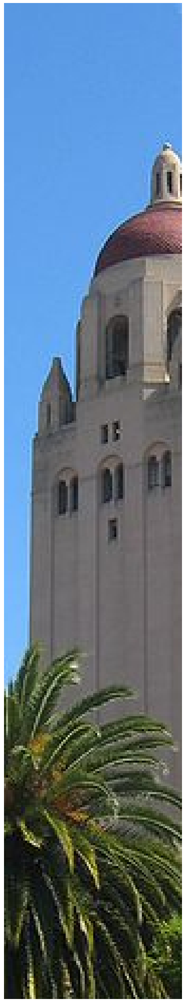

Growth and Ideas
Romer (1990) and Jones (2005)

Chad Jones

Stanford GSB

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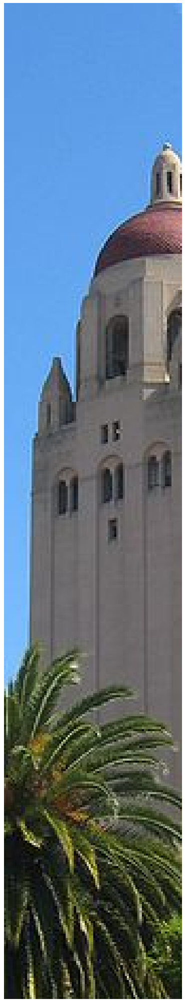
Outline: How do we understand frontier growth?

- A Discussion of Ideas
- Simple Model
- Full Model, and various resource allocations
- Applications



A Discussion of Ideas

Solow and Romer

- Robert Solow (1950s)
  - Capital versus Labor
  - Cannot sustain long-run growth
- Paul Romer (1990s)
  - Objects versus Ideas
  - Sustains long-run growth
  - Wide-ranging implications for intellectual property, antitrust policy, international trade, the limits to growth, sources of “catch-up” growth

Romer’s insight: Economic growth is sustained by discovering better and better ways to use the finite resources available to us

The Idea Diagram

Ideas $\rightarrow$ Nonrivalry $\rightarrow$ Increasing returns $\nearrow$ Long-run growth

$\searrow$ Problems with pure competition

The Essence of Romer's Insight

- **Question:** In generalizing from the neoclassical model to incorporate ideas (A), why do we write the PF as

$$Y = AK^{\alpha}L^{1-\alpha} \qquad (*)$$

instead of

$$Y = A^{\alpha}K^{\beta}L^{1-\alpha-\beta}$$

- Does A go inside the CRS or outside?
  - The "default" (*) is sometimes used, e.g. 1960s
  - 1980s: Griliches et al. put knowledge capital inside CRS

IRS and the Standard Replication Argument

- Familiar notation, but now let A_t denote the "stock of knowledge" or ideas:

$$Y_t = F(K_t, L_t, A_t) = A_t K_t^{\alpha} L_t^{1-\alpha}$$

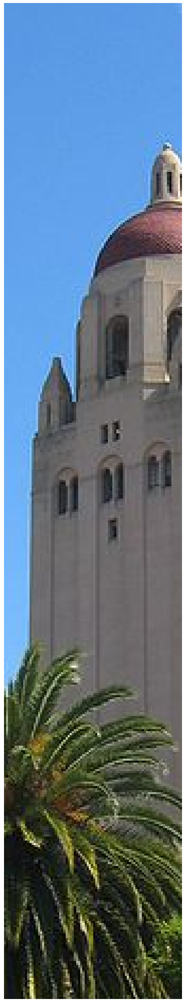
- Constant returns to scale in K and L holding knowledge fixed. Why?

$$F(\lambda K, \lambda L, A) = \lambda \times F(K, L, A)$$

- But therefore increasing returns in K, L, and A together!

$$F(\lambda K, \lambda L, \lambda A) > F(\lambda K, \lambda L, A)$$

- Economics is quite straightforward:
  - Replication argument implies CRS to objects
  - Therefore there must be IRS to objects and ideas



A Simple Model

The Simple Model

Production of final good	$Y_t = A_t^{\sigma} L_{Yt}$
Production of ideas	$\dot{A}_t = \nu(A_t) L_{At} = \nu L_{At} A_t^{\phi}$
Resource constraint	$L_{Yt} + L_{At} = L_t = L_0 e^{nt}$
Allocation of labor	$L_{At} = \bar{s} L_t, \quad 0 < \bar{s} < 1$

$\phi > 0$: Standing on shoulders

$\phi < 0$: "Fishing out"

Solving

$$(1)\ Y_t = A_t^{\sigma} L_{Yt}$$

$$(2)\ \dot{A}_t = \nu(A_t) L_{At} = \nu L_{At} A_t^{\phi}$$

$$(3)\ L_{Yt} + L_{At} = L_t = L_0 e^{nt}$$

$$(4)\ L_{At} = \bar{s} L_t, \quad 0 < \bar{s} < 1$$

Discussion: $g_y = \frac{\sigma n}{1-\phi}$

- Growth rate is the product of (1) degree of increasing returns and (2) rate at which scale is rising.
- More people $\Rightarrow$ more ideas $\Rightarrow$ more income per capita.
- But China is huge while Hong Kong is tiny?
- But Africa has fast population growth while Europe has slow?
- What happens if $\bar{s}$ permanently increases?

From IRS to Growth

- Objects: Add one computer $\Rightarrow$ make one worker more productive.

  Output per worker $\sim$ # of computers per worker

- Ideas: Add one new idea $\Rightarrow$ make everyone better off.
  - E.g. computer code for 1st spreadsheet or the software protocols for the internet itself

  Income per person $\sim$ the aggregate stock of knowledge, not on the number of ideas per person.

But it is easy to make aggregates grow: population growth!

IRS $\Rightarrow$ bigger is better.

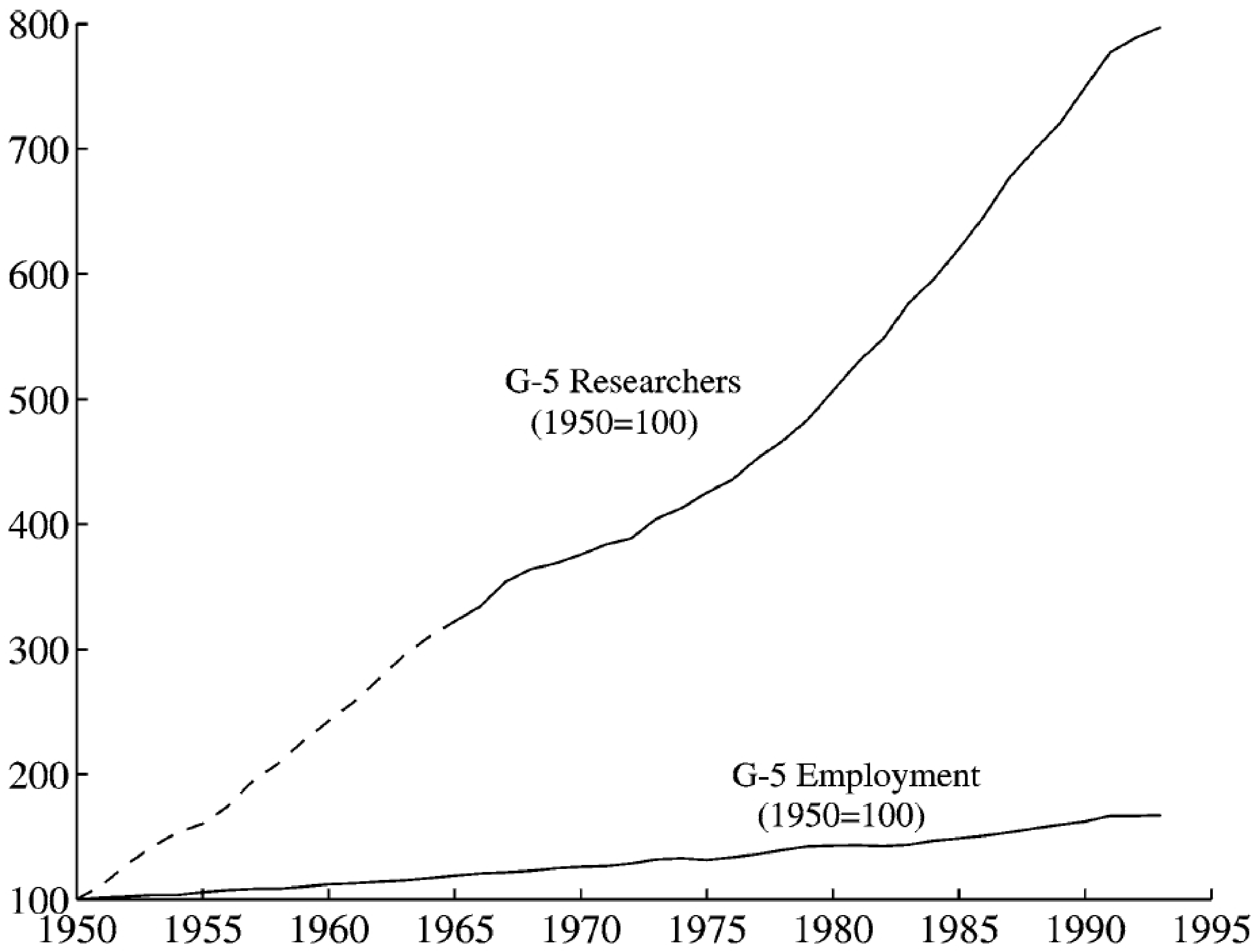
Romer (1990) and Scale Effects

- Romer (1990) / Aghion-Howitt (1992) / Grossman-Helpman (1991) have $\phi = 1$

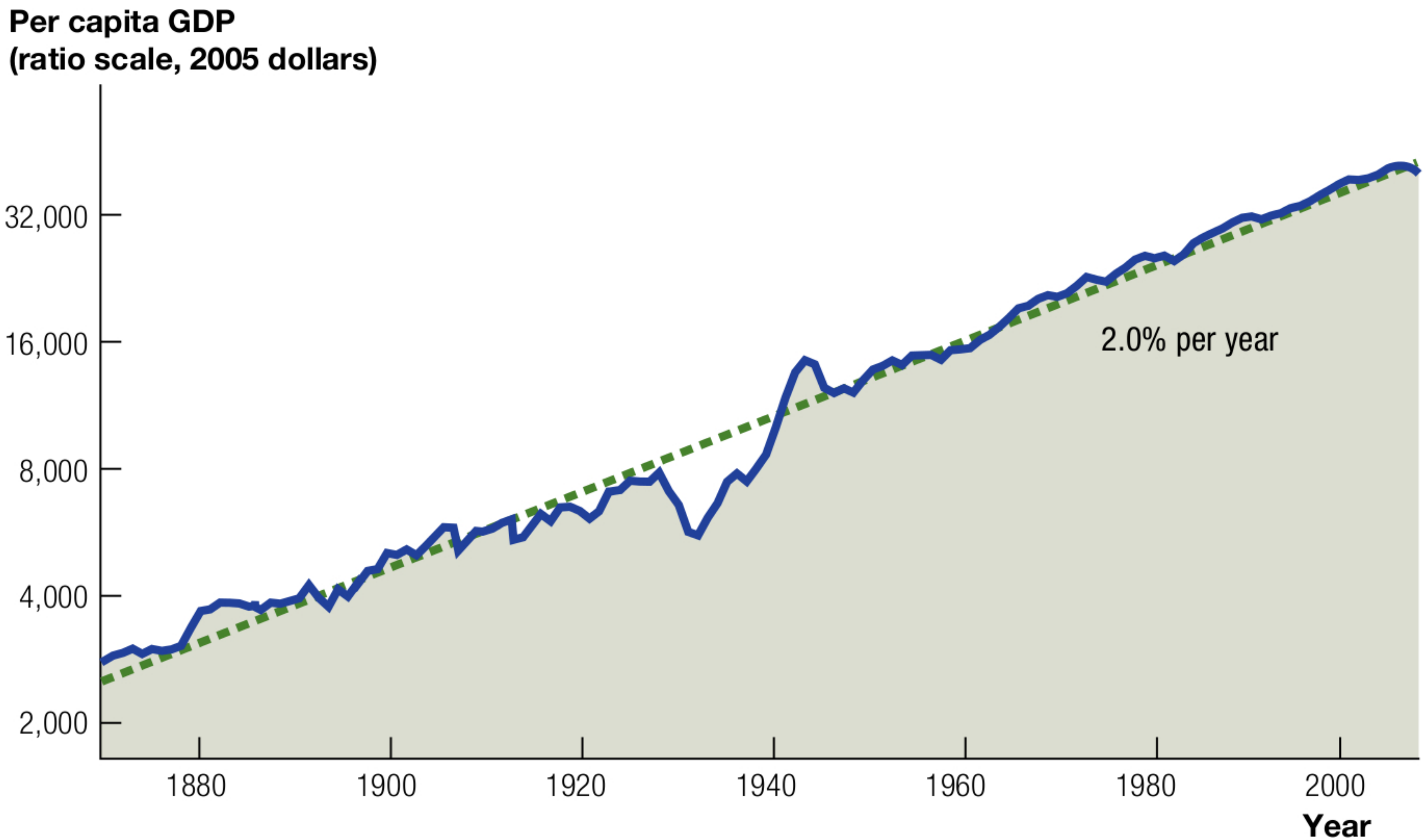
$$\frac{\dot{A}_t}{A_t} = \nu L_{At} = \nu \bar{s} L_t$$

- Policy effect: $\uparrow \bar{s}$ raises long-run growth.
- Problem (Jones 1995): Growth in number of researchers (or population) implies accelerating growth
  - True over the very long run (Kremer 1993)
  - But not in the 20th century for the United States.

Researchers in Advanced Countries



U.S. GDP per Person



Scale Effects

- Strong versus weak (versus none)
  - Strong: $\phi = 1$ — scale affects the growth rate in LR
  - Weak: $\phi < 1$ — scale affects the level in LR
- Literature
  - Young (1998), Peretto (1998), Dinopoulos-Thompson (1998), Howitt (1999); discussed in Jones (1999 AEAPP)
- No role for scale? What about nonrivalry?

What happens if $\phi > 1$?

What happens if $\phi > 1$?

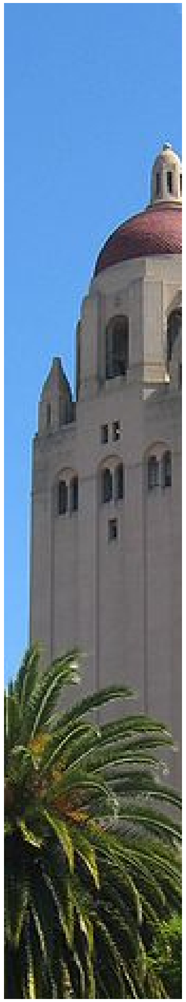
- Let $\phi = 1 + \epsilon$ and assume population is constant.

$$\dot{A}_t = \nu L A_t^{1+\epsilon}$$

- Integrating this differential equation from 0 to t gives

$$A_t = \left(\frac{1}{A_0^{-\epsilon} - \epsilon \nu L t} \right)^{1/\epsilon}$$

$\Rightarrow$ There exists a finite date t^* when A becomes infinite!



The Full Romer Model

Overview of Romer Model

- All of the key insights regarding nonrivalry and increasing returns.
- In addition, a significant troubling question was how to decentralize the allocation of resources
  - With increasing returns, why doesn't one firm come to dominate?
  - Imperfect competition, a la Spence (1976), Dixit-Stiglitz (1977), and Ethier (1982, production side)

The Economic Environment

Final good
$$Y_t = \left(\int_0^{A_t} x_{it}^{\theta}\, di\right)^{\alpha/\theta} L_{Yt}^{1-\alpha}, \quad 0 < \theta < 1$$

Capital
$$\dot{K}_t = I_t - \delta K_t, \quad C_t + I_t = Y_t$$

Production of ideas
$$\dot{A}_t = \nu L_{At}^{\lambda} A_t^{\phi}, \quad \phi < 1$$

Resource constraint (capital)
$$\int_0^{A_t} x_{it}\, di = K_t$$

Resource constraint (labor)
$$L_{Yt} + L_{At} = L_t = L_0 e^{nt}$$

Preferences
$$U_t = \int_t^{\infty} L_s u(c_s) e^{-\rho(s-t)}\, ds, \quad u(c) = \frac{c^{1-\zeta}-1}{1-\zeta}$$

Allocating Resources

- Environment features
  - 9 unknowns: $Y, A, \{x_i\}, L_Y, L_A, K, C, I, L$
  - 6 1/2 equations (use $\int x_{it} di = K$ to get $\bar{x}_t$ below)

  Need 2 1/2 more equations to complete

- A rule of thumb allocation in this economy features

$$I_t / Y_t = \bar{s}_K \in (0, 1)$$

$$L_{At} / L_t = \bar{s}_A \in (0, 1)$$

$$x_{it} = \bar{x}_t \text{ for all } i \in [0, A_t].$$

Balanced Growth Path

A balanced growth path in this economy is a situation in which all variables grow at constant exponential rates (possibly zero) forever.

Result 1 (Rule of Thumb)

(a) Symmetry of capital goods implies

$$Y_t = A_t^{\sigma} K_t^{\alpha} L_{Yt}^{1-\alpha}, \quad \sigma \equiv \alpha(\frac{1}{\theta} - 1)$$

(b) Along BGP, y depends on the total stock of ideas:

$$y_t^* = \left(\frac{\bar{s}_K}{n + g_k + \delta}\right)^{\frac{\alpha}{1-\alpha}} A_t^{*\frac{\sigma}{1-\alpha}}.$$

(c) Along BGP, the stock of ideas depends on the number of researchers

$$A_t^* = \left(\frac{\nu}{g_A}\right)^{\frac{1}{1-\phi}} L_{At}^{*\frac{\lambda}{1-\phi}}$$

Result 1 (continued)

(d) Combining these last two results(b) and (c),

$$y_t^* \propto L_{At}^{*\gamma} = (\bar{s}_A L_t)^{\gamma}, \quad \gamma \equiv \frac{\sigma}{1-\alpha} \cdot \frac{\lambda}{1-\phi}$$

(e) Finally, TLAD gives the growth rates

$$g_y = g_k = \frac{\sigma}{1-\alpha} g_A = \gamma g_{L_A} = g \equiv \gamma n.$$

The Optimal Allocation

- The optimal allocation features time paths $\{c_t, s_{At}, \{x_{it}\}\}_{t=0}^{\infty}$ that maximize utility U_t at each point in time given the economic environment.
- Using symmetry of x_{it}:

$$\max_{\{c_t, s_{At}\}} \int_0^{\infty} L_t u(c_t) e^{-\rho t}\, dt$$

subject to

$$y_t = A_t^{\sigma} k_t^{\alpha} (1 - s_{At})^{1-\alpha}$$

$$\dot{A}_t = \nu (s_{At} L_t)^{\lambda} A_t^{\phi}$$

$$\dot{k}_t = y_t - c_t - (n + \delta) k_t$$

The Maximum Principle in the Romer Model

- The Hamiltonian for the optimal allocation is

$$\mathcal{H}_t = u(c_t) + \mu_{1t}(y_t - c_t - (n+\delta)k_t) + \mu_{2t}\nu s_{At}^{\lambda} N_t^{\lambda} A_t^{\phi},$$

  where $y_t = A_t^{\sigma} k_t^{\alpha}(1 - s_{At})^{1-\alpha}$.

- First order necessary conditions

$$\partial\mathcal{H}_t/\partial c_t = 0, \quad \partial\mathcal{H}_t/\partial s_{At} = 0$$

$$\bar{\rho} = \frac{\partial\mathcal{H}_t/\partial k_t}{\mu_{1t}} + \frac{\dot{\mu}_{1t}}{\mu_{1t}}, \quad \bar{\rho} = \frac{\partial\mathcal{H}_t/\partial A_t}{\mu_{2t}} + \frac{\dot{\mu}_{2t}}{\mu_{2t}}$$

$$\lim_{t\to\infty} \mu_{1t} e^{-\bar{\rho}t} k_t = 0, \quad \lim_{t\to\infty} \mu_{2t} e^{-\bar{\rho}t} A_t = 0$$

Result 2 (Optimal Allocation)

(a) All of Result 1 continues to hold.
   - Same long-run growth rate!

(b) Optimal consumption satisfies the Euler equation

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\zeta}\left(\frac{\partial y_t}{\partial k_t} - \delta - \rho\right)$$

(c) Optimal saving along BGP

$$s_K^{op} = \frac{\alpha(n+g+\delta)}{\rho+\delta+\zeta g}$$

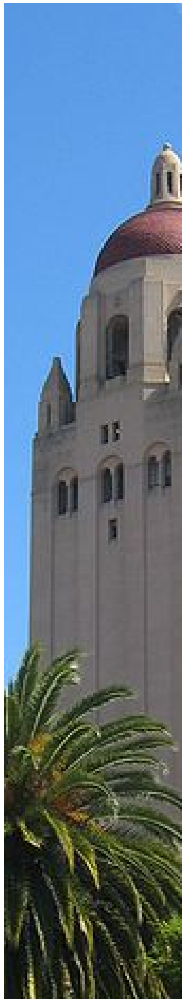
Result 2 (Optimal Allocation — continued)

(d) Optimal labor allocation equates marginal products

$$\frac{s_{At}^{op}}{1 - s_{At}^{op}} = \frac{\frac{\mu_{2t}}{\mu_{1t}} \lambda \dot{A}_t}{(1-\alpha) y_t}$$

Along the BGP, this implies

$$\frac{s_{A}^{op}}{1 - s_{A}^{op}} = \frac{\frac{\sigma Y_t / A_t}{r^* - (g_Y - g_A) - \phi g_A} \lambda \dot{A}_t}{(1-\alpha) Y_t}$$



Equilibrium in the Romer Model

Market Equilibrium with Imperfect Competition

- Increasing returns means a perfectly competitive equilibrium does not exist.
- Romer (1990) adds infinitely lived patents on ideas to set up an equilibrium with imperfect competition as in Spence (1976), Dixit-Stiglitz (1977), Ethier (1982).
- Partial excludability associated with patents leads individuals to exert effort to discover new ideas.
- We define the decision problems first and then the equilibrium.

Households: Problem (HH)

Taking the time path of $\{w_t, r_t\}$ as given, HHs solve

$$\max_{\{c_t\}} \int_0^\infty u(c_t)e^{-\bar{\rho}t}\, dt$$

subject to

$$\dot{v}_t = (r_t - n)v_t + w_t - c_t, \quad v_0 \;\; given$$

$$\lim_{t\to\infty} v_t e^{-\int_0^t (r_s - n)ds} \geq 0$$

where v_t is financial wealth, w_t is the wage, and r_t is the interest rate.

Final Goods Producers: Problem (FG)

- Perfectly competitive
- At each t, taking w_t, A_t, and $\{p_{it}\}$ as given, the representative firm chooses L_{Yt} and $\{x_{it}\}$ to solve

$$\max_{\{x_{it}\}, L_{Yt}} \left(\int_0^{A_t} x_{it}^{\theta} \, di \right)^{\alpha/\theta} L_{Yt}^{1-\alpha} - w_t L_{Yt} - \int_0^{A_t} p_{it} x_{it} di.$$

Capital Goods Producers: Problem (CG)

- Monopolistic competition — a patent gives a single firm the exclusive right to produce each variety.
- At each t and for each capital good i, taking r_t and $x(\cdot)$ as given, a monopolist solves

$$\max_{p_{it}} \pi_{it} \equiv (p_{it} - r_t - \delta)x(p_{it})$$

where $x(p_{it})$ is the (constant elasticity) demand from the final goods sector for variety i — from the FOC in Problem (FG).

Idea Producers: Problem (R&D)

- Perfectly competitive, seeing the idea production function as

$$\dot{A}_t = \bar{\nu}_t L_{At}$$

- The representative research firm solves

$$\max_{L_{At}} P_{At}\bar{\nu}_t L_{At} - w_t L_{At}$$

taking the price of an idea P_{At}, research productivity $\bar{\nu}_t$, and the wage rate w_t as given.

Market Equilibrium

An equilibrium with imperfect competition is $\{c_t, \{x_{it}\}, Y_t, K_t, I_t, v_t, \{\pi_{it}\}, L_{Yt}, L_{At}, L_t, A_t, \bar{\nu}_t\}_{t=0}^{\infty}$ and prices $\{w_t, r_t, \{p_{it}\}, P_{At}\}_{t=0}^{\infty}$ such that for all t:

1. c_t, v_t solve Problem (HH).
2. $\{x_{it}\}$ and L_{Yt} solve Problem (FG).
3. p_{it} and π_{it} solve Problem (CG) for all $i \in [0, A_t]$.
4. L_{At} solves Problem (R&D).
5. (r_t) The capital market clears: $V_t \equiv v_t L_t = K_t + P_{At} A_t$.
6. (w_t) The labor market clears: $L_{Yt} + L_{At} = L_t$.
7. $(\bar{\nu}_t)$ The idea production function is satisfied: $\bar{\nu}_t = \nu L_{At}^{\lambda-1} A_t^{\phi}$.
8. (K_t) The capital resource constraint is satisfied: $\int_0^{A_t} x_{it} di = K_t$.
9. (P_{At}) Assets have equal returns: $r_t = \frac{\pi_{it}}{P_{At}} + \frac{\dot{P}_{At}}{P_{At}}$.
10. Y_t, A_t,L_t, I_t are determined by their production functions

(16 equations, 16 unknowns. Goods market clears by Walras' Law: $C_t + I_t = Y_t$)

Result 3 (Market Equilibrium Allocation)

(a) All of Result 1 continues to hold:
- Same long-run growth rate!

(b) The same Euler equation characterizes consumption

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\zeta}(r_t^{eq} - \rho)$$

Note: Since the growth rate is the same and the Euler equation is the same, $r^{eq} = r^*$ (steady state).

Result 3 (Market Equilibrium — continued)

(c) Capital goods are priced with the usual monopoly markup:

$$p_{it}^{eq} = p_t^{eq} \equiv \frac{1}{\theta}(r_t^{eq} + \delta)$$

As a result, capital is paid less than its marginal product:

$$r_t^{eq} = \alpha\theta \frac{Y_t}{K_t} - \delta.$$

Same growth rate $\Rightarrow$ same interest rate. Underpaying capital leads to low K/Y:

$$s_K^{eq} = \frac{\alpha\theta(n + g + \delta)}{\rho + \delta + \zeta g} = \theta s_K^{op}$$

Result 3 (Market Equilibrium — continued)

(d) Free flow of labor equates marginal products

$$\frac{s_{At}^{eq}}{1 - s_{At}^{eq}} = \frac{P_{At}\dot{A}_t}{(1-\alpha)Y_t}$$

Along BGP

$$\frac{s_{A}^{eq}}{1 - s_{A}^{eq}} = \frac{\frac{\sigma\theta Y_t/A_t}{r^{eq}-(g_Y-g_A)}\dot{A}_t}{(1-\alpha)Y_t}$$

Compare s_A^{op} and s_A^{eq}

$$\frac{s_A^{eq}}{1-s_A^{eq}} = \frac{\frac{\sigma\theta Y_t/A_t}{r^{eq}-(g_Y-g_A)}\dot{A}_t}{(1-\alpha)Y_t}, \qquad \frac{s_A^{op}}{1-s_A^{op}} = \frac{\frac{\sigma Y_t/A_t}{r^{*}-(g_Y-g_A)-\phi g_A}\lambda\dot{A}_t}{(1-\alpha)Y_t}$$

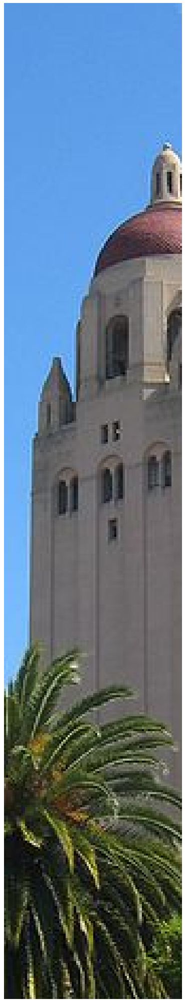
Three differences:

- $\lambda < 1$: Duplication externality
- $\phi \neq 0$: Knowledge spillover externality
- $\theta < 1$: Appropriability effect

Equilibrium may involve either too much or too little research.

Summary

- More people $\Rightarrow$ more ideas $\Rightarrow$ more per capita income (nonrivalry)
    - Log-difference $\Rightarrow$ per capita growth is proportional to population growth, where proportionality measures increasing returns
- Long-run growth rate is independent of the allocation of resources
    - Subsides or taxes on research affect growth along the transition path and have long-run level effects (like Solow)
    - In contrast, in Romer (1990), they lead to long-run growth effects



Applications

Several Applications of this Framework

1. Endogenizing population growth
2. Growth over the very long run
3. The linearity critique
4. Growth accounting

1. Endogenizing Population Growth (Jones 2003)

- Let fertility be a choice variable in a Barro and Becker (1989) kind of framework.
  - Converts this to a fully endogenous growth model

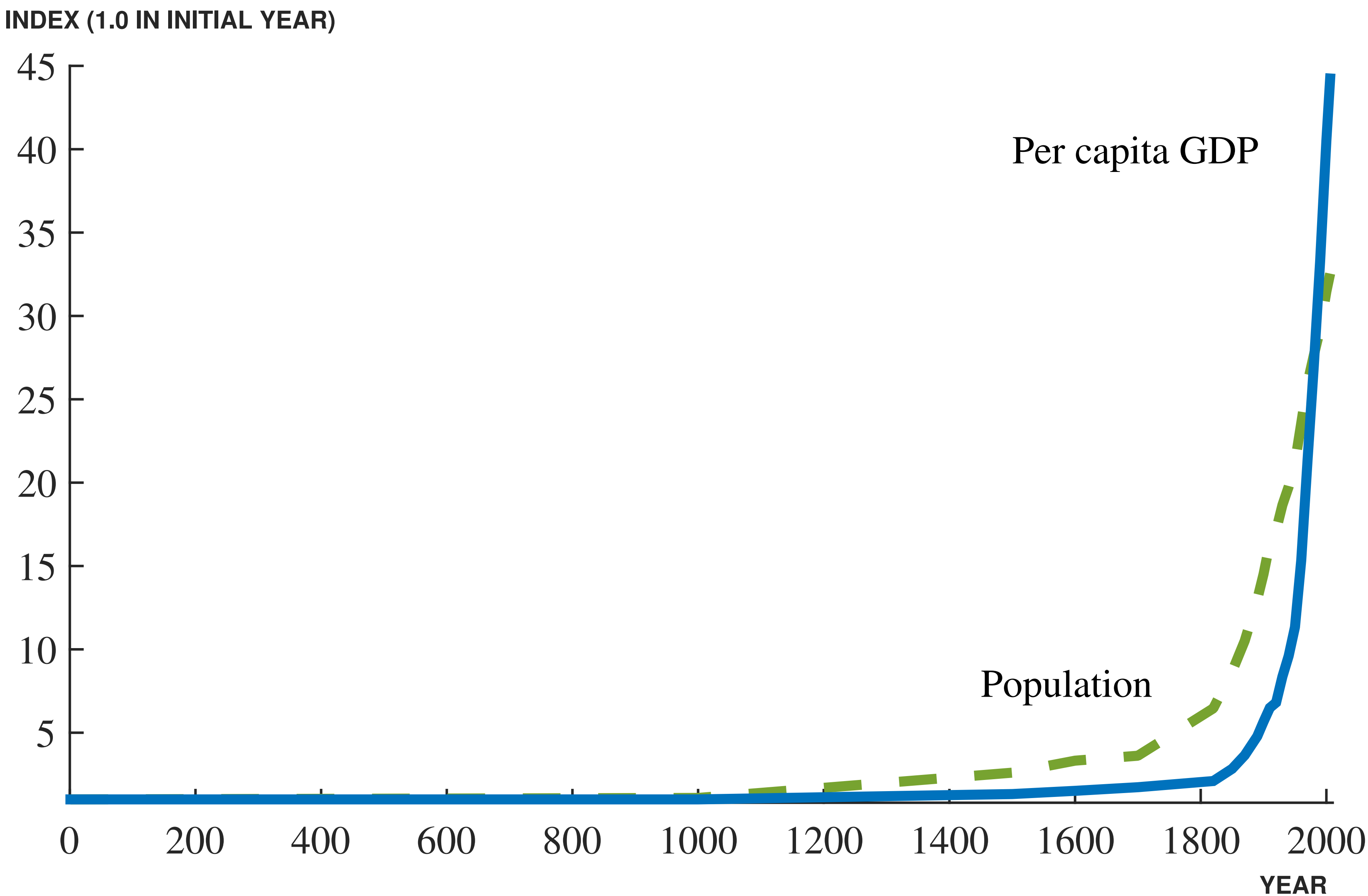
- But policy can have odd effects: a subsidy to research shifts labor toward producing ideas but away from producing kids $\Rightarrow$ lower long-run growth!

- A recent reference on some intriguing issues that arise along this line of research is Cordoba and Ripoll, "The Elasticity of Intergenerational Substitution, Parental Altruism, and Fertility Choice" (REStud 2019)

2. Growth over the Very Long Run

- Malthus: $c = y = AL^\alpha, \quad \alpha < 1$
  - Fixed supply of land: $\uparrow L \Rightarrow \downarrow c$ holding A fixed
- Story (Lee 1988, Kremer 1993):
  - 100,000 BC: small population $\Rightarrow$ ideas come very slowly
  - New ideas $\Rightarrow$ temporary blip in consumption, but permanently higher population
  - This means ideas come more frequently
  - Eventually, ideas arrive faster than Malthus can reduce consumption!
- People produce ideas and Ideas produce people
  - If nonrivarly $>$ Malthus, this leads to the hockey stick

Population and Per Capita GDP: the Very Long Run



3. The Linearity Critique

- The result of any successful growth model is an equation like $\dot{y}_t = \bar{g} y_t$, with a story about $\bar{g}$.
- An essential ingredient to getting this result is (essentially) some linear differential equation somewhere in the economic environment:

$$\dot{X}_t = \_\_ \; X_t$$

- Growth models differ according to what they call the X_t variable and how they fill in the blank.

Catalog of Growth Models: What is X_t?

Solow	$y_t = k^\alpha, \ \dot{k}_t = k_t^\alpha - c_t - \delta k_t$
Solow	$\dot{A}_t = \bar{g} A_t$
AK model	$Y_t = \bar{A} K_t, \ \ \dot{K}_t = A K_t - C_t - \delta K_t$
Lucas	$Y_t = K_t^\alpha (u_t h_t L)^{1-\alpha}, \ \dot{h}_t = (1 - u_t) h_t$
Romer ($\phi = 1$)	$\dot{A}_t = \nu L_{at} A_t$
Romer ($\phi < 1$)	$\dot{L}_t = \bar{n} L_t$

4. Growth Accounting

- Jones (2002): “Sources of U.S. Economic Growth in a World of Ideas”
- Puzzle (earlier graphs)
    - A straight line fits log U.S. GDP per person quite well
    - But human capital and R&D investment rates appear to be rising
- Human capital: Completion rates for adult population
    - 1940: 25% high school, 5% college
    - 1993: 80% high school, 20% college
- U.S. science/eng researchers as a fraction of labor force:
    - 1950: 1/4 of 1%
    - 1993: 3/4 of 1%

How to reconcile?

- Imagine a Solow model
  - What happens if the investment rate grows over time?
- Same thing could be going on in an idea model with $\phi < 1$
  - Only it is human capital and R&D investment rates that rise
  - Implication for the long run...

Key Equations of the Model

Production of final good	$Y_t = A_t^{\sigma} K_t^{\alpha} H_{yt}^{1-\alpha}$
Production of ideas	$\dot{A}_t = \nu H_{at}^{\lambda} A_t^{\phi}$
Efficiency units of labor	$H_{it} = h_t L_{it}, \quad h_t = e^{\psi \ell_{ht}}$
Resource constraint	$L_{yt} + L_{at} = (1 - \ell_{ht}) L_t$

- Rewriting the production function

$$y_t = \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \ell_{yt} h_t A_t^{\frac{\sigma}{1-\alpha}}$$

- Balanced growth path: $g_y = \gamma n$ where $\gamma \equiv \frac{\sigma}{1-\alpha} \cdot \frac{\lambda}{1-\phi}$

Accounting for Growth ($\gamma = 1/3$), 1950–2007

$$y^* \approx \left(\frac{K}{Y}\right)^{\beta} \cdot h \cdot (\text{R\&D intensity})^{\gamma} \cdot L^{\gamma}$$

	Solow	Lucas	Romer/AH/GH	J/K/S
2.0	0.0	0.4	1.2	0.4
(100%)	(0%)	(20%)	(58%)	(21%)

- Educational attainment rises $\approx$ 1 year per decade. With $\psi = .06 \Rightarrow$ about 0.6 percentage points of growth per year.
- Transition dynamics are 80 percent of growth.
- "Steady state" growth is only 20 percent of recent growth!
- Numbers from "The Future of U.S. Economic Growth" (with Fernald)

Alternative Futures?