Nonrivalry and the Economics of Data

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19 November 2018

Examples of Data

- Google, Facebook
- Amazon
- Tesla, Uber, Waymo
- Medical and genetic data
- Location history
- Speech records
- Physical action data

Canonical example: data as input into machine learning algorithm.
E.g. self-driving car.

Data is Nonrival

- Data is infinitely usable
  - Contrast with rival goods: coffee, computer, doctor
  - Multiple engineers/algorithms can use same data at same time (within and across firms)
- Key ways that data enters the economy:
  - Nonrivalry $\Rightarrow$ social gain from sharing data
  - Privacy
  - Firm: competitive advantage ("moat")
- Social planner and consumers only care about the first two. But firms care a lot about the last one $\Rightarrow$ inefficiency

Policies on Data Are Being Written Now

What policies governing data use maximize welfare?

- European General Data Protection Regulation (GDPR)
  - Privacy vs. social gain from sharing
  - "The protection of natural persons in relation to the processing of personal data is a fundamental right"
  - "The right . . . must be considered in relation to its function in society and be balanced against other fundamental rights. . . "
- The California Consumer Privacy Act of 2018
  - Allows consumers to opt out of having their data sold

Nonrivalry of Data $\Rightarrow$ Increasing Returns

- Nonrivalry implies increasing returns to scale: $Y = F(D, X)$
  - Constant returns to rival inputs: $F(D, \lambda X) = \lambda F(D, X)$
  - Increasing returns to data and rival inputs: $F(\lambda D, \lambda X) > \lambda F(D, X)$
- When firms hoard data, a firm learns only from its own consumers
- But when firms share data, all firms learn from all consumers
  - Firms, fearing creative destruction, will not do this
  - But if consumers own the data, they appropriately balance data sharing and privacy

Data Property Rights Matter

- Key point: allocations with different degrees of data sharing

  $\Rightarrow$ different output, welfare, etc.

- To illustrate, we assume (plausibly?) the Coase theorem fails
  - Consumers can't commit to selling data to just one firm
  - Firms can't commit to not using data they acquire
  - Useful for showing the role of data sharing

- How do different property rights affect the use of data?
  - "Firms own data" versus "consumers own data"

Data is Nonrival $\Rightarrow$ Interesting Questions

- Do markets produce the right amount of data?
- Why don't firms (always) sell their data?
- Who should own data as it's created?
- Implications of data nonrivalry for antitrust and economic growth?

We develop a framework for thinking through these questions

Outline

- Economic environment
- Allocations:
    - Optimal allocation
    - Firms own data
    - Consumers own data
    - Extreme privacy protection: outlaw data sharing
- Theory results and a numerical example

Basic Setup

Overview

- Representative consumer with a love for variety
- Innovation $\Rightarrow$ endogenous measure of varieties
- Nonrivalry of data $\Rightarrow$ increasing returns to scale
- How is data produced?
  - Learning by doing: each unit consumed $\rightarrow$ 1 unit of data
  - Alternative: separate PF (Tesla vs Google self-driving car)
- Any data equally useful in all firms $\Rightarrow$ one sector of economy
- Data depreciates fully each period

The Economic Environment

Utility	$\int_0^\infty e^{-\rho t} L_t u(c_t, x_{it}, \tilde{x}_{it}) dt$
Flow Utility	$u(c_t, x_{it}, \tilde{x}_{it}) = \log c_t - \frac{\kappa}{2} \frac{1}{N_t^2} \int_0^{N_t} x_{it}^2 \, di - \frac{\tilde{\kappa}}{2} \frac{1}{N_t} \int_0^{N_t} \tilde{x}_{it}^2 \, di$
Consumption per person	$c_t = \left(\int_0^{N_t} c_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$ with $\sigma > 1$
Data production	$J_{it} = c_{it} L_t$
Variety resource constraint	$c_{it} = Y_{it} / L_t$
Firm production	$Y_{it} = D_{it}^{\eta} L_{it}, \quad \eta \in (0, 1)$
Data used by firm i	$D_{it} \leq \alpha x_{it} J_{it} + (1 - \alpha) B_t$ (nonrivalry)
Data of firm i used by others	$D_{sit} \leq \tilde{x}_{it} J_{it}$
Data bundle	$B_t = \left(N_t^{-\frac{1}{\epsilon}} \int_0^{N_t} D_{sit}^{\frac{\epsilon-1}{\epsilon}} \, di \right)^{\frac{\epsilon}{\epsilon-1}}$ with $\epsilon > 1$
Innovation (new varieties)	$\dot{N}_t = \frac{1}{\chi} \cdot L_{et}$
Labor resource constraint	$L_{et} + \int_0^{N_t} L_{it} \, di = L_t$
Population growth (exogenous)	$L_t = L_0 e^{g_L t}$
Creative destruction	$\delta(\tilde{x}_{it}) = \frac{\delta_0}{2} \tilde{x}_{it}^2$ (equilibrium)

The Planner Problem (using symmetry of firms)

$$\max_{\{L_{pt}, x_{it}, \tilde{x}_{it}\}} \int_0^\infty e^{-\tilde{\rho} t} L_0 u(c_t, x_{it}, \tilde{x}_{it})\, dt, \quad \tilde{\rho} := \rho - g_L$$

subject to

$$\begin{aligned}
c_t &= Y_t / L_t \\
Y_t &= N_t^{\frac{1}{\sigma-1}} D_{it}^{\eta} L_{pt} \\
D_{it} &= \alpha x_{it} Y_{it} + (1-\alpha) N_t \tilde{x}_{it} Y_{it} \\
Y_{it} &= D_{it}^{\eta} \cdot \frac{L_{pt}}{N_t} \\
\dot{N}_t &= \frac{1}{\chi}(L_t - L_{pt}) \\
L_t &= L_0 e^{g_L t}
\end{aligned}$$

- More sharing ⇒ negative utility cost but more consumption
- Balance labor across production and entry/innovation

Scale Effect from Sharing Data

$$D_{it} = \alpha x_{it} J_{it} + (1-\alpha)\left(N_t^{-\frac{1}{\epsilon}} \int_0^{N_t} (\tilde{x}_{it} J_{it})^{\frac{\epsilon-1}{\epsilon}}\, di\right)^{\frac{\epsilon}{\epsilon-1}}$$

$$\begin{aligned} D_{it} &= \alpha x_{it} Y_{it} + (1-\alpha) N_t \tilde{x}_{it} Y_{it} \\ &= [\alpha x_{it} + (1-\alpha)\tilde{x}_{it} N_t] Y_{it} \end{aligned}$$

- No sharing versus sharing:
  - No sharing: Only the αx_t term = no scale effect
  - Sharing: The $(1-\alpha)\tilde{x}_t N_t$ term = extra scale effect

  Source of Scale Effect: N_t scales with L_t

- Plugging into production function:

$$Y_{it} = \left([\alpha x_t + (1-\alpha)\tilde{x}_t N_t]^{\eta} L_{it}\right)^{\frac{1}{1-\eta}}$$

The Optimal Allocation on BGP (asymptotic)

$$\tilde{x}_{it} = \tilde{x}_{sp} = \left(\frac{1}{\tilde{\kappa}} \cdot \frac{\eta}{1-\eta}\right)^{1/2} \tag{1}$$

$$x_{it} = x_{sp} = \frac{\alpha}{1-\alpha} \frac{\tilde{\kappa}}{\kappa} \left(\frac{1}{\tilde{\kappa}} \cdot \frac{\eta}{1-\eta}\right)^{1/2} \tag{2}$$

$$L_{it}^{sp} = \chi\rho \cdot \frac{\sigma - 1}{1-\eta} := \nu_{sp} \tag{3}$$

$$N_t^{sp} = \frac{L_t}{\chi\left(g_L + \nu_{sp}\right)} := \psi_{sp} L_t \tag{4}$$

$$L_{pt}^{sp} = \nu_{sp}\psi_{sp}L_t \tag{5}$$

$$Y_t^{sp} = \left(\nu_{sp}(1-\alpha)^{\eta}\tilde{x}_{sp}^{\eta}\right)^{\frac{1}{1-\eta}} \left(\psi_{sp}L_t\right)^{\frac{1}{\sigma-1}+\frac{1}{1-\eta}} \tag{6}$$

$$c_t^{sp} = \frac{Y_t}{L_t} = \left(\nu_{sp}(1-\alpha)^{\eta}\tilde{x}_{sp}^{\eta}\right)^{\frac{1}{1-\eta}} \left(\psi_{sp}L_t\right)^{\frac{1}{\sigma-1}+\frac{\eta}{1-\eta}} \tag{7}$$

$$g_c^{sp} = \left(\frac{1}{\sigma-1} + \frac{\eta}{1-\eta}\right) g_L \tag{8}$$

$$D_i^{sp} = \left((1-\alpha)\tilde{x}_{sp}\nu_{sp}\psi_{sp}L_t\right)^{\frac{1}{1-\eta}} \tag{9}$$

$$D^{sp} = ND_i = \left((1-\alpha)\tilde{x}_{sp}\nu_{sp}\right)^{\frac{1}{1-\eta}} \left(\psi_{sp}L_t\right)^{1+\frac{1}{1-\eta}} \tag{10}$$

$$Y_{it}^{sp} = \left(\nu_{sp}(1-\alpha)^{\eta}\tilde{x}_{sp}^{\eta}\right)^{\frac{1}{1-\eta}} \left(\psi_{sp}L_t\right)^{\frac{\eta}{1-\eta}} \tag{11}$$

The Optimal Allocation: Data, Firm Size, Variety

$$\tilde{x}_{sp} = \left(\frac{1}{\tilde{\kappa}} \cdot \frac{\eta}{1-\eta} \right)^{1/2}$$

$$L_{it}^{sp} = \chi\rho \cdot \frac{\sigma - 1}{1-\eta} := \nu_{sp}$$

$$N_t^{sp} = \frac{L_t}{\chi g_L + \nu_{sp}} := \psi_{sp} L_t$$

- Data shared increasing in data production elasticity and decreasing in privacy cost
- Firm size constant on BGP. N has opposite comparative statics
- Higher entry cost, time preference, population growth, and elasticity of substitution raise firm size and reduce varieties
- Higher η raises firm size and reduces varieties: Entry does not create data

The Optimal Allocation: GDP per person

$$c_t^{sp} = \frac{Y_t}{L_t} = \left(\nu_{sp}(1-\alpha)^\eta \tilde{x}_{sp}^\eta\right)^{\frac{1}{1-\eta}} \left(\psi_{sp} L_t\right)^{\frac{1}{\sigma-1}+\frac{\eta}{1-\eta}}$$

$$g_c^{sp} = \left(\frac{1}{\sigma-1} + \frac{\eta}{1-\eta}\right) g_L$$

- Scale effect: $\underbrace{\frac{1}{\sigma-1}}_{\text{Love of Variety}} + \underbrace{\frac{\eta}{1-\eta}}_{\text{Data}}$
- More people make more data and all firms use all shared data

Firms Own Data

Firms Own Data: Consumer Problem

- Firms own data and choose one data policy $(x_{it}, \tilde{x}_{it})$ applied to all consumers

- Consumers just choose consumption:

$$U_0 = \max_{\{c_{it}\}} \int_0^\infty e^{-\tilde{\rho} t} L_0 u(c_t, x_{it}, \tilde{x}_{it}) dt$$

$$\text{s.t.} \quad c_t = \left(\int_0^{N_t} c_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

$$\dot{a}_t = (r_t - g_L) a_t + w_t - \int_0^{N_t} p_{it} c_{it} di$$

Firms own Data: Data Decisions

- Firms buy D_{bit} data from intermediary at given price p_b
- Firms sell D_{sit} data to intermediary at chosen price p_{si}
  - Perfect competition inconsistent with nonrival data!
  - Monopolistically competitive with own data
  - See the intermediary's downward-sloping demand curve and set price
- How much data to use / sell?
  - x_{it}: Use all of own data $\Rightarrow x_{it} = 1$
  - $\tilde{x}_{it}$: Trade off = selling data versus creative destruction

    $\delta(\tilde{x}_{it})$ = Poisson rate transferring ownership of variety

Firms own the Data: Incumbent Firm Problem

- Monopolistically competitive firm takes demand for variety as given (from FOC of consumer problem): $p_{it} = \left(\frac{c_t}{c_{it}}\right)^{\frac{1}{\sigma}} = \left(\frac{Y_t}{Y_{it}}\right)^{\frac{1}{\sigma}}$

$$r_t V_{it} = \max_{L_{it}, D_{bit}, x_{it}, \tilde{x}_{it}} \left(\frac{Y_t}{Y_{it}}\right)^{\frac{1}{\sigma}} Y_{it} - w_t L_{it} - p_{bt} D_{bit} + p_{st} \tilde{x}_{it} Y_{it} + \dot{V}_{it} - \delta(\tilde{x}_{it}) V_{it}$$

$$\text{s.t.} \quad Y_{it} = D_{it}^{\eta} L_{it}$$

$$D_{it} = \alpha x_{it} Y_{it} + (1-\alpha) D_{bit}$$

$$x_{it} \in [0,1], \tilde{x}_{it} \in [0,1]$$

$$p_{sit} = \lambda_{DI} N_t^{-\frac{1}{\epsilon}} \left(\frac{B_t}{\tilde{x}_{it} Y_{it}}\right)^{\frac{1}{\epsilon}}$$

- Data Intermediary $(p_{bt}, p_{st}, D_{bit})$ and Free Entry complete eqm.

Firms own the Data: Data Intermediary Problem

- A monopolist takes data purchase price as given and sees the downward sloping demand curve for data $p_{bt}(D_{bit})$:

$$\max_{p_{bt}, D_{sit}} \quad p_{bt} \int_0^{N_t} D_{bit}\, di - p_{st} \int_0^{N_t} D_{sit}\, di$$

s.t.

$$D_{bit} \leq B_t = \left(N_t^{-\frac{1}{\epsilon}} \int_0^{N_t} D_{sit}^{\frac{\epsilon-1}{\epsilon}}\, di \right)^{\frac{\epsilon}{\epsilon-1}}$$

$$p_{bt} \leq p^*_{bt}$$

- Free entry at zero cost $\Rightarrow$ zero profits
- Problem incorporates data nonrivalry
  - Buys data once from each firm
  - But can sell the same bundle multiple times

Entry: Innovation Creates a New Variety

- χ units of labor needed to create an additional variety
- Free entry condition:

$$\chi w_t = V_{it} + \frac{\int_0^{N_t} \delta(\tilde{x}_{it}) V_{it} \, di}{\dot{N}_t}$$

- The value of a new variety and the per-entrant share of business stealing from creative destruction

Firms Own the Data: Definition of Equilibrium

The equilibrium in which firms own the data consists of quantities $\{c_t, Y_t, c_{it}, x_{it}, \tilde{x}_{it}, a_t,$ $Y_{it}, L_{it}, D_{it}, D_{bit}, B_t, D_{sit}, N_t, L_{pt}, L_{et}\}$ and prices $\{p_{it}, p_{bt}, p_{sit}, w_t, r_t, V_{it}\}$ such that

1. $\{c_t, c_{it}, a_t\}$ solve the Household Problem.
2. $\{L_{it}, Y_{it}, p_{it}, p_{sit}, D_{bit}, D_{it}, x_{it}, \tilde{x}_{it}, V_{it}\}$ solve the Firm Problem.
3. (D_{sit}, B_t) Data markets clear: $D_{bit} = B_t$ and $D_{sit} = \tilde{x}_{it} Y_{it}$
4. (p_{bt}) Free entry into data intermediation gives zero profits there (constrains p_b as a function of p_s)
5. (L_{et}) Free entry into producing a new variety leads to zero profits: $\chi w_t = V_{it} + + \frac{\int_0^{N_t} \delta(\tilde{x}_{it}) V_{it} \; di}{\dot{N}_t}$
6. Definition of L_{pt}: $L_{pt} = \int_0^{N_t} L_{it} di$
7. w_t clears the labor market: $L_{pt} + L_{et} = L_t$
8. r_t clears the asset market: $a_t = \int_0^{N_t} V_{it} di / L_t$
9. N_t follows its law of motion: $\dot{N}_t = \frac{1}{\chi}(L_t - L_{pt})$
10. $Y_t := c_t L_t$ denotes aggregate output.

Firms Own Data: A “No Trade” Law

- What if the government, in an attempt to protect consumers privacy, makes data sharing illegal?
- Government chooses
  - $x_{it} \in (0, 1]$
  - $\tilde{x}_{it} = 0$
- We call this the “Outlaw Sharing” allocation

Consumers Own Data

Consumers own Data: Consumer Problem

- Consumers own data, so now choose how much to share $(x_{it}, \tilde{x}_{it})$:

$$U_0 = \max_{\{c_{it}, x_{it}, \tilde{x}_{it}\}} \int_0^\infty e^{-\tilde{\rho}t} L_0 u(c_t, x_{it}, \tilde{x}_{it}) dt$$

$$\text{s.t.} \quad c_t = \left(\int_0^{N_t} c_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

$$\dot{a}_t = (r_t - g_L) a_t + w_t - \int_0^{N_t} p_{it} c_{it} di + \int_0^{N_t} x_{it} p_{st}^a c_{it} di + \int_0^{N_t} \tilde{x}_{it} p_{st}^b c_{it} di$$

- Firm problem similar to before, but now takes $x, \tilde{x}$ as given, can't sell data, and has to buy "own" data

Consumers own the Data: Incumbent Firm Problem

- Monopolistically competitive firm takes demand for variety as given (from FOC of consumer problem):
$q_{it} = \left(\frac{c_t}{c_{it}}\right)^{\frac{1}{\sigma}} = \left(\frac{Y_t}{Y_{it}}\right)^{\frac{1}{\sigma}} = p_{it} - x_{it}p^a_{st} - \tilde{x}_{it}p^b_{st}$

- Firm buys data on its own variety (D_{ait}) and data on other firms varieties (D_{bit})

$$r_t V_{it} = \max_{L_{it}, D_{ait}, D_{bit}} \left[\left(\frac{Y_t}{Y_{it}}\right)^{\frac{1}{\sigma}} + p^a_{st}x_{it} + p^b_{st}\tilde{x}_{it} \right] Y_{it} - w_t L_{it}$$
$$- p_{at}D_{ait} - p_{bt}D_{bit} + \dot{V}_{it} - \delta(\tilde{x}_t)V_{it}$$

s.t. $Y_{it} = D^{\eta}_{it} L_{it}$

$$D_{it} = \alpha D_{ait} + (1-\alpha) D_{bit}$$

$$D_{ait} \geq 0, \quad D_{bit} \geq 0$$

Consumers own the Data: Data Intermediary Problem

- The DI chooses the price at which it sells a firm its own data and the price of other firms data, given its purchase price

$$\max_{p_{ait}, p_{bit}, D^a_{cit}, D^b_{cit}} \int_0^{N_t} (p_{ait} D_{ait} + p_{bit} D_{bit}) di - \int_0^{N_t} (p^a_{st} D^a_{cit} + p^b_{st} D^b_{cit}) di$$

s.t.

$$D_{ait} \leq D^a_{cit} \quad \forall i$$

$$D_{bit} \leq B_t = \left(N_t^{-\frac{1}{\epsilon}} \int_0^{N_t} (D^b_{cit})^{\frac{\epsilon - 1}{\epsilon}} \, di \right)^{\frac{\epsilon}{\epsilon - 1}} \quad \forall i$$

$p_{ait} \leq p^*_{ait}$ and $p_{bit} \leq p^*_{bit}$

- Can not sell more data on firm i than it buys from consumers
- Can sell all data purchased as “type-b” data to each firm (nonrivalry)

Consumers own the Data: Equilibrium

An equilibrium in which consumers own data consists of quantities $\{c_t, Y_t, c_{it}, x_{it}, \tilde{x}_{it}, a_t, Y_{it}, L_{it}, D_{it}, D_{ait}, D_{bit}, D^a_{cit}, D^b_{cit}, B_t, N_t, L_{pt}, L_{et}\}$ and prices $\{q_{it}, p_{it}, p_{ait}, p_{bit}, p^a_{st}, p^b_{st}, w_t, r_t, V_{it}\}$ such that

1. $\{c_t, c_{it}, x_{it}, \tilde{x}_{it}, a_t\}$ solve the Household Problem.
2. $\{L_{it}, Y_{it}, p_{it}, D_{ait}, D_{bit}, D_{it}, V_{it}\}$ solve the Firm Problem.
3. (q_{it}) The effective consumer price is $q_{it} = p_{it} - x_{it}p^a_{st} - \tilde{x}_{it}p^b_{st}$
4. D^a_{cit}, D^b_{cit}, B_t, p_{ait}, and p_{bit} solve the Data Intermediary Problem (with zero profits).
5. p^a_{st} clears the data market: $D^a_{cit} = x_{it}c_{it}L_t$.
6. p^b_{st} clears the data market: $D^b_{cit} = \tilde{x}_{it}c_{it}L_t$.
7. (L_{et}) Free entry into new varieties leads to zero profits:
   $\chi w_t = V_{it} + + \frac{\int_0^{N_t} \delta(\tilde{x}_{it}) V_{it} \, di}{\dot{N}_t}$
8. Definition of L_{pt}: $L_{pt} = \int_0^{N_t} L_{it} di$
9. w_t clears the labor market: $L_{pt} + L_{et} = L_t$
10. r_t clears the asset market: $a_t = \int_0^{N_t} V_{it} di / L_t$
11. N_t follows its law of motion: $\dot{N}_t = \frac{1}{\chi}(L_t - L_{pt})$
12. $Y_t := c_t L_t$ denotes aggregate GDP.

Key Forces: Consumers vs. Firms vs. Outlaw Sharing

- Firms
  - use all data on own variety, ignoring consumer privacy
  - restrict data sharing because of creative destruction
- Consumers
  - respect their own privacy concerns
  - sell data broadly, ignoring creative destruction
- Outlaw sharing
  - maximizes privacy gains
  - missing scale effect reduces consumption

Results: Comparing Allocations

1. Planner Problem
2. Firms Own Data
3. Outlaw Data Sharing
4. Consumers Own Data

Results —Preview of Scale Effect

- Recall: Role of data sharing

$$D_{it} = \alpha x_t Y_{it} + (1-\alpha) N_t \tilde{x}_t Y_{it}$$
$$= [\alpha x_t + (1-\alpha)\tilde{x}_t N_t] Y_{it}$$

- Plugging into production function:

$$Y_{it} = \left([\alpha x_t + (1-\alpha)\tilde{x}_t N_t]^{\eta} L_{it}\right)^{\frac{1}{1-\eta}}$$

- Scale Effect from Data Sharing:
  - $(1-\alpha)\tilde{x}_t N_t$ from data sharing and N_t scales with L_t
  - Unless $\tilde{x}_t = 0$ (outlaw sharing)

Key Allocations: $alloc \in \{sp, f, c, ns\}$

- Firm size: $L_i^{alloc} = L_{pt}/N_t = \nu_{alloc}$

$$\nu_{sp} := \chi\rho \cdot \frac{\sigma - 1}{1 - \eta}$$
$$\nu_{os} := \chi\rho \cdot \frac{\sigma - 1}{1 - \sigma\eta}$$
$$\nu_{c} := \chi g_L \cdot \frac{\rho + \delta(\tilde{x}_c)}{g_L + \delta(\tilde{x}_c)} \cdot \frac{\sigma - 1}{1 - \sigma\eta}$$
$$\nu_{f} := \chi g_L \cdot \frac{\rho + \delta(\tilde{x}_f)}{g_L + \delta(\tilde{x}_f)} \cdot \frac{\sigma - 1}{1 - \sigma\eta\frac{\epsilon - 1}{\epsilon}}$$

- Number of firms: $N_t^{alloc} = \psi_{alloc} L_t$

$$\psi_{alloc} := \frac{1}{\chi g_L + \nu_{alloc}}$$

Data Sharing

Own Firm Data	Sharing with Other Firms
$x_{sp} = \frac{\alpha}{1-\alpha}\frac{\tilde{\kappa}}{\kappa}\left(\frac{1}{\tilde{\kappa}} \cdot \frac{\eta}{1-\eta}\right)^{1/2}$	$\tilde{x}_{sp} = \left(\frac{1}{\tilde{\kappa}} \cdot \frac{\eta}{1-\eta}\right)^{1/2}$
$x_f = 1$	$\tilde{x}_f = \left(\frac{\Gamma\rho}{(2-\Gamma)\delta_0}\right)^{1/2}, \Gamma := \frac{\eta(\sigma-1)}{\frac{\epsilon}{\epsilon-1}-\sigma\eta}$
$x_{os} \in (0,1]$	$\tilde{x}_{os} = 0$
$x_c = \frac{\alpha}{1-\alpha}\frac{\tilde{\kappa}}{\kappa}\left(\frac{1}{\tilde{\kappa}} \cdot \frac{\eta}{1-\eta} \cdot \frac{\sigma-1}{\sigma}\right)^{1/2}$	$\tilde{x}_c = \left(\frac{1}{\tilde{\kappa}} \cdot \frac{\eta}{1-\eta} \cdot \frac{\sigma-1}{\sigma}\right)^{1/2}$

- Firms fear creative destruction and share less than planner (δ_0)
- Consumers share less than planner because of mark up
- No sharing law restricts data even more
- Firms use more own-variety data compared to consumer/planner

Output

- For $alloc \in \{sp, c, f\}$:

$$Y_t^{alloc} = \left[\nu_{alloc}(1-\alpha)^{\eta}\tilde{x}_{alloc}^{\eta}\right]^{\frac{1}{1-\eta}} \left(\psi_{alloc}L_t\right)^{1+\frac{1}{\sigma-1}+\frac{\eta}{1-\eta}}$$

- For Outlaw Sharing:

$$Y_t^{os} = \left[\nu_{os}\alpha^{\eta}x_{os}^{\alpha}\right]^{\frac{1}{1-\eta}} \left(\psi_{os}L_t\right)^{1+\frac{1}{\sigma-1}}$$

- Two source of increasing returns to scale:
    - Standard variety effect: $\frac{\sigma}{\sigma-1}$
    - Data sharing: $\frac{\eta}{1-\eta}$
- Recall $\tilde{x}_t > 0$ from data sharing $\Rightarrow$ scale effect

Consumption per person and Growth

- Consumption per person:

For $alloc \in \{sp, c, f\}$: $\quad c_t^{alloc} = Const_{alloc} \cdot L_t^{\frac{1}{\sigma-1}+\frac{\eta}{1-\eta}}$

For outlaw sharing: $\quad c_t^{os} = Const_{os} \cdot L_t^{\frac{1}{\sigma-1}}$

- Per capita growth:

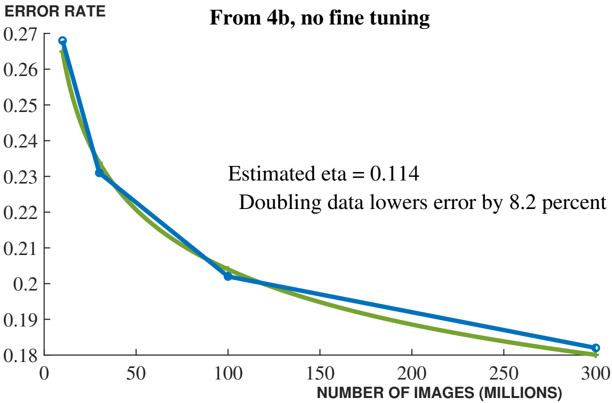
$$g_c^{sp} = g_c^f = g_c^c = \left(\frac{1}{\sigma-1} + \frac{\eta}{1-\eta}\right) g_L$$

$$g_c^{os} = \left(\frac{1}{\sigma-1}\right) g_L$$

Intuition: No sharing means you learn from 10 workers (constant firm size), sharing means you learn from the entire population

Numerical Example: How large is η?

- Error rate is proportional to $M^{-\eta}$. Productivity = 1/(error rate)



- Average $\eta = 0.08$. Double data $\Rightarrow$ 6% reduction in error rate

Numerical Example: Other Parameters

Description	Parameter	Value
Importance of data	η	0.08
Elasticity of substitution	σ	5
Weight on privacy	$\kappa = \tilde{\kappa}$	0.20
Population level	L_0	100
Population growth rate	g_L	0.02
Rate of time preference	ρ	0.03
Labor cost of entry	χ	0.01
Creative destruction	δ_0	0.4
Weight on own data	α	1/2
Use of own data in NS	$\bar{x}$	1

Numerical Example: Consumption Equivalent Welfare

$$U_{ss}^{alloc} = \frac{1}{\tilde{\rho}} \left(\log c_0^{alloc} - \frac{\tilde{\kappa}}{2} \tilde{x}_{alloc}^2 + \frac{g_c^{alloc}}{\tilde{\rho}} \right).$$

Let $U_{ss}^{alloc}(\lambda)$ denote steady-state welfare when we perturb the allocation of consumption by some proportion λ:

$$U_{ss}^{alloc}(\lambda) = \frac{1}{\tilde{\rho}} \left(\log(\lambda c_0^{alloc}) - \frac{\tilde{\kappa}}{2} \tilde{x}_{alloc}^2 + \frac{g_c^{alloc}}{\tilde{\rho}} \right).$$

Define consumption equivalent welfare as λ^{alloc}:

$$U_{ss}^{sp}(\lambda^{alloc}) = U_{ss}^{alloc}(1) \text{ with}$$

$$\log \lambda^{alloc} = \underbrace{\log c_0^{alloc} - \log c_0^{sp}}_{\text{Level term}} - \underbrace{\frac{\tilde{\kappa}}{2} \left(\tilde{x}_{alloc}^2 - \tilde{x}_{sp}^2 \right)}_{\text{Privacy term}} + \underbrace{\frac{g_c^{alloc} - g_c^{sp}}{\tilde{\rho}}}_{\text{Growth term}}$$

Note: The x_{it} terms drop out because scaled by $1/N$

Allocations

Allocation	Data Sharing "own" x	"others" $\tilde{x}$	Firm size ν	Variety $N/L = \psi$	Consu-mption c	Growth g	Creative Destruct. δ
Social Planner	0.66	0.66	1304	665	18.6	0.67%	0.0870
Consumers Own Data	0.59	0.59	1482	594	18.3	0.67%	0.0696
Firms Own Data	1	0.16	1838	491	16.0	0.67%	0.0052
Outlaw Sharing	1	0	2000	455	7.3	0.50%	0

- Firms overuse their own data and undershare with others
- Consumers share less data than planner, but not by much
- Growth rate scale effect is modest, level differences are large

Consumption Equivalent Welfare

Allocation	Welfare λ	$\log \lambda$	Level term	Privacy term	Growth term
Optimal Allocation	1	0	..	..	..
Consumers Own Data	0.9886	-0.0115	-0.0202	0.0087	0.0000
Firms Own Data	0.8917	-0.1146	-0.1555	0.0409	0.0000
Outlaw Sharing	0.3429	-1.0703	-0.9399	0.0435	-0.1739

- Outlaw sharing: particularly harmful law (66 percent worse!)
- Firms own data: substantially lower welfare (11 percent worse)
- Consumers own data: nearly optimal (1 or 2 percent worse)

Implications for IO

- Firms that use data might grow fast compared to those that don't
- Firms would like to merge into one single economy-wide firm
  - Implications for antitrust
- Targeted mandatory sharing?
  - E.g., airplane safety (after a crash)
- What are the costs of forced sharing?
  - Disincentive to create data
    (in MS, data is aggregate, good approx?)
  - Data as a barrier to entry
    (extension to quality ladder model)

Data versus Ideas: Excludability

- Maybe technologically easier to transmit data than idea (usb key vs. education) . . .
- But data can be encrypted and monitored
- Data seems highly excludable
  - Idea: use machine learning to train self-driving car algorithm
  - ML needs lots of data. Each firm gathering own data

The Boundaries of Data Diffusion: Firms and Countries

- How does data diffuse across firms and countries?
  - Ideas eventually diffuse across firms or countries, so no country scale effect (e.g. HK vs China)
  - What about data?
  - Intellectual property rights
- Scale effects and country size
  - Larger countries may have an important advantage as data grows in importance
- Scale effects and institutions
  - What if China mandates data sharing across Chinese firms and U.S. has no such policy

Conclusion

- Nonrival data $\Rightarrow$ large social gain from sharing data
- If firms own data, they may:
  - privately use more data than consumers/planner would
  - share less data across firms than consumers/planner would
- Nonrivalry $\Rightarrow$ Laws that outlaw sharing could be very harmful
- Consumers owning data good at balancing privacy and sharing