Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Philadelphia
Based on data through August 24, 2020
Outline of Slides

• Basic data from Johns Hopkins CSSE (raw and smoothed)
• Brief summary of the model
• Baseline results \((\delta = 1.0\%, \gamma = 0.2, \theta = 0.1)\)
• Simulation of re-opening – possibilities for raising \(R_0\)
• Results with alternative parameter values:
  o Lower mortality rate, \(\bar{\delta} = 0.8\%\)
  o Higher mortality rate, \(\bar{\delta} = 1.2\%\)
  o Infections last longer, \(\gamma = 0.15\)
  o Cases resolve more quickly, \(\theta = 0.2\)
  o Cases resolve more slowly, \(\theta = 0.07\)
• Data underlying estimates of \(R_0(t)\)
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Philadelphia: Daily Deaths per Million People

Philadelphia

Daily deaths per million people

Apr May Jun Jul Aug 2020
Brief Summary of Model

• See the paper for a full exposition

• A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

- Inferred from daily deaths, and
- the change in daily deaths, and
- the change in (the change in daily deaths)
(see end of slide deck for this data)
Philadelphia: Estimates of $R_0(t)$

Philadelphia

$\delta = 0.010 \quad \theta=0.10 \quad \gamma=0.20$
Philadelphia: Percent Currently Infectious

Philadelphia

Peak I/N = 1.04%  Final I/N = 0.11%  \( \delta = 0.010 \)  \( \theta = 0.10 \)  \( \gamma = 0.20 \)
Philadelphia: Growth Rate of Daily Deaths over Past Week (percent)

Philadelphia

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

• **Warning:** Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

• 7 days of forecasts: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  - Black = current
  - **Red** = oldest, **Orange** = second oldest, **Yellow** = third oldest...
  - **Violet (purple)** = one day earlier

• For robustness graphs, same idea
  - Black = baseline (e.g. \( \delta = 1.0\% \))
  - **Red** = lowest parameter value (e.g. \( \delta = 0.8\% \))
  - **Green** = highest parameter value (e.g. \( \delta = 1.2\% \))
How does $R_0$ change over time?

- Inferred from death data when we have it

- For future, two approaches:
  
  1. Alternatively, we fit this equation:

\[
\log R_0(t) = a_0 - \alpha(Daily\ Deaths)
\]

  \[\Rightarrow \alpha \approx .05\]

  $R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline

- Robustness: Assume $R_0(t) =$ final empirical value. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Philadelphia (7 days): Daily Deaths per Million People ($\alpha = 0.05$)

Philadelphia

$R_0 = 2.1/1.2/1.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \% \text{Infect} = 11/12/15$

DATA THROUGH 24-AUG-2020

Daily deaths per million people

Apr May Jun Jul Aug Sep Oct Nov Dec Jan 2020
Philadelphia (7 days): Cumulative Deaths per Million (Future, $\alpha = .05$)

Philadelphia

$R_0 = 2.1/1.2/1.2$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 11/12/15

DATA THROUGH 24-AUG-2020
Philadelphia (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0.05$)

Philadelphia

$R_0 = 2.1/1.2/1.2\;\delta = 0.010\;\alpha = 0.05\;\theta = 0.1\;\%\text{Infect} = 11/12/15$
Robustness to Mortality Rate, $\delta$
Philadelphia: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

Philadelphia

$R_0 = 2.1/1.2/1.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 11/12/15$

DATA THROUGH 24-AUG-2020
Philadelphia: Daily Deaths per Million People ($\delta = .01/ .008/ .012$)

Philadelphia

$R_0 = 2.1/1.2/1.2$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$ Infect $= 11/12/15$

DATA THROUGH 24-AUG-2020
Philadelphia: Cumulative Deaths per Million ($\delta = 0.01/0.008/0.012$)

Philadelphia

$R_0 = 2.1/1.2/1.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \% \text{Infect} = 11/12/15$

DATA THROUGH 24-AUG-2020
Reopening and Herd Immunity

– Black: assumes $R_0(\text{today})$ remains in place forever
– Red: assumes $R_0(\text{suppress}) = 1/s(\text{today})$
– Green: we move 25% of the way from $R_0(\text{today})$ back to initial $R_0 = \text{“normal”}$
– Purple: we move 50% of the way from $R_0(\text{today})$ back to initial $R_0 = \text{“normal”}$

NOTE: Lines often cover each other up
Philadelphia: Re-Opening ($\alpha = .05$)

Philadelphia

$R_0(t)=1.2$, $R_0(\text{suppress})=1.1$, $R_0(25/50)=1.4/1.6$, $\delta = 0.010$, $\alpha=0.05$
Philadelphia: Re-Opening ($\alpha = 0$)

Philadelphia

$R_0(t)=1.3$, $R_0(\text{suppress})=1.1$, $R_0(25/50)=1.5/1.7$, $\delta = 0.010$, $\alpha=0.00$
Results for alternative parameter values
Philadelphia (7 days): Daily Deaths per Million People ($\alpha = 0$)

Philadelphia

$R_0 = 2.1/1.3/1.3 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infect} = 11/13/21$

DATA THROUGH 24-AUG-2020
Philadelphia (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

Philadelphia

$R_0 = 2.1/1.3/1.3 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infect}=11/13/21$

DATA THROUGH 24-AUG-2020
Philadelphia (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Philadelphia

$R_0=2.1/1.3/1.3 \quad \delta = 0.010 \quad \alpha=0.00 \quad \theta=0.1 \quad \%\text{Infect}=11/13/21$

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New York City

Italy
Philadelphia: Daily Deaths per Million People ($\delta = 0.8\%$)

Philadelphia

$R_0=2.1/1.3/1.2$  $\delta = 0.008$  $\theta=0.1$  $\gamma=0.2$  $\%$Infect=14/15/18
Philadelphia: Cumulative Deaths per Million ($\delta = 0.8\%$)

Philadelphia

$R_0 = 2.1/1.3/1.2$  $\delta = 0.008$  $\theta = 0.1$  $\gamma = 0.2$  %Infect = 14/15/18
Philadelphia: Daily Deaths per Million People ($\delta = 1.2\%$)

Philadelphia

$R_0 = 2.1/1.2/1.1$  \  $\delta = 0.012$  \  $\theta = 0.1$  \  $\gamma = 0.2$  \  $\%\text{Infect} = 9/10/12$
Philadelphia: Cumulative Deaths per Million ($\delta = 1.2\%$)

Philadelphia

$R_0=2.1/1.2/1.1$  $\delta = 0.012$  $\theta=0.1$  $\gamma=0.2$  %Infect= 9/10/12
Philadelphia: Daily Deaths per Million People ($\gamma = .2/.15$)

Philadelphia

$R_0 = 2.1/1.2/1.2$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$Infect = 11/12/15

DATA THROUGH 24-AUG-2020
Philadelphia: Cumulative Deaths per Million $\gamma = .2/.15$)

Philadelphia

$R_0=2.1/1.2/1.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect}=11/12/15$

$\gamma = 0.15 \quad \gamma = 0.2$

DATA THROUGH 24-AUG-2020
Philadelphia: Daily Deaths per Million People ($\theta = .1/\cdot .07/\cdot .2$)

Philadelphia

$R_0=2.1/1.2/1.2 \: \delta = 0.010 \: \alpha=0.05 \: \theta=0.1 \: \%\text{Infect}=11/12/15$

DATA THROUGH 24-AUG-2020
Philadelphia: Cumulative Deaths per Million People ($\theta = .1/.07/.2$)

Philadelphia

$R_0 = 2.1/1.2/1.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect}=11/12/15$

DATA THROUGH 24-AUG-2020

Cumulative deaths per million people
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Philadelphia: Daily Deaths, Actual and Smoothed

Philadelphia: Daily deaths, d
\( \delta = 0.010 \ \ \theta = 0.10 \ \ \gamma = 0.20 \)
Philadelphia: Change in Smoothed Daily Deaths

Philadelphia: Delta d
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Philadelphia: Change in (Change in Smoothed Daily Deaths)

Philadelphia: Delta (Delta d)
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]