Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Eswatini
Based on data through August 24, 2020
Outline of Slides

- Basic data from Johns Hopkins CSSE (raw and smoothed)
- Brief summary of the model
- Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
- Simulation of re-opening – possibilities for raising $R_0$
- Results with alternative parameter values:
  - Lower mortality rate, $\delta = 0.8\%$
  - Higher mortality rate, $\delta = 1.2\%$
  - Infections last longer, $\gamma = 0.15$
  - Cases resolve more quickly, $\theta = 0.2$
  - Cases resolve more slowly, $\theta = 0.07$
- Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
### Brief Summary of Model

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Eswatini: Estimates of $R_0(t)$

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Eswatini: Percent Currently Infectious

Eswatini

Peak I/N = 0.09%  Final I/N = 0.09%  δ = 0.010  θ = 0.10  γ = 0.20
Eswatini: Growth Rate of Daily Deaths over Past Week (percent)

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

- **Warning**: Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

- 7 days of forecasts: Rainbow color order!
  - ROY-G-BIV (old to new, low to high)
    - Black = current
    - Red = oldest, Orange = second oldest, Yellow = third oldest...
    - Violet (purple) = one day earlier

- For robustness graphs, same idea
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - Red = lowest parameter value (e.g. $\delta = 0.8\%$)
  - Green = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:
  
  1. Alternatively, we fit this equation:

$$\log R_0(t) = a_0 - \alpha(Daily \ Deaths)$$

  \[\Rightarrow \alpha \approx .05\]

  $R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline

- Robustness: Assume $R_0(t) = \text{final empirical value}$. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Eswatini (7 days): Daily Deaths per Million People ($\alpha = .05$)

Eswatini

$R_0 = 1.1/1.0/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 1/1/5$

DATA THROUGH 24-AUG-2020
Eswatini (7 days): Cumulative Deaths per Million (Future, $\alpha = .05$)

R$_0$=1.1/1.0/1.0 $\delta = 0.010$ $\alpha=0.05$ $\theta=0.1$ %Infect= 1/1/5

DATA THROUGH 24-AUG-2020
Eswatini (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = .05$)

\[ R_0 = 1.1/1.0/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 1/1/5 \]
Robustness to Mortality Rate, $\delta$
Eswatini: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

DATA THROUGH 24-AUG-2020

Eswatini

$R_0=1.1/1.0/1.0$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $%Infect= 1/ 1/ 5$

Cumulative deaths per million people


2020
Eswatini: Daily Deaths per Million People ($\delta = .01/.008/.012$)

$R_0=1.1/1.0/1.0 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%Infect= 1/1/5$

DATA THROUGH 24-AUG-2020
Eswatini: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

Eswatini

$R_0=1.1/1.0/1.0$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%Infect=1/1/5$

DATA THROUGH 24-AUG-2020
Reopening and Herd Immunity

– Black: assumes $R_0(\text{today})$ remains in place forever
– Red: assumes $R_0(\text{suppress}) = 1/s(\text{today})$
– Green: we move 25% of the way from $R_0(\text{today})$
  back to initial $R_0 = “\text{normal}”$
– Purple: we move 50% of the way from $R_0(\text{today})$
  back to initial $R_0 = “\text{normal}”$

NOTE: Lines often cover each other up
Eswatini: Re-Opening ($\alpha = 0.05$)

Eswatini

$R_0(t) = 1.0$, $R_0(\text{suppress}) = 1.0$, $R_0(25/50) = 1.3/1.5$, $\delta = 0.010$, $\alpha = 0.05$
Eswatini: Re-Opening ($\alpha = 0$)

Eswatini

$R_0(t) = 1.0$, $R_0\text{(suppress)} = 1.0$, $R_0\text{(25/50)} = 1.3/1.5$, $\delta = 0.010$, $\alpha = 0.00$
Results for alternative parameter values
Eswatini (7 days): Daily Deaths per Million People ($\alpha = 0$)

Eswatini

$R_0 = 1.1/1.0/1.0$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  %Infect $= 1/1/5$

DATA THROUGH 24-AUG-2020
Eswatini (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

Eswatini

$R_0=1.1/1.0/1.0$ $\delta = 0.010$ $\alpha=0.00$ $\theta=0.1$ $%\text{Infect}= 1/1/5$

DATA THROUGH 24-AUG-2020
Eswatini (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

$R_0 = 1.1/1.0/1.0$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  %Infect = 1/1/5
Eswatini: Daily Deaths per Million People ($\delta = 0.8\%$)

Eswatini

$R_0=1.1/1.0/1.0$ $\delta = 0.008$ $\theta=0.1$ $\gamma=0.2$ $\%$ Infect $= 1/2/6$
Eswatini: Cumulative Deaths per Million ($\delta = 0.8\%$)

Eswatini

$R_0 = 1.1/1.0/1.0$, $\delta = 0.008$, $\theta = 0.1$, $\gamma = 0.2$, %Infect = 1/2/6
Eswatini: Daily Deaths per Million People ($\delta = 1.2\%$)

Eswatini

$R_0=1.1/1.0/1.0$  $\delta = 0.012$  $\theta=0.1$  $\gamma=0.2$  $\%\text{Infect}=1/1/4$
Eswatini: Cumulative Deaths per Million ($\delta = 1.2\%$)

Eswatini

$R_0=1.1/1.0/1.0$  $\delta = 0.012$  $\theta=0.1$  $\gamma=0.2$  $\%$Infect $= 1/1/4$
Eswatini: Daily Deaths per Million People ($\gamma = 0.2/0.15$)

Eswatini

$R_0 = 1.1/1.0/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%Infect = 1/1/5$

DATA THROUGH 24-AUG-2020
Eswatini: Cumulative Deaths per Million $\gamma = .2/.15$)

Eswatini

$R_0 = 1.1/1.0/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 1/1/5$

DATA THROUGH 24-AUG-2020

Cumulative deaths per million people


$\gamma = 0.15$

$\gamma = 0.2$
Eswatini: Daily Deaths per Million People ($\theta = .1/.07/.2$)

Eswatini

$R_0 = 1.1/1.0/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 1/1/5$

DATA THROUGH 24-AUG-2020
Eswatini: Cumulative Deaths per Million People ($\theta = .1/.07/.2$)

Eswatini

$R_0=1.1/1.0/1.0$ $\delta = 0.010$ $\alpha=0.05$ $\theta=0.1$ %Infect= 1/ 1/ 5

DATA THROUGH 24-AUG-2020

\[
\theta = 0.07 \\
\theta = 0.1 \\
\theta = 0.2
\]
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Eswatini: Daily Deaths, Actual and Smoothed

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Eswatini: Change in Smoothed Daily Deaths

Eswatini: Delta d
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Eswatini: Change in (Change in Smoothed Daily Deaths)

Eswatini: Delta (\(\Delta d\))
\[
\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20
\]