Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Pennsylvania
Based on data through October 9, 2020
Outline of Slides

• Basic data from Johns Hopkins CSSE (raw and smoothed)
• Brief summary of the model
• Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
• Simulation of re-opening – possibilities for raising $R_0$
• Results with alternative parameter values:
  o Lower mortality rate, $\delta = 0.8\%$
  o Higher mortality rate, $\delta = 1.2\%$
  o Infections last longer, $\gamma = 0.15$
  o Cases resolve more quickly, $\theta = 0.2$
  o Cases resolve more slowly, $\theta = 0.07$
• Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Pennsylvania: Daily Deaths per Million People

Pennsylvania
**Brief Summary of Model**

- See the [paper](#) for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Pennsylvania: Estimates of $R_0(t)$

Pennsylvania

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Pennsylvania: Percent Currently Infectious

Pennsylvania
Peak I/N = 0.58%  Final I/N = 0.06%  δ = 0.010  θ = 0.10  γ = 0.20
Pennsylvania: Growth Rate of Daily Deaths over Past Week (percent)

Pennsylvania
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

• **Warning:** Results are often very uncertain; this can be seen by comparing across multiple graphs. See the *original paper*.

• 7 days of forecasts: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  - **Black** = current
  - **Red** = oldest, **Orange** = second oldest, **Yellow** = third oldest...
  - **Violet** (purple) = one day earlier

• For robustness graphs, same idea
  - **Black** = baseline (e.g. $\delta = 1.0\%$)
  - **Red** = lowest parameter value (e.g. $\delta = 0.8\%$)
  - **Green** = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:
  1. Alternatively, we fit this equation:

$$\log R_0(t) = a_0 - \alpha(Daily\ Deaths)$$

$$\Rightarrow \alpha \approx .05$$

$R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline.

- Robustness: Assume $R_0(t) =$ final empirical value. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Pennsylvania (7 days): Daily Deaths per Million People ($\alpha = .05$)

Pennsylvania

$R_0=1.8/1.1/1.1$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%Infect=7/7/9$

DATA THROUGH 09-OCT-2020
Pennsylvania (7 days): Cumulative Deaths per Million (Future, $\alpha = .05$)

Pennsylvania

$R_0 = 1.8/1.1/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 7/7/9$

DATA THROUGH 09-OCT-2020
Pennsylvania (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 1$)

Pennsylvania

$R_0 = 1.8/1.1/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 7/7/9
Robustness to Mortality Rate, $\delta$
Pennsylvania: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

\[ R_0 = 1.8/1.1/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 7/7/9 \]

DATA THROUGH 09-OCT-2020
Pennsylvania: Daily Deaths per Million People ($\delta = .01/.008/.012$)

Pennsylvania

$R_0 = 1.8/1.1/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 7/7/9

DATA THROUGH 09-OCT-2020
Pennsylvania: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

Data Through 09-Oct-2020

$R_0 = 1.8/1.1/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 7/7/9
Reopening and Herd Immunity

– Black: assumes $R_0$(today) remains in place forever
– Red: assumes $R_0$(suppress) = $1/s(today)$
– Green: we move 25% of the way from $R_0$(today) back to initial $R_0$ = “normal”
– Purple: we move 50% of the way from $R_0$(today) back to initial $R_0$ = “normal”

NOTE: Lines often cover each other up
Pennsylvania: Re-Opening ($\alpha = .05$)

Pennsylvania
$R_0(t)=1.1, \ R_0\ (\text{suppress})=1.1, \ R_0\ (25/50)=1.4/1.6, \ \delta = 0.010, \ \alpha=0.05$

(Light bars = New York City, for comparison)
Pennsylvania: Re-Opening ($\alpha = 0$)

Pennsylvania

$R_0(t)=1.1$, $R_0(\text{suppress})=1.1$, $R_0(25/50)=1.4/1.6$, $\delta = 0.010$, $\alpha=0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Pennsylvania (7 days): Daily Deaths per Million People ($\alpha = 0$)

Pennsylvania

$R_0=1.8/1.1/1.1$  $\delta = 0.010$  $\alpha=0.00$  $\theta=0.1$  $\%$Infect= 7/7/10

DATA THROUGH 09-OCT-2020
Pennsylvania (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

$R_0 = 1.8/1.1/1.1$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  $\%$ Infect = 7/7/10

DATA THROUGH 09-OCT-2020
Pennsylvania (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Pennsylvania

$R_0 = 1.8/1.1/1.1$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  $\%$Infect$ = 7/7/10$

Cumulative deaths per million people

Pennsylvania: Daily Deaths per Million People ($\delta = 0.8\%$)

Pennsylvania

$R_0=1.8/1.2/1.1$  $\delta = 0.008$  $\theta=0.1$  $\gamma=0.2$  %Infect= 8/ 9/11
Pennsylvania: Cumulative Deaths per Million ($\delta = 0.8\%$)

Pennsylvania

$R_0 = 1.8/1.2/1.1 \quad \delta = 0.008 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 8/9/11$
Pennsylvania: Daily Deaths per Million People ($\delta = 1.2\%$)

Pennsylvania

$R_0=1.8/1.1/1.1$  $\delta = 0.012$  $\theta=0.1$  $\gamma=0.2$  $\%\text{Infect}=6/6/8$
Pennsylvania: Cumulative Deaths per Million ($\delta = 1.2\%$)

Pennsylvania

$R_0 = 1.8/1.1/1.1$  \hspace{1em} $\delta = 0.012$  \hspace{1em} $\theta = 0.1$  \hspace{1em} $\gamma = 0.2$  \hspace{1em} $%\text{Infect} = 6/6/8$
Pennsylvania: Daily Deaths per Million People ($\gamma = .2/.15$)

Pennsylvania

$R_0 = 1.8/1.1/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%Infect = 7/7/9$

DATA THROUGH 09-OCT-2020
Pennsylvania: Cumulative Deaths per Million $\gamma = .2/.15$)

Pennsylvania

$R_0=1.8/1.1/1.1$ $\delta = 0.010$ $\alpha=0.05$ $\theta=0.1$ $\%$Infect= 7/7/9

\[ \gamma \equiv 0.15 \]

DATA THROUGH 09-OCT-2020
Pennsylvania: Daily Deaths per Million People ($\theta = .1/.07/.2$)

Pennsylvania

$R_0=1.8/1.1/1.1 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect} = 7/7/9$

DATA THROUGH 09-OCT-2020
Pennsylvania: Cumulative Deaths per Million People ($\theta = .1/.07/.2$)

Pennsylvania
$R_0=1.8/1.1/1.1$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%\text{Infect}=7/7/9$

DATA THROUGH 09-OCT-2020
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Pennsylvania: Daily Deaths, Actual and Smoothed

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Pennsylvania: Change in Smoothed Daily Deaths

Pennsylvania: Delta d
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Pennsylvania: Change in (Change in Smoothed Daily Deaths)

Pennsylvania: Delta (Delta d)

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]