Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

Jesús Fernández-Villaverde and Chad Jones

Extended results for Paris, France
Based on data through September 11, 2020
Outline of Slides

• Basic data from Johns Hopkins CSSE (raw and smoothed)
• Brief summary of the model
• Baseline results \((\delta = 1.0\%, \gamma = 0.2, \theta = 0.1)\)
• Simulation of re-opening – possibilities for raising \(R_0\)
• Results with alternative parameter values:
  o Lower mortality rate, \(\delta = 0.8\%\)
  o Higher mortality rate, \(\delta = 1.2\%\)
  o Infections last longer, \(\gamma = 0.15\)
  o Cases resolve more quickly, \(\theta = 0.2\)
  o Cases resolve more slowly, \(\theta = 0.07\)
• Data underlying estimates of \(R_0(t)\)
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Paris, France: Daily Deaths per Million People

Paris, France

Daily deaths per million people

Apr May Jun Jul Aug Sep
0 5 10 15 20 25 30 35

2020
Paris, France: Daily Deaths per Million People (Smoothed)
Brief Summary of Model

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t / \gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Paris, France: Estimates of $R_0(t)$

Paris, France

$\delta = 0.010$  $\theta = 0.10$  $\gamma = 0.20$
Paris, France: Percent Currently Infectious

Paris, France
Peak I/N = 1.33%  Final I/N = 0.04%  δ = 0.010  θ = 0.10  γ = 0.20
Paris, France: Growth Rate of Daily Deaths over Past Week (percent)

Paris, France

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

• **Warning:** Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

• 7 days of forecasts: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  - Black = current
  - Red = oldest, Orange = second oldest, Yellow = third oldest...
  - Violet (purple) = one day earlier

• For robustness graphs, same idea
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - Red = lowest parameter value (e.g. $\delta = 0.8\%$)
  - Green = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:
  
  1. Alternatively, we fit this equation:

     \[
     \log R_0(t) = a_0 - \alpha(Daily\ Deaths)
     \]

     \[\Rightarrow \alpha \approx 0.05\]

     $R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline

- Robustness: Assume $R_0(t) =$ final empirical value. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Paris, France (7 days): Daily Deaths per Million People ($\alpha = 0.05$)

Paris, France

$R_0 = 2.1/1.2/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 9/9/10$

DATA THROUGH 11-SEP-2020
Paris, France (7 days): Cumulative Deaths per Million (Future, $\alpha = 0.05$)

Paris, France

$R_0 = 2.1/1.2/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$Infect = 9/9/10

DATA THROUGH 11-SEP-2020
Paris, France (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = \ldots$)

Paris, France
$R_0=2.1/1.2/1.1$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%\text{Infect}=9/9/10$
Robustness to Mortality Rate, $\delta$
Paris, France: Cumulative Deaths per Million ($\delta = .01/0.008/0.012$)

Paris, France

$R_0 = 2.1/1.2/1.1$  \( \delta = 0.010 \)  \( \alpha = 0.05 \)  \( \theta = 0.1 \)  \%Infect\% = 9/9/10

DATA THROUGH 11-SEP-2020
Paris, France: Daily Deaths per Million People ($\delta = .01/.008/.012$)

Paris, France

$R_0 = 2.1/1.2/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 9/9/10$

DATA THROUGH 11-SEP-2020
Paris, France: Cumulative Deaths per Million ($\delta = 0.01/0.008/0.012$)

Paris, France
$R_0 = 2.1/1.2/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 9/9/10

DATA THROUGH 11-SEP-2020

Cumulative deaths per million people

0 100 200 300 400 500 600 700 800 900 1000
Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan 2020

$\delta = 0.008$
Reopening and Herd Immunity

- **Black**: assumes $R_0(today)$ remains in place forever
- **Red**: assumes $R_0(suppress) = 1/s(today)$
- **Green**: we move 25% of the way from $R_0(today)$ back to initial $R_0 = “normal”$
- **Purple**: we move 50% of the way from $R_0(today)$ back to initial $R_0 = “normal”$

**NOTE**: Lines often cover each other up
Paris, France: Re-Opening ($\alpha = .05$)

Paris, France

$R_0(t)=1.2$, $R_0(\text{suppress})=1.1$, $R_0(25/50)=1.4/1.6$, $\delta = 0.010$, $\alpha=0.05$

(Light bars = New York City, for comparison)
Paris, France: Re-Opening ($\alpha = 0$)

Paris, France

$R_0(t)=1.2$, $R_0$ (suppress) = 1.1, $R_0 (25/50)=1.4/1.6$, $\delta = 0.010$, $\alpha=0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Paris, France (7 days): Daily Deaths per Million People ($\alpha = 0$)

Paris, France

$R_0 = 2.1/1.2/1.2$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  $\%Infect = 9/9/10$

DATA THROUGH 11-SEP-2020
Paris, France (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

Paris, France

$R_0 = 2.1/1.2/1.2$, $\delta = 0.010$, $\alpha = 0.00$, $\theta = 0.1$, $\%$Infect = 9/9/10

DATA THROUGH 11-SEP-2020
Paris, France (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Paris, France
$R_0=2.1/1.2/1.2 \ \delta = 0.010 \ \alpha=0.00 \ \theta=0.1 \ \%\text{Infect}=9/9/10$

Cumulative deaths per million people

New York City
Italy
Paris, France: Daily Deaths per Million People ($\delta = 0.8\%$)

Paris, France

$R_0 = 2.1/1.2/1.1 \quad \delta = 0.008 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 11/11/12$
Paris, France: Cumulative Deaths per Million (δ = 0.8%)
Paris, France: Daily Deaths per Million People \( (\delta = 1.2\%) \)

\[
R_0 = 2.1/1.1/1.1 \quad \delta = 0.012 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 7/7/8
\]
Paris, France: Cumulative Deaths per Million ($\delta = 1.2\%$)

Paris, France

$R_0 = 2.1/1.1/1.1$  $\delta = 0.012$  $\theta = 0.1$  $\gamma = 0.2$  $%\text{Infect} = 7/7/8$
Paris, France: Daily Deaths per Million People ($\gamma = .2 / .15$)

Paris, France

$R_0 = 2.1 / 1.2 / 1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$ Infect = 9 / 9 / 10

DATA THROUGH 11-SEP-2020

Daily deaths per million people
Paris, France: Cumulative Deaths per Million $\gamma = .2 / .15$.

Paris, France
$R_0 = 2.1/1.2/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$Infect = 9/9/10

DATA THROUGH 11-SEP-2020
Paris, France: Daily Deaths per Million People \((\theta = .1/.07/.2)\)

Paris, France

\[R_0=2.1/1.2/1.1 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}= 9/9/10\]

DATA THROUGH 11-SEP-2020
Paris, France: Cumulative Deaths per Million People ($\theta = .1/.07/.2$)

Paris, France

$R_0=2.1/1.2/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 9/9/10

DATA THROUGH 11-SEP-2020
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Paris, France: Daily Deaths, Actual and Smoothed

Paris, France: Daily deaths, d
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Paris, France: Change in Smoothed Daily Deaths

Paris, France: Delta $d$

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Paris, France: Change in (Change in Smoothed Daily Deaths)

Paris, France: Delta (Delta d)
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]