The Direction of Technical Change

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Uzawa’s Theorem

Suppose a NGM with $Y_t = F(K_t, L_t, t)$ exhibits a BGP with $\frac{\dot{y}_t}{y_t} = g > 0$ starting at date 0. Then $\forall t > 0$,

$$Y_t = F(K_t, A_t L_t, 0)$$

where $\frac{\dot{A}_t}{A_t} = g$.

- If a NGM exhibits a BGP, then technical change must be “labor augmenting” along that path.
- Intuition: By CRS,

$$1 = F\left(\frac{K_t}{Y_t}, \frac{L_t}{Y_t}, t\right)$$

$K_t/Y_t$ constant, so technical change must exactly neutralize the fall in $L_t/Y_t$. 
The Direction of Technical Change: Why?

- Why in a NGM should technical change be labor augmenting? (Acemoglu 2003)

- To understand changes in the ratio of wages for college graduates to high school graduates, Katz and Murphy (1992) and a huge follow-on literature invoke skill-biased technical change (SBTC). Why should it be this way? (Acemoglu 1998)

- How do environmental problems and resource depletion affect the direction of technical change, sustainability, and growth? (Acemoglu, Aghion, Bursztyn, and Hemous).
Key Properties of CES Production Functions

\[ Y_t = F(M_tK_t, N_tL_t) = (\alpha(M_tK_t)^\rho + (1 - \alpha)(N_tL_t)^\rho)^{1/\rho} \]

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( E_{ofS} = \frac{1}{1-\rho} )</th>
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</thead>
<tbody>
<tr>
<td>Cobb-Douglas</td>
<td>0</td>
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<tr>
<td>Leontief: ( \min(K,L) )</td>
<td>(-\infty)</td>
</tr>
<tr>
<td>Perfect Subst: ( Y=K+L )</td>
<td>1</td>
</tr>
<tr>
<td>Low ( E_{ofS} )</td>
<td>( \rho &lt; 0 )</td>
</tr>
<tr>
<td>High ( E_{ofS} )</td>
<td>( 0 &lt; \rho &lt; 1 )</td>
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\(-\infty < \rho < 1 \)

\( 0 < \sigma < \infty \)

- Isoquants – \( K,L \) that produce a fixed amount of \( Y \).
CES Properties (continued)

• Simple way to compute marginal products (memorize)

\[
\frac{F_K K}{Y} = \alpha \left( \frac{MK}{Y} \right) ^ \rho
\]

\[
F_K = \alpha \frac{Y}{K} \cdot \left( \frac{MK}{Y} \right) ^ \rho
\]

• Key applications of CES in growth models
  ◦ Katz and Murpy (1992 QJE) Skill-biased tech. change
  ◦ LJones and Manuelli (1990 JPE): AK behavior asymptotically \( \sigma > 1 \)
  ◦ Acemoglu — various
  ◦ Caselli and Coleman (2006 AER): Development accounting with CES.
How Factor Shares Change with Scarcity

\[
\frac{F_K K}{Y} = \alpha \left( \frac{MK}{Y} \right)^\rho
\]

- \( \sigma = 1 \ (\rho = 0) \): Cobb-Douglas, constant factor shares
- \( \sigma < 1 \ (\rho < 0) \): Hard to substitute \( \Rightarrow \) price changes more than quantity \( \Rightarrow \) Scarcer factor gets rising share
- \( \sigma > 1 \ (\rho > 0) \): Easy to substitute \( \Rightarrow \) price changes less than quantity \( \Rightarrow \) Plentiful factor gets rising share
  - Example: LJones and Manuelli: \( \sigma > 1 \) \( \Rightarrow \) Capital share rises to one as capital accumulates \( \Rightarrow \) asymptotically production is like \( Y = MK \).
U.S. Factor Shares

- Labor share
- Capital share

YEAR
- 1950
- 1960
- 1970
- 1980
- 1990
- 2000
- 2010

PERCENT
- 20
- 30
- 40
- 50
- 60
- 70
- 80

Direction of Tech Change – p. 7
Acemoglu (2003):
Labor- and Capital-
Augmenting Technical Change
Overview

• Why should technical change be labor augmenting?
  ◦ Study a two-dimensional Romer model, where
    \[ Y = F(MK, NL) \]
  ◦ R&D can raise \( M \) or \( N \). What happens?

• Old literature in 1960s (Hicks, Samuelson, Kennedy, Fellner, Drandakis/Phelps).
  ◦ Specify an frontier tradeoff \( \frac{\dot{M}_t}{M_t} \) versus \( \frac{\dot{N}_t}{N_t} \).
  ◦ Maximize cost reduction instead of welfare
  ◦ No true R&D model, no microfoundations
  ◦ Sometimes got the Uzawa result
Economic Environment

Final output

\[ Y = \left( \gamma Y_L^\frac{1-\epsilon}{\epsilon} + (1 - \gamma) Y_K^\frac{1-\epsilon}{\epsilon} \right)^\frac{\epsilon}{1-\epsilon} \]

Capital

\[ \dot{K} = I \]

Labor goods

\[ Y_L = \left( \int_0^\infty y_\ell(i)^\beta di \right)^{1/\beta}, \quad 0 < \beta < 1 \]

Capital goods

\[ Y_K = \left( \int_0^\infty y_k(i)^\beta di \right)^{1/\beta} \]

Production

\[ y_\ell(i) = \ell(i), \quad y_k(i) = k(i) \]

Resource constraints

\[ \int_0^n \ell(i) di = L, \quad \int_0^m k(i) di = K, \]

Idea PF

\[ \frac{\dot{n}_t}{n_t} = b_\ell S_\ell - \delta, \quad \frac{\dot{m}_t}{m_t} = b_k S_k - \delta \]

Resource constraint

\[ S_\ell + S_k = \bar{S} \]

Preferences

\[ \int_0^\infty \frac{C_t^{1-1/\sigma}}{1-1/\sigma} e^{-\rho t} dt \]
Social Planner Allocation

Symmetry: $Y_L = NL$, $Y_K = MK$, $N \equiv n^{1/\beta-1}$, $M \equiv m^{1/\beta-1}$

\[
\max_{\{C_t, v_t\}} \int_0^\infty u(C_t) e^{-\rho t} \text{ s.t.} \quad Y_t = (\gamma(M_tK_t)\eta + (1 - \gamma)(N_tL_t)\eta)^{1/\eta} \]

\[\dot{K}_t = Y_t - C_t\]

\[\frac{\dot{N}_t}{N_t} = b_n v_t \bar{S} - \delta\]

\[\frac{\dot{M}_t}{M_t} = b_m (1 - v_t) \bar{S} - \delta\]
Hamiltonian

\[ H = u(C_t) + \lambda_t(Y_t - C_t) + \mu_{nt}(b_nv_t\bar{S}N_t - \delta N_t) + \mu_{mt}(b_m(1 - v_t)\bar{S}M_t - \delta M_t) \]

FOC:

1. \( H_c = 0 \):
   \[ u'(C_t) = \lambda_t \]

2. \( H_v = 0 \):
   \[ \mu_{nt}b_n\bar{S}N_t = \mu_{mt}b_m\bar{S}M_t \]

3. Arbitrage(N):
   \[ \rho = \frac{\mu_{nt}}{\mu_{nt}} + \frac{1}{\mu_n} \left[ \lambda_t \frac{\partial Y_t}{\partial N_t} + \mu_{nt} \frac{\dot{N}_t}{N_t} \right] \]

4. Arbitrage(M):
   \[ \rho = \frac{\mu_{mt}}{\mu_{mt}} + \frac{1}{\mu_m} \left[ \lambda_t \frac{\partial Y_t}{\partial M_t} + \mu_{mt} \frac{\dot{M}_t}{M_t} \right] \]

5. Arbitrage(K):
   \[ \rho = \frac{\dot{\lambda}_t}{\lambda_t} + \frac{1}{\lambda_t} \left[ \lambda_t \frac{\partial Y_t}{\partial K_t} \right] \]

and transversality conditions.
Solving for BGP

- (1) + (5) \( \Rightarrow \frac{\dot{C}_t}{C_t} = \sigma \left( \frac{\partial Y}{\partial K} - \rho \right) \Rightarrow \frac{\partial Y}{\partial K} \) constant

- \( Y = C + I \) and \( \dot{K} = I \) \( \Rightarrow g_Y = g_C = g_I = g_K \) along BGP.

- What is \( \frac{\partial Y}{\partial K} \)?

\[
\frac{\partial Y}{\partial K} = (1 - \gamma) \left( \frac{MK}{Y} \right)^\eta \frac{Y}{K}
\]

\( \Rightarrow M_t \) must be constant along a BGP!
Now, solve rest of model to make sure a constant $M$ is okay

\[
\frac{\dot{M}}{M} = 0 \ \Rightarrow \ b_m(1 - v_t)\bar{S} = \delta \ \Rightarrow \\
\]

\[
v^* = 1 - \frac{\delta}{b_m\bar{S}}
\]

Growth: $g_Y = g_C = g_K = g_I = g_N$

\[
g_N = b_n v^* \bar{S} - \delta
\]
as long as $b_n$ is sufficiently large.

Great! Acemoglu provides microfoundations where researchers endogenously choose LATC.
‘Lab Equipment’ Version?

- Suppose idea PF uses $K$ and $L$ as inputs, not just labor (Rivera-Batiz and Romer, 1991)

- New economic environment:

\[
C + I + R_m + R_n = Y
\]

\[
\dot{N} = b_n s_n Y - \delta N, \quad R_{nt} = s_{nt} Y_t
\]

\[
\dot{M} = b_m s_m Y - \delta M, \quad R_{mt} = s_{mt} Y_t
\]
Hamiltonian

\[ H = u(C) + \lambda((1 - s_n - s_m)Y - C) + \mu_n(b_n s_n Y - \delta N) + \mu_m(b_m s_m M - \delta M) \]

FOC: (use (2) and (3) to simply arbitrage results)

1. \( H_c = 0 \):
   \[ u'(C) = \lambda \]

2. \( H_{s_n} = 0 \):
   \[ \lambda Y = \mu_n b_n Y \]

3. \( H_{s_m} = 0 \):
   \[ \lambda Y = \mu_m b_m Y \]

3. Arbitrage(N):
   \[ \rho = \frac{\dot{\mu}_n}{\mu_n} + \frac{\lambda}{\mu_n} \frac{\partial Y}{\partial N} - \delta \]

4. Arbitrage(M):
   \[ \rho = \frac{\dot{\mu}_m}{\mu_m} + \frac{\lambda}{\mu_m} \frac{\partial Y}{\partial M} - \delta \]

5. Arbitrage(K):
   \[ \rho = \frac{\dot{\lambda}}{\lambda} + \frac{\partial Y}{\partial K} \]

and transversality conditions.
Solving for BGP

- As before Euler eqn $\Rightarrow$ MPK constant $\Rightarrow M$ constant. But now, this will pose problems!
- FOC (2) and (3) $\Rightarrow \frac{\mu_n}{\mu_m} = \frac{b_m}{b_n}$ constant. (Why?)
- But (4) and (5) $\Rightarrow$

$$
\mu_n = \frac{\lambda \frac{\partial Y}{\partial N}}{\rho - g \mu_n + \delta}, \quad \mu_m = \frac{\lambda \frac{\partial Y}{\partial M}}{\rho - g \mu_m + \delta}
$$

- Therefore $\frac{\mu_n}{\mu_m}$ constant $\Rightarrow \frac{\partial Y/\partial N}{\partial Y/\partial M}$ constant

$$
\frac{\partial Y/\partial N}{\partial Y/\partial M} = \frac{\gamma}{1 - \gamma} \left( \frac{LN}{MK} \right)^\eta \frac{M}{N}
$$

- So $\frac{\partial Y/\partial N}{\partial Y/\partial M}$ falls at rate $g_N \Rightarrow$ No BGP!
Comparing the models

• In both, MPK constant $\Rightarrow M$ constant.

• Moreover, the benefit of creating ideas depends on

$$\frac{\partial Y/\partial N}{\partial Y/\partial M} = \frac{\gamma}{1 - \gamma} \left( \frac{LN}{MK} \right)^\eta \frac{M}{N}$$

which falls at rate $g_N$.

• Therefore, for a BGP to exist, the relative cost of creating ideas must fall at rate $g_N$ as well...
Comparing the models (continued)

Does the relative cost of creating $N$ versus $M$ fall at rate $g_N$?

Model 1:  
\[
\dot{N} = b_n S_\ell N - \delta N \\
\dot{M} = b_m S_k M - \delta M
\]

Model 2:  
\[
\dot{N} = b_n v Y - \delta N \\
\dot{M} = b_m (1 - v) Y - \delta M
\]

Model 3:  
\[
\dot{N} = b_n S_\ell^\lambda N^\phi - \delta N \\
\dot{M} = b_m S_k^\lambda M^\phi - \delta M
\]

Model 4:  
\[
\dot{N} = b_n S_\ell N^\alpha M^\beta - \delta N \\
\dot{M} = b_m S_k N^\lambda M^\theta - \delta M
\]
• Great idea for a paper!

• One can write down a model with microfoundations that leads to the LATC result and a BGP

• However, that model is quite fragile.

• This paper offers an intriguing possibility, but in general there’s no real reason here to think that economic forces will lead to LATC.
Additional Work

- Jones (2005 QJE): Houthakker + Kortum =
  - Exponential growth
  - Cobb-Douglas (global) production function
  - Labor-augmenting technical change.

- Karabarbounis and Neiman (2014 QJE)
  - “Declining Labor Shares and the Global Rise of Corporate Savings”
  - Great data on labor shares in 51 countries
  - Many show declines

- Robots? Agriculture?
  - Acemoglu and Restrepo, “The Race between Man and Machine...” in progress
Further Directions after AABH

• Dell, Jones, Olken (2011) “Temperature Shocks and Economic Growth: Evidence from the Last Half Century”

• Per Krusell, Tony Smith, John Hassler, Golosov, Tsyvinski — recent papers on climate, pollution, and growth.


• Aghion et al (Hemous/JVR), (2015 JPE) “Carbon taxes, path dependency and directed technical change: evidence from the auto industry”

• How to move the model closer to empirics — wide range of outcomes are optimal in current setup. \( \epsilon, \psi \)?

• Apply to developing countries (China, India)?