Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

Jesús Fernández-Villaverde and Chad Jones

Extended results for Wisconsin
Based on data through October 9, 2020
Outline of Slides

• Basic data from Johns Hopkins CSSE (raw and smoothed)
• Brief summary of the model
• Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
• Simulation of re-opening – possibilities for raising $R_0$
• Results with alternative parameter values:
  o Lower mortality rate, $\delta = 0.8\%$
  o Higher mortality rate, $\delta = 1.2\%$
  o Infections last longer, $\gamma = 0.15$
  o Cases resolve more quickly, $\theta = 0.2$
  o Cases resolve more slowly, $\theta = 0.07$
• Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
**Brief Summary of Model**

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

- Inferred from daily deaths, and
- the change in daily deaths, and
- the change in (the change in daily deaths)
(see end of slide deck for this data)
Wisconsin: Estimates of $R_0(t)$

Wisconsin

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Wisconsin: Percent Currently Infectious

Wisconsin
Peak I/N = 0.15%  Final I/N = 0.15%  δ = 0.010  θ = 0.10  γ = 0.20
Wisconsin: Growth Rate of Daily Deaths over Past Week (percent)

Wisconsin
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

- **Warning**: Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

- **7 days of forecasts**: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  - Black = current
  - Red = oldest, Orange = second oldest, Yellow = third oldest...
  - Violet (purple) = one day earlier

- For robustness graphs, same idea
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - Red = lowest parameter value (e.g. $\delta = 0.8\%$)
  - Green = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:
  
  1. Alternatively, we fit this equation:

     \[
     \log R_0(t) = a_0 - \alpha \text{(Daily Deaths)}
     \]

     \[\Rightarrow \alpha \approx 0.05\]

     \(R_0\) declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline

- Robustness: Assume $R_0(t) = \text{final empirical value}$. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Wisconsin (7 days): Daily Deaths per Million People ($\alpha = .05$)

Wisconsin

$R_0 = 1.5/1.1/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  \%Infect= 3/4/8

DATA THROUGH 09-OCT-2020
Wisconsin (7 days): Cumulative Deaths per Million (Future, \( \alpha = 0.05 \))

Wisconsin

\[ R_0 = 1.5/1.1/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 3/4/8 \]

DATA THROUGH 09-OCT-2020
Wisconsin (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0.05$)

Wisconsin

$R_0 = 1.5/1.1/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 3/4/8$

Cumulative deaths per million people

New York City

Italy

Robustness to Mortality Rate, $\delta$
Wisconsin: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

Wisconsin

$R_0=1.5/1.1/1.1$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect= 3/4/8

DATA THROUGH 09-OCT-2020
Wisconsin: Daily Deaths per Million People ($\delta = 0.01/0.008/0.012$)

Wisconsin

$R_0 = 1.5/1.1/1.1$ \hspace{1cm} $\delta = 0.010$ \hspace{1cm} $\alpha = 0.05$ \hspace{1cm} $\theta = 0.1$ \hspace{1cm} %Infect = 3/4/8

DATA THROUGH 09-OCT-2020
Wisconsin: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

Wisconsin

$R_0=1.5/1.1/1.1$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect= 3/4/8

DATA THROUGH 09-OCT-2020
Reopening and Herd Immunity

– **Black**: assumes $R_0(today)$ remains in place forever
– **Red**: assumes $R_0(suppress) = 1/s(today)$
– **Green**: we move 25% of the way from $R_0(today)$ back to initial $R_0 = “normal”$
– **Purple**: we move 50% of the way from $R_0(today)$ back to initial $R_0 = “normal”$

NOTE: Lines often cover each other up
Wisconsin: Re-Opening ($\alpha = .05$)

Wisconsin

$R_0(t)=1.1$, $R_0$ (suppress) = 1.0, $R_0(25/50)=1.4/1.6$, $\delta = 0.010$, $\alpha=0.05$

(Light bars = New York City, for comparison)
Wisconsin: Re-Opening ($\alpha = 0$)

Wisconsin

$R_0(t) = 1.2$, $R_0(\text{suppress}) = 1.1$, $R_0(25/50) = 1.4/1.6$, $\delta = 0.010$, $\alpha = 0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Wisconsin (7 days): Daily Deaths per Million People ($\alpha = 0$)

Wisconsin

$R_0=1.5/1.2/1.2 \quad \delta = 0.010 \quad \alpha=0.00 \quad \theta=0.1 \quad \%Infect= 3/ 5/19$

DATA THROUGH 09-OCT-2020
Wisconsin (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

\[
R_0 = 1.5/1.2/1.2 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infect} = 3/5/19
\]

DATA THROUGH 09-OCT-2020
Wisconsin (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Wisconsin

$R_0 = 1.5/1.2/1.2 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%Infect = 3/5/19$
Wisconsin: Daily Deaths per Million People ($\delta = 0.8\%$)

Wisconsin

$R_0 = 1.5 / 1.1 / 1.1$  $\delta = 0.008$  $\theta = 0.1$  $\gamma = 0.2$  %Infect = 4/6/10
Wisconsin: Cumulative Deaths per Million ($\delta = 0.8\%$)

Wisconsin

$R_0 = 1.5/1.1/1.1 \quad \delta = 0.008 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 4/6/10$
Wisconsin: Daily Deaths per Million People ($\delta = 1.2\%$)

Wisconsin

$R_0=1.5/1.1/1.1$  $\delta = 0.012$  $\theta=0.1$  $\gamma=0.2$  %Infect=$3/ 4/ 7$
Wisconsin: Cumulative Deaths per Million ($\delta = 1.2\%$)

Wisconsin

$R_0 = 1.5/1.1/1.1$  \( \delta = 0.012 \)  \( \theta = 0.1 \)  \( \gamma = 0.2 \)  \%Infect= 3/ 4/ 7
Wisconsin: Daily Deaths per Million People ($\gamma = .2/.15$)

Wisconsin

$R_0=1.5/1.1/1.1$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect= 3/ 4/ 8

DATA THROUGH 09-OCT-2020
Wisconsin: Cumulative Deaths per Million $\gamma = 0.2/0.15$

Wisconsin

$R_0 = 1.5/1.1/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \% \text{Infect} = 3/4/8$

$\gamma = 0.15$

$\gamma = 0.2$

DATA THROUGH 09-OCT-2020
Wisconsin: Daily Deaths per Million People ($\theta = 0.1/0.07/0.2$)

Wisconsin

$R_0=1.5/1.1/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect$= 3/4/8$

DATA THROUGH 09-OCT-2020
Wisconsin: Cumulative Deaths per Million People ($\theta = .1 / .07 / .2$)

Wisconsin

$R_0 = 1.5/1.1/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 3/4/8$

DATA THROUGH 09-OCT-2020
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Wisconsin: Daily Deaths, Actual and Smoothed

Wisconsin: Daily deaths, $d$

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Wisconsin: Change in Smoothed Daily Deaths

Wisconsin: Delta \( d \)

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Wisconsin: Change in (Change in Smoothed Daily Deaths)

Wisconsin: Delta (Delta \(d\))
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]