Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for North Dakota
Based on data through October 9, 2020
Outline of Slides

• Basic data from Johns Hopkins CSSE (raw and smoothed)
• Brief summary of the model
• Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
• Simulation of re-opening – possibilities for raising $R_0$
• Results with alternative parameter values:
  o Lower mortality rate, $\delta = 0.8\%$
  o Higher mortality rate, $\delta = 1.2\%$
  o Infections last longer, $\gamma = 0.15$
  o Cases resolve more quickly, $\theta = 0.2$
  o Cases resolve more slowly, $\theta = 0.07$
• Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
North Dakota: Daily Deaths per Million People

North Dakota

Daily deaths per million people
North Dakota: Daily Deaths per Million People (Smoothed)

Daily deaths per million people (smoothed)
**Brief Summary of Model**

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
North Dakota: Estimates of $R_0(t)$

North Dakota

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
North Dakota: Percent Currently Infectious

North Dakota
Peak I/N = 0.15%  Final I/N = 0.09%  δ = 0.010  θ = 0.10  γ = 0.20
North Dakota: Growth Rate of Daily Deaths over Past Week (percent)

North Dakota

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

• Warning: Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

• 7 days of forecasts: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  ○ Black = current
  ○ Red = oldest, Orange = second oldest, Yellow = third oldest...
  ○ Violet (purple) = one day earlier

• For robustness graphs, same idea
  ○ Black = baseline (e.g. $\delta = 1.0\%$)
  ○ Red = lowest parameter value (e.g. $\delta = 0.8\%$)
  ○ Green = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:
  1. Alternatively, we fit this equation:

\[
\log R_0(t) = a_0 - \alpha (\text{Daily Deaths})
\]

$\Rightarrow \alpha \approx .05$

*R_0* declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline.

- Robustness: Assume $R_0(t) =$ final empirical value. Constant in future, so no $\alpha$ adjustment $\Rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
North Dakota (7 days): Daily Deaths per Million People ($\alpha = .05$)

North Dakota

$R_0=1.3/0.2/0.2$ $\delta = 0.010$ $\alpha=0.05$ $\theta=0.1$ $\%$Infect= 1/ 1/ 1

DATA THROUGH 09-OCT-2020
North Dakota (7 days): Cumulative Deaths per Million (Future, $\alpha = 0.05$)

North Dakota
$R_0=1.3/0.2/0.2$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect= 1/ 1/ 1

DATA THROUGH 09-OCT-2020
North Dakota (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0.05$)

North Dakota

$R_0 = 1.3/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 1/1/1$

New York City

Italy
Robustness to Mortality Rate, $\delta$
North Dakota: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

North Dakota

$R_0=1.3/0.2/0.2 \, \delta = 0.010 \, \alpha = 0.05 \, \theta = 0.1 \, \%\text{Infect} = 1/1/1$

DATA THROUGH 09-OCT-2020
North Dakota: Daily Deaths per Million People ($\delta = .01/.008/.012$)

North Dakota

$R_0 = 1.3/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 1/1/1$

DATA THROUGH 09-OCT-2020
North Dakota: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

North Dakota

$R_0 = 1.3/0.2/0.2$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$ Infection = 1/1/1

DATA THROUGH 09-OCT-2020
Reopening and Herd Immunity

- **Black**: assumes $R_0(\text{today})$ remains in place forever
- **Red**: assumes $R_0(\text{suppress}) = 1/s(\text{today})$
- **Green**: we move 25% of the way from $R_0(\text{today})$ back to initial $R_0 = \text{“normal”}$
- **Purple**: we move 50% of the way from $R_0(\text{today})$ back to initial $R_0 = \text{“normal”}$

**NOTE:** Lines often cover each other up
North Dakota: Re-Opening ($\alpha = .05$)

North Dakota

$R_0(t) = 0.2$, $R_0(\text{suppress}) = 1.0$, $R_0(25/50) = 0.7/1.1$, $\delta = 0.010$, $\alpha = 0.05$

(Light bars = New York City, for comparison)
North Dakota: Re-Opening ($\alpha = 0$)

North Dakota

$R_0(t)=0.2, \ R_0(\text{suppress})=1.0, \ R_0(25/50)=0.7/1.1, \ \delta = 0.010, \ \alpha=0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
North Dakota (7 days): Daily Deaths per Million People ($\alpha = 0$)

North Dakota

$R_0 = 1.3/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infect} = 1/1/1$

DATA THROUGH 09-OCT-2020
North Dakota (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

North Dakota

$R_0=1.3/0.2/0.2 \quad \delta = 0.010 \quad \alpha=0.00 \quad \theta=0.1 \quad \%\text{Infect}=1/1/1$

DATA THROUGH 09-OCT-2020

Cumulative deaths per million people

North Dakota
North Dakota (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

North Dakota

$R_0 = 1.3/0.2/0.2$, $\delta = 0.010$, $\alpha = 0.00$, $\theta = 0.1$, $\%\text{Infect} = 1/1/1$

New York City

Italy

Cumulative deaths per million people

North Dakota: Daily Deaths per Million People ($\delta = 0.8\%$)

North Dakota

$R_0=1.3/0.2/0.2 \quad \delta = 0.008 \quad \theta=0.1 \quad \gamma=0.2 \quad \%\text{Infect} = 2/2/2$

SOME ERRORS IN ESTIMATION...
North Dakota: Cumulative Deaths per Million ($\delta = 0.8\%$)

North Dakota

$R_0=1.3/0.2/0.2$ \quad $\delta = 0.008$ \quad $\theta=0.1$ \quad $\gamma=0.2$ \quad %Infect= 2/ 2/ 2

SOME ERRORS IN ESTIMATION...
North Dakota: Daily Deaths per Million People ($\delta = 1.2\%$)

North Dakota

$R_0 = 1.3/0.2/0.2$  $\delta = 0.012$  $\theta = 0.1$  $\gamma = 0.2$  $\%$Infect = 1/1/1

SOME ERRORS IN ESTIMATION...
North Dakota: Cumulative Deaths per Million ($\delta = 1.2\%$)

North Dakota

$R_0 = 1.3/0.2/0.2$  $\delta = 0.012$  $\theta = 0.1$  $\gamma = 0.2$  $\%$Infect = 1/1/1

SOME ERRORS IN ESTIMATION...
North Dakota: Daily Deaths per Million People ($\gamma = 0.2/0.15$)

North Dakota

$R_0 = 1.3/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 1/1/1$

DATA THROUGH 09-OCT-2020
North Dakota: Cumulative Deaths per Million $\gamma = .2/.15$)

North Dakota

$R_0=1.3/0.2/0.2$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect= 1/1/1

DATA THROUGH 09-OCT-2020
North Dakota: Daily Deaths per Million People ($\theta = 0.1/0.07/0.2$)

North Dakota
$R_0 = 1.3/0.2/0.2$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 1/1/1

DATA THROUGH 09-OCT-2020
North Dakota: Cumulative Deaths per Million People ($\theta = .1/.07/.2$)

North Dakota
$R_0=1.3/0.2/0.2 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}=1/1/1$

DATA THROUGH 09-OCT-2020

Cumulative deaths per million people


$\theta = 0.2$

$\theta = 0.07$
Data Underlying Estimates of Time-Varying $R_0$

- Inferred from daily deaths, and
- the change in daily deaths, and
- the change in (the change in daily deaths)
North Dakota: Daily Deaths, Actual and Smoothed

North Dakota: Daily deaths, d
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
North Dakota: Change in Smoothed Daily Deaths

North Dakota: Delta $d$

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
North Dakota: Change in (Change in Smoothed Daily Deaths)

North Dakota: Delta (Delta d)
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]