Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Philadelphia
Based on data through September 11, 2020
Outline of Slides

- Basic data from Johns Hopkins CSSE (raw and smoothed)
- Brief summary of the model
- Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
- Simulation of re-opening – possibilities for raising $R_0$
- Results with alternative parameter values:
  - Lower mortality rate, $\delta = 0.8\%$
  - Higher mortality rate, $\delta = 1.2\%$
  - Infections last longer, $\gamma = 0.15$
  - Cases resolve more quickly, $\theta = 0.2$
  - Cases resolve more slowly, $\theta = 0.07$
- Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Philadelphia: Daily Deaths per Million People

Philadelphia
Philadelphia: Daily Deaths per Million People (Smoothed)
Brief Summary of Model

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Philadelphia: Estimates of $R_0(t)$

Philadelphia

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Philadelphia: Percent Currently Infectious

Philadelphia

Peak I/N = 1.04%  Final I/N = 0.03%  δ = 0.010  θ = 0.10  γ = 0.20
Philadelphia: Growth Rate of Daily Deaths over Past Week (percent)

Philadelphia

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Notes on Interpreting Results
Guide to Graphs

• **Warning:** Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

• 7 days of forecasts: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  
  - **Black** = current
  
  - **Red** = oldest, **Orange** = second oldest, **Yellow** = third oldest...
  
  - **Violet** (purple) = one day earlier

• For robustness graphs, same idea
  
  - **Black** = baseline (e.g. \( \delta = 1.0\% \))
  
  - **Red** = lowest parameter value (e.g. \( \delta = 0.8\% \))
  
  - **Green** = highest parameter value (e.g. \( \delta = 1.2\% \))
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:
  
  1. Alternatively, we fit this equation:

     $\log R_0(t) = a_0 - \alpha (\text{Daily Deaths})$

     $\Rightarrow \alpha \approx .05$

     $R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline

- Robustness: Assume $R_0(t) = \text{final empirical value}$. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Philadelphia (7 days): Daily Deaths per Million People ($\alpha = .05$)

Philadelphia

$R_0=2.1/0.6/0.6 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}=11/11/11$

DATA THROUGH 11-SEP-2020
Philadelphia (7 days): Cumulative Deaths per Million (Future, $\alpha = .05$)

Philadelphia

$R_0 = 2.1/0.6/0.6$, $\delta = 0.010$, $\alpha = 0.05$, $\theta = 0.1$, $\%$Infect$=11/11/11$

DATA THROUGH 11-SEP-2020
Philadelphia (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0.05$)

Philadelphia

$R_0 = 2.1/0.6/0.6$, $\delta = 0.010$, $\alpha = 0.05$, $\theta = 0.1$, $\%$Infect = 11/11/11
Robustness to Mortality Rate, $\delta$
Philadelphia: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

Philadelphia

$R_0=2.1/0.6/0.6$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect=11/11/11

DATA THROUGH 11-SEP-2020
Philadelphia: Daily Deaths per Million People ($\delta = .01/.008/.012$)

<table>
<thead>
<tr>
<th>Month</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
<th>Jan</th>
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</thead>
<tbody>
<tr>
<td>Daily deaths per million people</td>
<td></td>
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</table>

Philadelphia

$R_0 = 2.1/0.6/0.6$, $\delta = 0.010$, $\alpha = 0.05$, $\theta = 0.1$, %Infect = 11/11/11

DATA THROUGH 11-SEP-2020
Philadelphia: Cumulative Deaths per Million ($\delta = 0.01/0.008/0.012$)

Philadelphia

$R_0 = 2.1/0.6/0.6$  \( \delta = 0.010 \)  \( \alpha = 0.05 \)  \( \theta = 0.1 \)  \%Infect = 11/11/11

DATA THROUGH 11-SEP-2020
Reopening and Herd Immunity

– **Black**: assumes $R_0^{(today)}$ remains in place forever
– **Red**: assumes $R_0^{(suppress)} = 1/s^{(today)}$
– **Green**: we move 25% of the way from $R_0^{(today)}$ back to initial $R_0 = “normal”$
– **Purple**: we move 50% of the way from $R_0^{(today)}$ back to initial $R_0 = “normal”$

**NOTE**: Lines often cover each other up
Philadelphia: Re-Opening ($\alpha = 0.05$)

Philadelphia

$R_0(t)=0.6$, $R_0(\text{suppress})=1.1$, $R_0(25/50)=1.0/1.3$, $\delta = 0.010$, $\alpha=0.05$

(Light bars = New York City, for comparison)
Philadelphia: Re-Opening ($\alpha = 0$)

Philadelphia

$R_0(t)=0.6, \ R_0\text{ (suppress)}=1.1, \ R_0(25/50)=1.0/1.3, \ \delta = 0.010, \ \alpha=0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Philadelphia (7 days): Daily Deaths per Million People ($\alpha = 0$)

Philadelphia

$R_0=2.1/0.6/0.6 \quad \delta = 0.010 \quad \alpha=0.00 \quad \theta=0.1 \quad \%\text{Infect}=11/11/11$

DATA THROUGH 11-SEP-2020
Philadelphia (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

Philadelphia

$R_0=2.1/0.6/0.6$  $\delta = 0.010$  $\alpha=0.00$  $\theta=0.1$  $%\text{Infect}=11/11/11$

DATA THROUGH 11-SEP-2020

Cumulative deaths per million people

Mar  Apr  May  Jun  Jul  Aug  Sep  Oct  Nov  Dec  Jan  2020
Philadelphia (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

New York City

Italy

Philadelphia

$R_0=2.1/0.6/0.6 \quad \delta = 0.010 \quad \alpha=0.00 \quad \theta=0.1 \quad \%\text{Infect}=11/11/11$

Cumulative deaths per million people

Cumulative deaths per million people

Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan 2020
Philadelphia: Daily Deaths per Million People ($\delta = 0.8\%$)

Philadelphia

$R_0 = 2.1/0.6/0.7 \quad \delta = 0.008 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 14/14/14$
Philadelphia: Cumulative Deaths per Million ($\delta = 0.8\%$)

Philadelphia
$R_0=2.1/0.6/0.7$ $\delta = 0.008$ $\theta=0.1$ $\gamma=0.2$ $\%$Infect$=14/14/14$
Philadelphia: Daily Deaths per Million People ($\delta = 1.2\%$)

Philadelphia

$R_0 = 2.1/0.6/0.6 \quad \delta = 0.012 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 9/9/9$
Philadelphia: Cumulative Deaths per Million ($\delta = 1.2\%$)

Philadelphia

$R_0 = 2.1/0.6/0.6$  $\delta = 0.012$  $\theta = 0.1$  $\gamma = 0.2$  $%$Infect = 9/9/9
Philadelphia: Daily Deaths per Million People \( (\gamma = .2 / .15) \)

\[ R_0 = 2.1/0.6/0.6 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \% \text{Infect} = 11/11/11 \]

DATA THROUGH 11-SEP-2020
Philadelphia: Cumulative Deaths per Million $\gamma = .2/ .15$)

Philadelphia

$R_0 = 2.1/ 0.6/ 0.6 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 11/11/11$

$\gamma = 0.25$
Philadelphia: Daily Deaths per Million People ($\theta = .1/.07/.2$)

Philadelphia

$R_0=2.1/0.6/0.6$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect=11/11/11

DATA THROUGH 11-SEP-2020
Philadelphia: Cumulative Deaths per Million People ($\theta = .1/.07/.2$)

Philadelphia

$R_0=2.1/0.6/0.6$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%$ Infect=11/11/11

DATA THROUGH 11-SEP-2020

$\theta = 0.07$
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Philadelphia: Daily Deaths, Actual and Smoothed

Philadelphia: Daily deaths, $d$

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Philadelphia: Change in Smoothed Daily Deaths

Philadelphia: Delta $d$

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Philadelphia: Change in (Change in Smoothed Daily Deaths)

Philadelphia: Delta (Delta d)
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]