaths per Million People ($\delta = 0.8\%$)

$R_0=1.5/1.0/1.2$  $\delta = 0.008$  $\theta=0.1$  $\gamma=0.2$  $\%$Infect$=14/15/16$
Mississippi: Cumulative Deaths per Million ($\delta = 0.8\%$)

Mississippi

$R_0=1.5/1.0/1.2$  $\delta = 0.008$  $\theta=0.1$  $\gamma=0.2$  $\%Infect=14/15/16$
Mississippi: Daily Deaths per Million People ($\delta = 1.2\%$)

Mississippi

$R_0 = 1.5/1.0/1.1$  $\delta = 0.012$  $\theta = 0.1$  $\gamma = 0.2$  $\%$ Infect = 9/10/11
Mississippi: Cumulative Deaths per Million ($\delta = 1.2\%$)

Mississippi

$R_0 = 1.5/1.0/1.1$  $\delta = 0.012$  $\theta = 0.1$  $\gamma = 0.2$  $\%\text{Infect} = 9/10/11$
Mississippi: Daily Deaths per Million People ($\gamma = 0.2/0.15$)

**Mississippi**

$R_0 = 1.5/1.0/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $%\text{Infect} = 11/12/13$

DATA THROUGH 09-OCT-2020
Mississippi: Cumulative Deaths per Million $\gamma = .2/ .15$)

**Mississippi**

$R_0 = 1.5/1.0/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$ Infect $= 11/12/13$

DATA THROUGH 09-OCT-2020

Cumulative deaths per million people

Mississippi: Daily Deaths per Million People ($\theta = .1/.07/.2$)

DATA THROUGH 09-OCT-2020

Mississippi

$R_0=1.5/1.0/1.1$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%Infect=11/12/13$
Mississippi: Cumulative Deaths per Million People ($\theta = .1/.07/.2$)

Mississippi
$R_0=1.5/1.0/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect}=11/12/13$

DATA THROUGH 09-OCT-2020
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Mississippi: Daily Deaths, Actual and Smoothed

Mississippi: Daily deaths, d
\(\delta = 0.010\)  \(\theta=0.10\)  \(\gamma=0.20\)
Mississippi: Change in Smoothed Daily Deaths

Mississippi: Delta $d$

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Mississippi: Change in (Change in Smoothed Daily Deaths)

Mississippi: Delta (Delta d)
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]