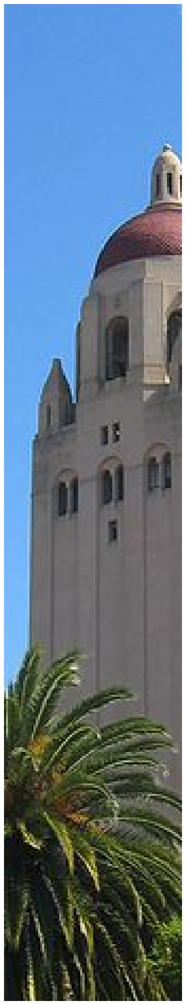

# Kortum (1997 Ema): “Research, Patenting, and Technological Change”

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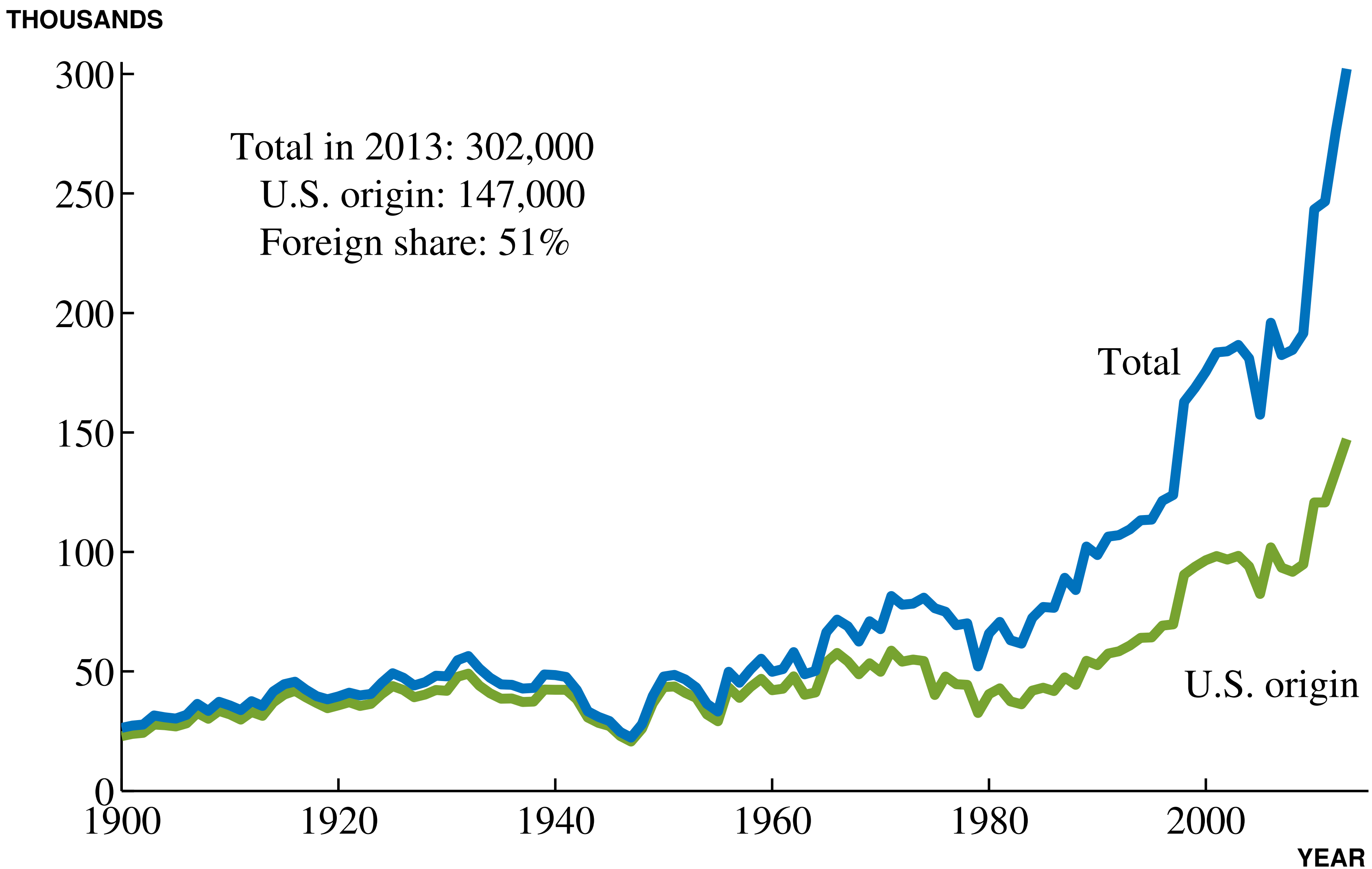
# Overview

- Construct a growth model consistent with these facts:
  - Exponential growth in research (scientists)
  - No growth in the number of patents granted to U.S. inventors

    $\Rightarrow$ large decline in Patents per Researcher
  - Exponential growth in output per worker

# Patents in the U.S.




1915 = 1950 = 1990 ≈ 40,000

## How does Kortum do this?

- Quality ladder model, a la Aghion and Howitt (1992)
  - Each idea is a proportional improvement in productivity (ten percent rather than ten units). E.g. $q \equiv 1.10$

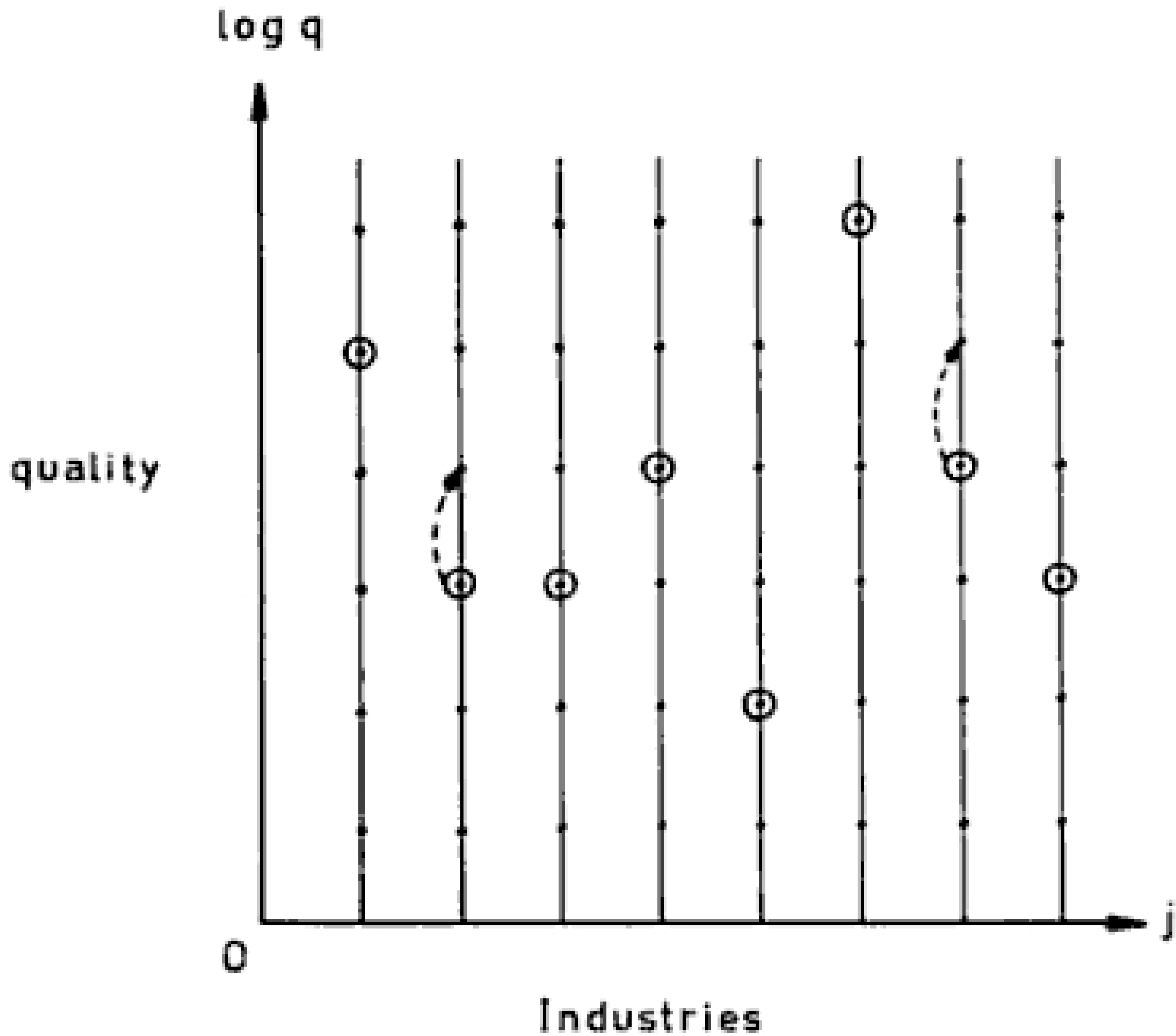
$$Y_t = q^{N_t} K_t^{\alpha} L_t^{1-\alpha}, \quad A_t \equiv q^{N_t}$$

$$\log A_t = N_t \log q$$

$$\Rightarrow \quad \frac{\dot{A}_t}{A_t} = \dot{N}_t \log q$$

- Also, make ideas harder to obtain over time $\Rightarrow$ it takes more and more researchers to discover the next idea
  - So TFP growth tied to growth in number of researchers. (Also, Segerstrom 1998 AER)

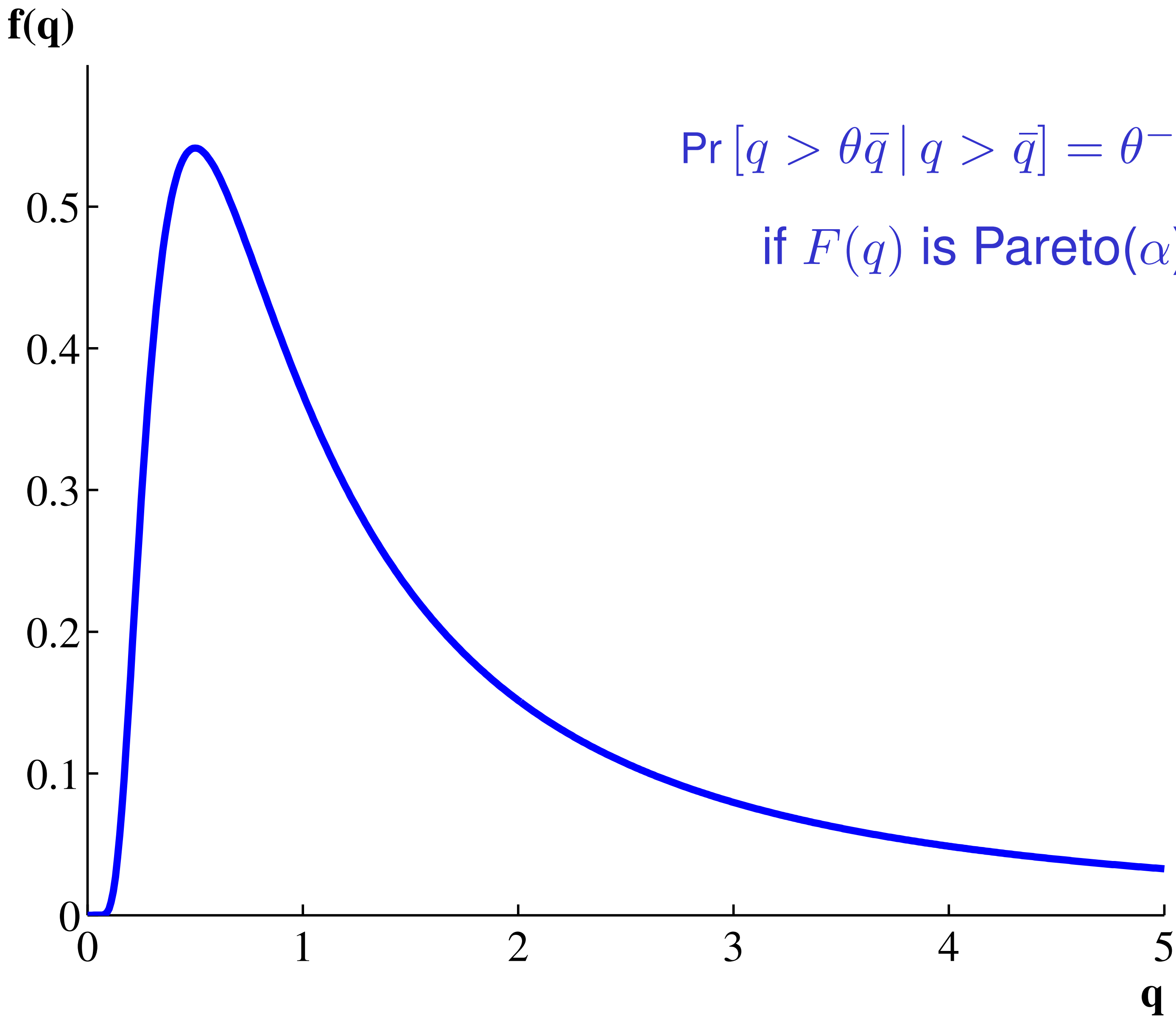
# Quality Ladders (Aghion-Howitt / Grossman-Helpman)



## Other Insights

- Search model
  - Ideas = draws from a probability distribution
  - All you care about is the best idea (Evenson and Kislev, 1976)
- Technical: Extreme Value Theory and Pareto Distributions
  - Key to exponential growth is that the stationary part of the search distribution have a Pareto upper tail
  - The probability of drawing a new idea that is 2% better than the frontier is invariant to the level of the frontier
  - Incomes versus heights

# Drawing Ideas from a Distribution




Pr $\left[q > \theta\bar{q} \,|\, q > \bar{q}\right] = \theta^{-\alpha}$

if $F(q)$ is Pareto($\alpha$)

## Extreme Value Theory

- Let $N$ be the number of draws from a distribution, and consider the distribution of the largest draw as $N \to \infty$.
- For a distribution with unbounded support, the max will go to infinity, so we have to “normalize” it somehow.
- Extreme Value Theorem (e.g. Galambos 1987) If a limiting distribution exists, then it takes one of three forms: Fréchet, Weibull, Gumbel.
- Kind of like the Central Limit Theorem (normalized mean is asymptotically normal).

## Fundamental Example

- Suppose $x^*$ is your income, equal to the maximum of $N$ iid draws from some distribution $F(\cdot)$.
- What is the distribution of $x^*$?

$$\begin{aligned} G(z) &\equiv \text{Pr}\left[x* < z\right] \\ &= \text{Pr}\left[x_1 < z\right] \cdot \text{Pr}\left[x_2 < z\right] \cdot \ldots \cdot \text{Pr}\left[x_N < z\right] \\ &= (F(z))^N. \end{aligned}$$

- Suppose $F(\cdot)$ is Pareto: $F(z) = 1 - (z/\gamma)^{-\alpha}$.

$$G(z) = \left(1 - (z/\gamma)^{-\alpha}\right)^N$$

But this goes to zero as $N \to \infty$. So we need to normalize somehow.

- Guess:

$$\text{Pr}\left[x^* < zN^\beta\right] = G(zN^\beta) = \left(1 - (zN^\beta/\gamma)^{-\alpha}\right)^N$$

- Recall $e^y \equiv \lim_{N\to\infty}(1 + y/N)^N \Rightarrow$ choose $\beta = 1/\alpha$:
- Therefore

$$\begin{aligned} G(zN^{1/\alpha}) &= \text{Pr}\left[x^* < zN^{1/\alpha}\right] = (1 - y/N)^N \\ &\to e^{-y}. \end{aligned}$$

where $y \equiv (z/\gamma)^{-\alpha}$.

- So as $N$ gets large

$$\text{Pr}\left[x^* < N^{1/\alpha}z\right] = e^{-(z/\gamma)^{-\alpha}} \quad = e^{-(1-F(z))}$$

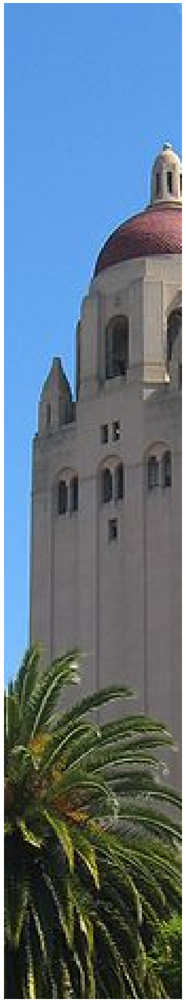
$$\Rightarrow \quad \text{Pr}\left[N^{-1/\alpha}x^* < z\right] = e^{-(z/\gamma)^{-\alpha}}$$

- And this is the *Fréchet* distribution!
- Therefore, as $N$ gets large

$$E[N^{-1/\alpha}x^*] = \gamma\Gamma(1 - 1/\alpha)$$

$$\Rightarrow E[x^*] \approx N^{1/\alpha}\gamma\Gamma(1 - 1/\alpha)$$

- So the maximum value scales as $N^{1/\alpha}$
- Note: If $F(x)$ does not have a Pareto upper tail, then the scaling is less than a power function of $N$.



# Model

# The Economic Environment

Preferences $$U_0 = \int_0^\infty e^{-\rho t} \exp(\int_0^1 \log C_{jt}\, dj)\, dt$$

Production $$C_{jt} = q_{jt}\ell_{jt}$$

Resource constraint $$\int_0^1 \ell_{jt} dj + R_t = L_t = L_0 e^{nt}$$

Research Poisson process, next slide

## Research and New Ideas

- An idea is a quality $q \sim F(q;K)$ and a sector $j \sim$ Uniform[0,1]
- Discovery is a Poisson process

| | |
|---|---|
| $R_t$ | Researchers |
| $R_t dt$ | Flow of new ideas per unit time |
| $R_t(1-F(q;K))dt$ | Flow of ideas that exceed quality level $q$ |

  - The length of time until an innovation occurs is exponentially distributed with parameter $R_t$
- $K$ is cumulative stock of research ("knowledge")

$$\dot{K}_t = R_t$$

# Key Assumption 2.1

$$Pr(Q \leq q; K) \equiv F(q; K) = 1 - S(K)(1 - F(q))$$

$$H(q) \equiv 1 - F(q) = Pr(Q > q)$$

$$\tilde{H}(q; K) \equiv 1 - F(q; K) = Pr(Q > q; K) = S(K)H(q)$$

- $S(K)$: Spillover function. Ex: $S(K) = K^{\gamma}$
- $F(q)$: Stationary search distribution
- If $\gamma = 0$, then $F(q; K) = F(q)$
- As $K$ grows, more of the mass is concentrated at higher values of $q$.

## Proposition 2.1

- The distribution $G_1$ of the state of the art productivity for producing in sector $j$ is, for a fixed $K$,

$$G_1(z;K) = \exp\{-(1 - F(z))\Sigma(K)\}$$

where $\Sigma(K) \equiv \int_0^K S(x)dx$ (cumulative spillovers).

- Remarks
  - $G_1(z;K)$ is an Extreme Value Distribution
  - Also the distribution of max productivity across sectors.
  - Research enters through $K$.
  - With Poisson process, things aggregate nicely.

# Proof

- $G_1(z)$ is probability frontier is less than $z$
- What is probability that no discovery occurs?

$$Pr(\text{No discovery}) = e^{-R(s)ds}$$

$$Pr(\text{No discovery} \geq z) = e^{-R(s)(1-F(z;K(s)))ds}$$

$\Rightarrow$

$$G_1(z;K(s+ds)) = G_1(z;K(s)) \cdot e^{-R(s)(1-F(z;K(s)))ds}$$

Prob < z tomorrow

Prob < z today

Prob no discovery > z

- Integrate this differential equation to get the result.

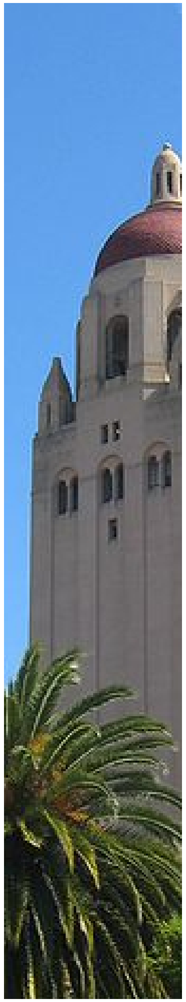
## Allocation of Resources

- An allocation in this economy is $\{R_t, \{\ell_{jt}\}\}$.
- To see many of the useful results, we can focus on a Rule of Thumb allocation:

$$R_t = \bar{s} L_t$$

$$\ell_{jt} = \bar{\ell} = (1 - \bar{s}) L_t$$

- Optimal to allocate labor equally across sectors given symmetry.



# Analyzing the Economy

Constant patents with growing research?

# Constant Patents??

- What fraction of new ideas are improvements (patentable)?

$$p(K) = \int_{q_0}^{\infty} \underbrace{(1 - F(z;K))}_{\text{prob idea exceeds } z} \, dG_1(z;K)$$

- Substituting for $G_1(\cdot)$ and making a change of variables $x \equiv S(K)(1 - F(z))$ when integrating gives

$$p(K) = \frac{S(K)}{\Sigma(K)} \cdot (1 - e^{-\Sigma(K)/S(K)})$$

- The fraction of new ideas that will be improvements depends on the spillover function
  - Independent of the stationary search distn $F(q)$!

## Remarks on $p(K) = \frac{S(K)}{\Sigma(K)} \cdot (1 - e^{-\Sigma(K)/S(K)})$

- Independence of $F(q)$ is wellknown in theory of recordbreaking (example: track and field)
  - Depends on the rate at which the stationary distribution shifts out (better shoes, track, nutrition)
  - Partial intuition: the distn of records itself depends on $F(\cdot)$, but what fraction get broken depends on how quickly we march down the tail
- Example: $S(K) = K^\gamma \Rightarrow \Sigma(K) = K^{1+\gamma}/(1+\gamma)$
  $\Rightarrow S/\Sigma = (1+\gamma)/K$

$$p(K) = \frac{1+\gamma}{K} \cdot (1 - e^{-K/(1+\gamma)})$$

$\Rightarrow$ Looks like $1/K$ for $K$ large and $\gamma = 0$...

## Glick (1978): Math of Record-Breaking

- Begins with a very simple example...
- Consider a sequence of daily weather observations — temperatures
- The first is obviously a record high
- The 2nd has a 50% chance of being a record (viewed before any data are recorded)
- Exchangeability: The probability that day $n$ is a record is $1/n$
- Independent of the distribution of temperatures.

## Patenting

- $R(t)$ ideas, $p(K)$ improve, so total patenting is

$$I_t = R_t \cdot p(K_t) = R_t \frac{S(K_t)}{\Sigma(K_t)} \left(1 - e^{-\Sigma(K_t)/S(K_t)}\right)$$

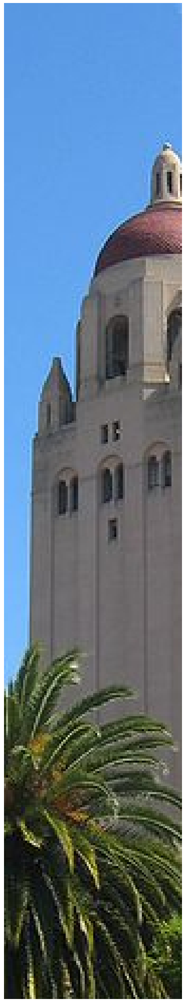
  So the rise in $R_t$ can be offset by a decline in $p(K_t)$.

- Proposition 3.1 says that for $I_t$ to be constant while $R$ grows at rate $n$, $S(K)$ must be a power function.

$$I_t = R_t \frac{1+\gamma}{K_t} (1 - e^{-K_t/(1+\gamma)})$$

$$\dot{K}_t = R_t \Rightarrow \frac{\dot{K}_t}{K_t} = \frac{R_t}{K_t} \rightarrow n$$

$$\boxed{I^* = n(1+\gamma)}$$



# Analyzing the Economy

Exponential income growth with constant patents?

## Productivity

- Given symmetry, a productivity index is

$$A(K_t) \equiv \int_{q_0}^{\infty} z dG_1(z; K_t)$$

  Proportional to output per worker.

- This does depend on the shape of $F(q)$. Examples:
  1. Pareto (incomes): $H(q) = q^{-1/\lambda}$
     $\Rightarrow A(K) = c_1 K^{\lambda(1+\gamma)}$
  2. Exponential (heights): $H(q) = e^{-q/\lambda}$
     $\Rightarrow A(K) = c_0 + c_1 \log K$
  3. Uniform (bounded): $H(q) = 1 - q/\lambda$
     $\Rightarrow A(K) = c_0 - \frac{c_1}{K^{1+\gamma}}$

## Growth Implications

- Pareto:

$$\frac{\dot{A}_t}{A_t} = \lambda(1+\gamma)\frac{\dot{K}_t}{K_t} \to \lambda(1+\gamma)n$$

  Sustained exponential growth! ($G_1(\cdot)$ is Fréchet).

- Exponential:

$$\dot{A}_t = c_1\frac{\dot{K}_t}{K_t} \to c_1 n \quad \Rightarrow \quad \frac{\dot{A}_t}{A_t} \to 0$$

  Arithmetic growth, but not exponential growth.

- Uniform:

$$K \to \infty \Rightarrow A \to c_0$$

  Stagnation — no long run growth.

# Growth (continued)

- Whether or not model can sustain growth depends on the shape of the upper tail of $f(q)$.

PROPOSITION 3.2: *As the stock of research K approaches infinity, the limiting form of the distribution of the technological frontier $G_1$ is either Fréchet, Gumbel, or Weibull (subject to normalizing sequences). In all three cases productivity satisfies* $\lim_{K\to\infty}\{((A_{k'K}/A_K)-1)/((A_{k'K}/A_K)-1)\}<\infty$ *for any* $0<k''<k'<\infty$. *If and only if the limiting form of $G_1$ is Fréchet does productivity satisfy*

$$\lim_{K\to\infty}(A_{kK}/A_K)=k^b$$

*for some $b>0$ and for all $k>0$. Stationary search distributions F leading to the Fréchet have unbounded upper support and satisfy* $\lim_{x\to\infty}\{(1-F(xa))/(1-F(x))\}=a^{-b}$, *for all $a>0$ and for some $b>0$.*

If and only if the upper tail of $f(q)$ is a power function, then exponential growth can be sustained. ($A(kK)/A(K)=k^b$).

## Remarks

- In a very different setup, Kortum gets the same result we got from the Romer model with $\phi < 1$: per capita income growth is tied to the rate of population growth.
  - $I_t = R_t \cdot p(K_t)$: growth in number of researchers is exactly offset by increased difficulty of finding a useful new idea.
  - If $F(q)$ is Pareto, then $H(q) = q^{-\alpha}$. So Pr(Idea is a 5% improvement | Idea is an improvement) is constant.
    $\Rightarrow$ Ideas are proportional improvements (a la quality ladder models)
    $\Rightarrow$ a constant flow of patents is consistent with exponential growth.
- What about $\phi = 1$ case? Kortum emphasizes that there is no $F(\cdot)$ such that the limiting distribution yields $A = e^{\lambda K}$.

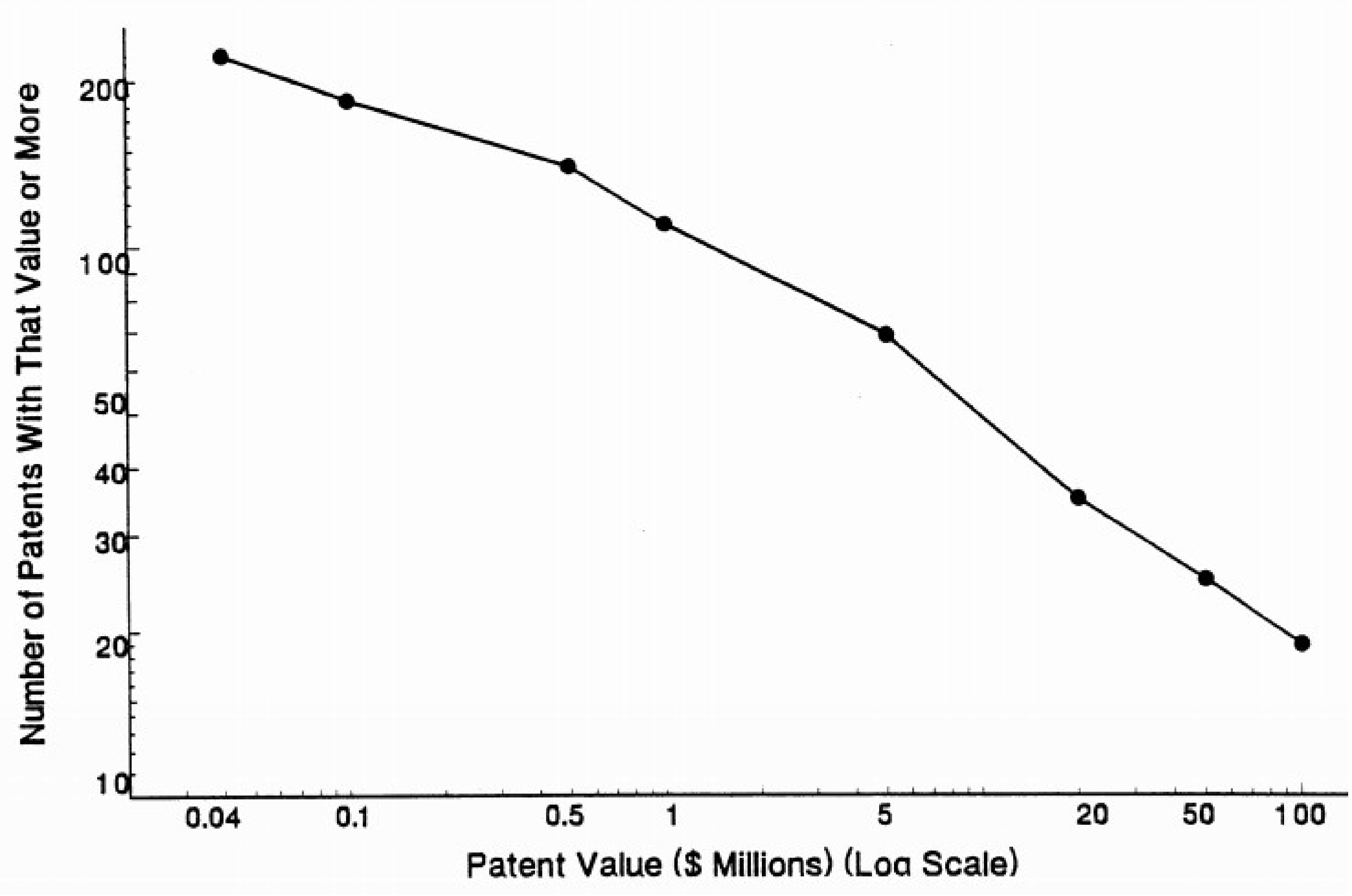
## Remarks (continued)

- Potential problem: total patents in U.S. is growing in recent decades?
- Kortum analyzes equilibrium and optimal allocations
  - Knowledge spillovers mean the equilibrium may feature too little investment in research.
  - Counterbalancing that is a business stealing effect: some of the new innovator's profits come at the expense of existing entrepreneurs.

## Applications

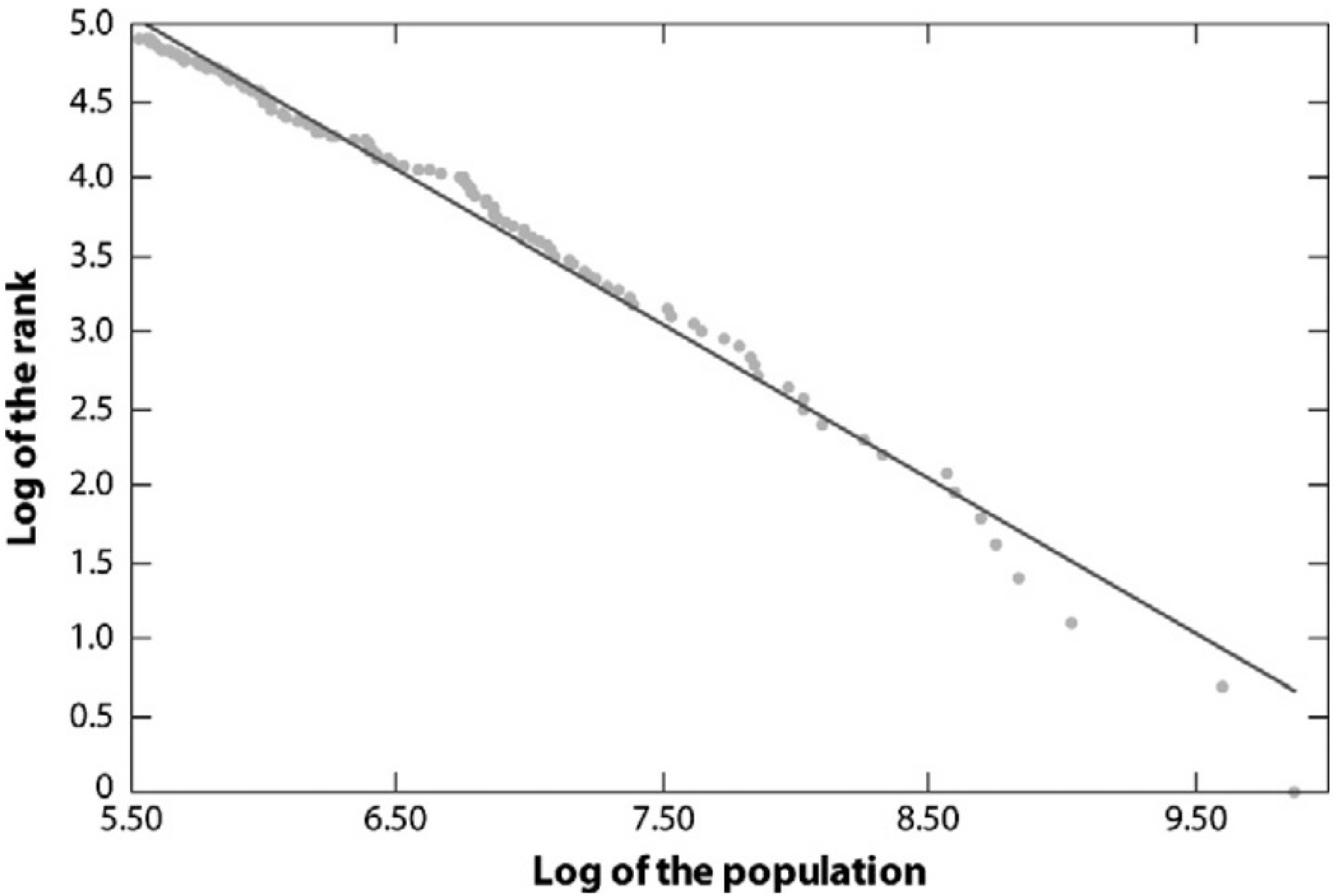
- Pareto and Fréchet distributions show up in many places now in economics
  - Zipf's Law: Size = 1/Rank for cities, etc. (Gabaix 1999). $\Pr\left[Size > s\right] \sim 1/s$, which is Pareto with parameter = 1. (Pareto $\Rightarrow$ exp growth and exp growth $\Rightarrow$ Pareto!)
  - Eaton and Kortum (2002 Ema) on trade
  - Erzo Luttmer (2010) on the size distribution of firms
  - Several recent Lucas papers (with Alvarez, Buera, and Moll); Perla and Tonetti (2014)
  - Hsieh, Hurst, Jones, and Klenow, "The Allocation of Talent and U.S. Economic Growth"
  - Jones and Kim (2018 JPE) on top income inequality

# Pareto Dist for U.S. Patent Values (Harhoff et al 2003)

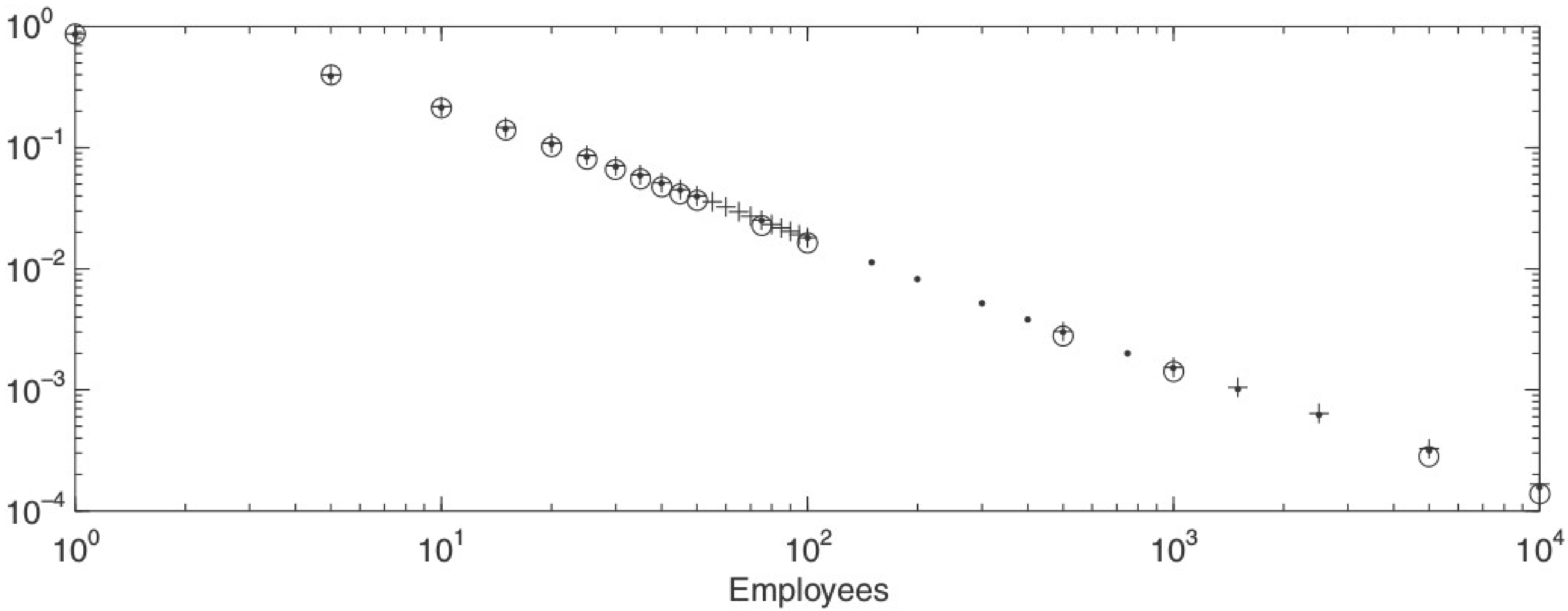



Harhoff, Scherer, Vopel “Exploring the tail...”

# Zipf's Law for U.S. Cities (Gabaix 1999)



# Zipf's Law for U.S. Firms (Luttmer 2010)




For 1992, 2000, and 2006

# Pareto Distribution for Top Incomes