Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for North Carolina
Based on data through September 11, 2020
Outline of Slides

• Basic data from Johns Hopkins CSSE (raw and smoothed)
• Brief summary of the model
• Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
• Simulation of re-opening – possibilities for raising $R_0$
• Results with alternative parameter values:
  o Lower mortality rate, $\delta = 0.8\%$
  o Higher mortality rate, $\delta = 1.2\%$
  o Infections last longer, $\gamma = 0.15$
  o Cases resolve more quickly, $\theta = 0.2$
  o Cases resolve more slowly, $\theta = 0.07$
• Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
North Carolina: Daily Deaths per Million People

North Carolina
North Carolina: Daily Deaths per Million People (Smoothed)
Brief Summary of Model

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t \ (\beta_t/\gamma)$</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
North Carolina: Estimates of $R_0(t)$

North Carolina

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
North Carolina: Percent Currently Infectious

North Carolina
Peak I/N = 0.14%  Final I/N = 0.14%  δ = 0.010  θ = 0.10  γ = 0.20
North Carolina: Growth Rate of Daily Deaths over Past Week (percent)

North Carolina
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

• **Warning:** Results are often very uncertain; this can be seen by comparing across multiple graphs. See the [original paper](#).

• 7 days of forecasts: Rainbow color order!

  ROY-G-BIV (old to new, low to high)
  
  - Black = current
  - *Red* = oldest, *Orange* = second oldest, *Yellow* = third oldest...
  - *Violet* (purple) = one day earlier

• For robustness graphs, same idea
  
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - *Red* = lowest parameter value (e.g. $\delta = 0.8\%$)
  - *Green* = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:
  1. Alternatively, we fit this equation:

\[
\log R_0(t) = a_0 - \alpha(Daily\ Deaths)
\]

\[\Rightarrow \alpha \approx .05\]

$R_0$ declines by 5 percent for each new daily death,
or rises by 5 percent when daily deaths decline

- Robustness: Assume $R_0(t) =$ final empirical value. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
North Carolina (7 days): Daily Deaths per Million People ($\alpha = .05$)

North Carolina

$R_0 = 1.3/1.0/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%\text{Infect} = 3/4/6$

DATA THROUGH 11-SEP-2020
North Carolina (7 days): Cumulative Deaths per Million (Future, $\alpha = .05$)

North Carolina

$R_0=1.3/1.0/1.1$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%\text{Infect}=3/4/6$

DATA THROUGH 11-SEP-2020
North Carolina (7 days): Cumulative Deaths per Million, Log Scale ($\alpha =$)

North Carolina

$R_0 = 1.3/1.0/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%Infect = 3/4/6$

Cumulative deaths per million people

New York City

Italy

Robustness to Mortality Rate, $\delta$
North Carolina: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

North Carolina

$R_0=1.3/1.0/1.1$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect= 3/4/6

DATA THROUGH 11-SEP-2020
North Carolina: Daily Deaths per Million People ($\delta = .01 / .008 / .012$)

North Carolina

$R_0=1.3/1.0/1.1$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  \%Infect= 3/ 4/ 6

Data through 11-Sep-2020
North Carolina: Cumulative Deaths per Million ($\delta = 0.01/0.008/0.012$)

North Carolina

$R_0 = 1.3/1.0/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 3/4/6

DATA THROUGH 11-SEP-2020

Cumulative deaths per million people

Reopening and Herd Immunity

– Black: assumes $R_0(today)$ remains in place forever
– Red: assumes $R_0(suppress) = 1/s(today)$
– Green: we move 25% of the way from $R_0(today)$ back to initial $R_0 = “normal”$
– Purple: we move 50% of the way from $R_0(today)$ back to initial $R_0 = “normal”$

NOTE: Lines often cover each other up
North Carolina: Re-Opening ($\alpha = .05$)

North Carolina

$R_0(t)=1.0, \ R_0(\text{suppress})=1.0, \ R_0(25/50)=1.3/1.5, \ \delta = 0.010, \ \alpha=0.05$

(Light bars = New York City, for comparison)
North Carolina: Re-Opening ($\alpha = 0$)

North Carolina

$R_0(t)=1.1$, $R_0(\text{suppress})=1.0$, $R_0(25/50)=1.3/1.5$, $\delta = 0.010$, $\alpha=0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
North Carolina (7 days): Daily Deaths per Million People ($\alpha = 0$)

North Carolina

$R_0 = 1.3/1.1/1.1$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  $\%$Infect $= 3/4/7$

DATA THROUGH 11-SEP-2020
North Carolina (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

North Carolina

$R_0 = 1.3/1.1/1.1 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infect} = 3/4/7$

DATA THROUGH 11-SEP-2020
North Carolina (7 days): Cumulative Deaths per Million, Log Scale

North Carolina
\[ R_0 = 1.3/1.1/1.1 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%Infect = 3/4/7 \]
North Carolina: Daily Deaths per Million People ($\delta = 0.8\%$)

North Carolina

$R_0=1.3/1.1/1.1 \quad \delta = 0.008 \quad \theta=0.1 \quad \gamma=0.2 \quad \%\text{Infect}=4/5/8$
North Carolina: Cumulative Deaths per Million ($\delta = 0.8\%$)

North Carolina

$R_0 = 1.3/1.1/1.1$  $\delta = 0.008$  $\theta = 0.1$  $\gamma = 0.2$  $%Infect = 4/5/8$
North Carolina: Daily Deaths per Million People ($\delta = 1.2\%$)

North Carolina

$R_0 = 1.3/1.0/1.0 \quad \delta = 0.012 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 3/4/5$
North Carolina: Cumulative Deaths per Million ($\delta = 1.2\%$)

North Carolina

$R_0 = 1.3/1.0/1.0$  $\delta = 0.012$  $\theta = 0.1$  $\gamma = 0.2$  %Infect = 3/4/5
North Carolina: Daily Deaths per Million People ($\gamma = .2/ .15$)

North Carolina

\[ R_0 = 1.3/1.0/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 3/ 4/ 6 \]
North Carolina: Cumulative Deaths per Million $\gamma = .2/.15$)

North Carolina

$R_0=1.3/1.0/1.1 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%Infect= 3/4/6$

$\gamma = 0.15$

DATA THROUGH 11-SEP-2020
North Carolina: Daily Deaths per Million People ($\theta = .1/0.07/0.2$)

North Carolina

$R_0=1.3/1.0/1.1$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%Infect= 3/4/6$

DATA THROUGH 11-SEP-2020
North Carolina: Cumulative Deaths per Million People ($\theta = .1/.07/.2$)

North Carolina

$R_0=1.3/1.0/1.1 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}= 3/4/6$

DATA THROUGH 11-SEP-2020
Data Underlying Estimates of Time-Varying $R_0$

- Inferred from daily deaths, and
- the change in daily deaths, and
- the change in (the change in daily deaths)
North Carolina: Daily Deaths, Actual and Smoothed

North Carolina: Daily deaths, $d$

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
North Carolina: Change in Smoothed Daily Deaths

North Carolina: Delta d
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
North Carolina: Change in (Change in Smoothed Daily Deaths)

North Carolina: Delta (Δd)

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]