

Intermediate Goods and Weak Links in the Theory of Economic Development

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August 31, 2010 – Version 5.0

Abstract

Per capita income in the richest countries of the world exceeds that in the poorest countries by more than a factor of 50. What explains these enormous differences? This paper returns to two old ideas in development economics and proposes that linkages and complementarity are a key part of the explanation. First, linkages between firms through intermediate goods deliver a multiplier similar to the one associated with capital accumulation in a neoclassical growth model. Because the intermediate goods share of gross output is about 1/2, this multiplier is substantial. Second, just as a chain is only as strong as its weakest link, problems at any point in a production chain can reduce output substantially if inputs enter production in a complementary fashion. This paper builds a model to quantify these forces and shows that they substantially amplify distortions to the allocation of resources, bringing us closer to understanding large income differences across countries.

*I would like to thank Daron Acemoglu, Andy Atkeson, Pol Antras, Sustanto Basu, Paul Beaudry, Roland Benabou, Olivier Blanchard, Antonio Ciccone, Bill Easterly, Xavier Gabaix, Luis Garicano, Pierre-Olivier Gourinchas, Chang Hsieh, Pete Klenow, John Leahy, Guido Lorenzoni, Kiminori Matsuyama, Ed Prescott, Andres Rodriguez, Dani Rodrik, Richard Rogerson, David Romer, Michele Tertilt, Alwyn Young and seminar participants at Berkeley, Brown, Chicago, the Chicago GSB, the European Economics Association, the LSE, the NBER EFG and growth meetings, Northwestern, NYU, Penn, Princeton, the San Francisco Fed, Stanford, Toulouse, UCLA, USC, and the World Bank for helpful comments. I am grateful to Urmila Chatterjee, On Jeasakul, and Mu-Jeung Yang for excellent research assistance, to the Hong Kong Institute for Monetary Research and the Federal Reserve Bank of San Francisco for hosting me during the early stages of this research, and to the National Science Foundation and the Toulouse Network for Information Technology for financial support.

1. Introduction

By the end of the 20th century, per capita income in the United States was more than 50 times higher than per capita income in Ethiopia and Tanzania. Dispersion across the 95th-5th percentiles of countries was more than a factor of 32. What explains these profound differences in incomes across countries?¹

This paper returns to two old ideas in the development economics literature and proposes that linkages and complementarity are a key part of the explanation. First, intermediate goods provide links between sectors that create a multiplier. Low productivity in electric power generation — for example because of theft, inferior technology, or misallocation — makes electricity more costly, which reduces output in banking and construction. But this in turn makes it harder to finance and build new dams and therefore further hinders electric power generation. This multiplier effect is similar to the multiplier associated with capital accumulation in a neoclassical growth model. In fact, intermediate goods are just another form of capital, albeit one that depreciates fully in production. Because the intermediate goods share of gross output is approximately 1/2, the intermediate goods multiplier is large.

Second, as a result of complementarity, high productivity in a firm requires a high level of performance along a large number of dimensions. Textile producers require raw materials, knitting machines, a healthy and trained labor force, knowledge of how to produce, security, business licenses, transportation networks, electricity, etc. These inputs enter in a complementary fashion, in the sense that problems with any input can substantially reduce overall output. Without electricity or production knowledge or raw materials or security or business licenses, production is likely to be severely curtailed.

The contribution of this paper is to build a model in which these ideas can be made precise. The multiplier that works through intermediate goods turns out to be readily quantified and large: incorporating intermediate goods into our models

¹Recent work on this topic includes Romer (1994), Klenow and Rodriguez-Clare (1997), Prescott (1998), Hall and Jones (1999), Parente and Prescott (1999), Howitt (2000), Parente, Rogerson and Wright (2000), Acemoglu, Johnson and Robinson (2001), Klenow and Rodriguez-Clare (2005), Manuelli and Seshadri (2005), Caselli and Coleman (2006), Armenter and Lahiri (2006), Erosa, Korshkova and Restuccia (2006), Marimon and Quadrini (2006), and Restuccia, Yang and Zhu (2006).

has a first-order impact on how we think about economic development. The effects of complementarity are more subtle and difficult to quantify — often turning out to be smaller than one might have expected — in part because they must constantly be weighed against various possibilities for substitution. In the end, however, these two forces substantially multiply the effects of distortions to the allocation of resources. Large income differences that are hard to explain in a traditional neoclassical setup appear within reach when the multipliers associated with intermediate goods and complementarity are taken into account.

The approach taken in this paper can be compared with the recent literature on political economy and institutions; for example, see Acemoglu and Johnson (2005) and Acemoglu and Robinson (2005). This paper is more about mechanics: can we develop a plausible mechanism for getting a big multiplier, so that whatever distortions exist lead to large income differences? The modern institutions approach builds up from political economy. This is crucial in explaining why the allocations in poor countries are inferior — for example, why investment rates in physical and human capital are so low — but the institutions approach ultimately still requires a large multiplier to explain income differences. As just one example, even if a political economy model explains observed differences in investment rates across countries, the model cannot explain 50-fold income differences if it is embedded in a neoclassical framework. The political economy approach explains why resources are misallocated; the approach here takes the extent of misallocation as given and explains how intermediate goods and weak links can amplify the effect of misallocation, potentially leading to large income differences. Clearly, both steps are needed to understand development.

2. Linkages and Complementarity

We begin by discussing briefly the key mechanisms at work in this paper. These mechanisms are conceptually distinct — one can have linkages without complementarity, for example — but they interact in important ways.

2.1. Linkages through Intermediate Goods

The notion that linkages across sectors can be central to economic performance dates back at least to Leontief (1936), which launched the field of input-output economics. Hirschman (1958) emphasized the importance of linkages (and complementarity) to economic development. A large subsequent empirical literature constructed input-output tables for many different countries and computed sectoral multipliers.

In what may prove to be an ill-advised omission, these insights have not generally be incorporated into modern growth theory. Linkages between sectors through intermediate goods deliver a multiplier very much like the multiplier associated with capital in the neoclassical growth model. More capital leads to more output, which in turn leads to more capital. This virtuous circle shows up mathematically as a geometric series which sums to a multiplier of $\frac{1}{1-\alpha}$ if α is capital's share of overall revenue. Because the capital share is only about 1/3, this multiplier is relatively small: differences in investment rates are too small to explain large income differences, and large total factor productivity residuals are required. This has led a number of authors to broaden the definition of capital, say to include human capital or organizational capital. It is generally recognized that if one can get the capital share up to something like 2/3 — so the multiplier is 3 — large income differences are much easier to explain without appealing to a large residual.²

Intermediate goods generate this same kind of multiplier. Problems in the financial services industry can reduce output in a range of sectors, including information technology, plastic manufacturing, and education. But this in turn feeds back and further reduces the output of financial services.

A simple example is quite helpful for understanding how intermediate goods generate a multiplier. Suppose gross output Q_t is produced using capital K_t , labor

²Mankiw, Romer and Weil (1992) is an early example of this approach to human capital. Chari, Kehoe and McGrattan (1997) introduced “organizational capital” for the same reason. Howitt (2000) and Klenow and Rodriguez-Clare (2005) use the accumulation of ideas to boost the multiplier. More recently, Manuelli and Seshadri (2005) and Erosa, Koreshkova and Restuccia (2006) have resurrected the human capital story in a more sophisticated fashion. The controversy in each of these stories is over whether or not the additional accumulation raises the multiplier sufficiently. Typically, the problem is that the magnitude of a key parameter is difficult to pin down.

L_t , and intermediate goods X_t :

$$Q_t = \bar{A} (K_t^\alpha L_t^{1-\alpha})^{1-\sigma} X_t^\sigma, \quad (1)$$

where σ and α are between zero and one. Gross output can be used for consumption or investment or it can be carried over to the next period and used as an intermediate good. To keep things simple, assume a constant fraction \bar{x} of gross output is used as an intermediate good. GDP in this economy is consumption plus investment, or output net of intermediate goods: $Y_t \equiv (1 - \bar{x})Q_t$. Assume a constant fraction \bar{s} of GDP is invested, so the laws of motion in this economy are

$$K_{t+1} = \bar{s}Y_t + (1 - \delta)K_t, \quad (2)$$

$$X_{t+1} = \bar{x}Q_t. \quad (3)$$

Let labor be exogenous and constant.

This model features a steady state where all key variables are constant and GDP is given by

$$Y = (\bar{A}\bar{m})^{\frac{1}{1-\sigma}} K^\alpha L^{1-\alpha}, \quad (4)$$

where $\bar{m} \equiv (1 - \bar{x})^{1-\sigma} \bar{x}^\sigma$. There are two important things to notice here. First, the allocation of resources — summarized by \bar{m} — affects total factor productivity. (Notice that \bar{m} is a hump-shaped function of \bar{x} that is maximized at $\bar{x} = \sigma$.) Second, the effect of changes in productivity \bar{A} and the misallocation of resources in \bar{m} are amplified by the presence of intermediates goods: there is a multiplier $\frac{1}{1-\sigma}$.

This second point can be better understood by solving further. In particular, in steady state, capital depends on investment, so GDP per worker is

$$y \equiv \frac{Y}{L} = \left(\bar{A}\bar{m} \left(\frac{\bar{s}}{\delta} \right)^{\alpha(1-\sigma)} \right)^{\frac{1}{(1-\alpha)(1-\sigma)}}. \quad (5)$$

Now the multiplier on misallocation \bar{m} and productivity \bar{A} is $\frac{1}{(1-\alpha)(1-\sigma)}$. In the absence of intermediate goods ($\sigma = 0$), this multiplier is just the familiar $\frac{1}{1-\alpha}$: an increase in productivity raises output, which leads to more capital, which leads to

more output, and so on. The cumulation of this virtuous circle is $1 + \alpha + \alpha^2 + \dots = \frac{1}{1-\alpha}$.

In the presence of intermediate goods, there is an additional multiplier: higher output leads to more intermediate goods, which raises output (and capital), and so on. The overall multiplier is therefore $\frac{1}{(1-\alpha)(1-\sigma)}$. Alternatively, let $\beta \equiv \alpha(1 - \sigma) + \sigma$ denote the total share of produced inputs. It is easy to show that the multiplier can also be expressed as $\frac{1}{1-\beta}$.

Quantitatively, the addition of intermediate goods has a large effect. For example, consider the multipliers using conventional parameter values, a capital exponent of $\alpha = 1/3$ and an intermediate goods exponent of $\sigma = 1/2$. (Evidence supporting this value will be discussed extensively below.) In this case, the share of produced factors in gross output is $\beta = \alpha(1 - \sigma) + \sigma = 1/6 + 1/2 = 2/3$.

In the absence of intermediate goods the multiplier is $\frac{1}{1-\alpha} = 3/2$, and a hypothetical doubling of TFP raises output by a factor of $2^{3/2} = 2.8$. But with intermediate goods, the multiplier is $\frac{1}{(1-\alpha)(1-\sigma)} = \frac{3}{2} \cdot 2 = 3$, and a doubling of TFP raises output by a factor of $2^3 = 8$. If we think of the standard neoclassical factors (like \bar{s} and \bar{x} in the example) as generating a 4-fold difference in incomes across rich and poor countries, then a hypothetical 2-fold difference in TFP leads to an 11.3-fold difference in the model with no intermediate goods, but to a 32-fold difference once intermediate goods are taken into account, close to what we see in the data.

At a basic level, this simple model captures the main contribution of the paper, and the rest is just elaboration. However, the elaboration turns out to be quite important in answering a key question that may be raised regarding the simple model. First, because the level of TFP is never observed directly but must be measured as a residual, there is a sense in which the calculation above may appear confusing. TFP can be measured using value-added or using gross output, and there is a one-to-one mapping between the two in this simple example. Let $\bar{B} \equiv \bar{A}^{1/(1-\sigma)}$. A 2-fold difference in \bar{A} corresponds to a 4-fold difference in \bar{B} in a value-added representation like equation (4). Does this observational equivalence mean there is no fundamental multiplier after all?

Instinctively, we know the answer to this question must be “no”: after all, the in-

junction from Diamond and Mirrlees (1971) about not taxing intermediate goods is based on this same multiplier. More specifically, this concern is addressed directly below by building a model in which distortions to the allocation of resources at the micro level such as theft or taxation aggregate up into TFP differences at the macro level. These micro-level distortions — which are in principle observable and have a definite magnitude (such as “10% of output gets stolen from the firm”) — are amplified by the intermediate goods multiplier. In the quantitative exercises at the end of the paper, one will see clearly that the presence or absence of intermediate goods plays a crucial role in determining the magnitude of income differences for a given set of micro-level distortions.

There is an alternative way to make this point. From Hsieh and Klenow (2009) and others, we know that distortions to the allocation of capital and labor can lead to aggregate TFP differences, and that these TFP differences are multiplied by the capital accumulation multiplier. This paper simply recognizes that production involves intermediate goods as well, and these too can be misallocated — recall the \bar{m} term in equation (5) above. Because intermediate goods are another produced factor of production, this misallocation gets amplified.

Another issue worth addressing now is vertical integration. Doesn't the extent of vertical integration influence the share of intermediate goods in gross output? If an economy were entirely vertically integrated, would there be no multiplier associated with intermediate goods? As the full model to be presented shortly will show, what matters is the extent to which first-order conditions in the economy get distorted. Consider an automobile manufacturer that vertically integrates, from the steel production and rubber manufacturing all the way through to the final sale of the automobile. Whether or not there is vertical integration, there are clearly many more first-order conditions that must be satisfied than just the ones involving capital and labor. The steel, rubber, sparkplugs, radios, and all the other parts have to be produced at the right time and delivered to the right place in the right quantity. Theft or corruption could affect any piece of this production chain. Similarly, once the cars are produced, they must be delivered to auto retailers around the country. Whether or not these are owned by the same entity as the producer does not really

affect the possibility that some cars or parts may be confiscated by corrupt officials along the way.³

Combining a neoclassical story of capital accumulation with a standard treatment of intermediate goods therefore delivers a very powerful engine for explaining income differences across countries. Related insights pervade the older development literature but have not had a large influence on modern growth theory. The main exception is Ciccone (2002), which appears to be underappreciated.⁴

2.2. The Role of Complementarity

Complementarity and linkages often go together, as in Hirschman (1958). This is in part because complementarity naturally arises when one considers intermediate goods: electricity, transportation, and raw materials are all essential inputs into production. This is one reason it is natural to consider the role of complementarity in this paper. The other is the large multiplier suggested by the O-ring story in Kremer (1993): the space shuttle Challenger and its seven-member crew are destroyed because of the failure of a single, inexpensive rubber seal.

In any production process, there are many things that can go wrong that will sharply reduce the value of production. In rich countries, there are enough substitution possibilities that these things do not often go wrong. In poor countries, on the other hand, any one of several problems can doom a project. Obtaining the instruction manual (the “knowledge”) for how to produce socks is not especially useful if the import of knitting equipment is restricted, if replacement parts are not

³To a great extent, the possibility of vertical integration simply raises a measurement issue. With a large amount of vertical integration, statistical authorities would understate the importance of intermediate goods — measured as purchases from other firms. As we will see later, however, there is a wealth of evidence suggesting that the intermediate goods share of gross output is approximately 1/2 across a range of economies at different levels of development. If this is understated, the multiplier associated with intermediate goods would be even larger.

⁴Ciccone develops the multiplier formula for intermediate goods and provides some quantitative examples illustrating that the multiplier can be large. The point may be overlooked by readers of his paper because the model also features increasing returns, externalities, and multiple equilibria. Yi (2003) argues that tariffs can multiply up in much the same way when goods get traded multiple times during the stages of production; see also Eaton and Kortum (2002). Interestingly, the intermediate goods multiplier shows up most clearly in the economic fluctuations literature; see Long and Plosser (1983), Basu (1995), Rotemberg and Woodford (1995), Horvath (1998), Dupor (1999), Conley and Dupor (2003), and Gabaix (2005). See also Hulten (1978).

readily available, if the electricity supply is erratic, if cotton and polyester threads cannot be obtained, if legal and regulatory requirements cannot be met, if property rights are not secure, or if the market to which these socks will be sold is unknown.

A moment's reflection is enough to convince nearly anyone of complementarity's potential for explaining income differences. This was certainly part of the original appeal of Kremer's paper. For reasons that are not entirely clear, those insights have not had a large influence on growth and development models of the last decade, and part of the goal of this paper is to explore these possibilities more carefully. Hence, complementarity is the second main ingredient in this paper.

2.3. Modeling Complementarity and Substitution

Kremer (1993) offers the basic insight that complementarity can generate a large multiplier by focusing on the extreme case in which all inputs combine in a Leontief fashion. In adding this second ingredient to our model, we choose a more flexible CES formulation that allows the degree of complementarity to be a parameter.⁵ As an illustration, suppose

$$Y = \left(\int_0^1 z_i^\eta di \right)^{1/\eta} \quad (6)$$

where z_i denote a firm's purchases of the i^{th} input, and a continuum of intermediate inputs are used for production. The elasticity of substitution among these activities is $\frac{1}{1-\eta}$, but this (or its inverse) could easily be called an elasticity of complementarity instead. For intermediate inputs, it is plausible to assume $\eta < 0$, so the elasticity of substitution is less than one. It is difficult to substitute electricity for transportation services or raw materials in production. Complementarity puts extra weight on the activities in which the firm is least successful. This is easy to see in the limiting case where $\eta \rightarrow -\infty$; in this case, the CES function converges to the minimum function, so output is equal to the smallest of the z_i .

⁵Kremer does not emphasize that his approach embodies a Leontief technology. Blanchard and Kremer (1997) formalize this interpretation and study a model of chains of production in order to understand the large declines in output in the former Soviet Union after 1989. Grossman and Maggi (2000), motivated in part by Kremer (1993), study trade between countries when production functions across sectors involve different degrees of complementarity. Other related papers include Murphy, Shleifer and Vishny (1989), Milgrom and Roberts (1990), Becker and Murphy (1992), Rodriguez-Clare (1996), and Rodrik (1996).

This intuition can be pushed further by noting that the CES combination in equation (6) is called the *power mean* of the underlying z_i in statistics. The power mean is just a generalized mean. For example, if $\eta = 1$, Y is the arithmetic mean of the z_i . If $\eta = 0$, output is the geometric mean (Cobb-Douglas). If $\eta = -1$, output is the harmonic mean, and if $\eta \rightarrow -\infty$, output is the minimum of the z_i . From a standard result in statistics, these means decline as η becomes more negative. Economically, a stronger degree of complementarity puts more weight on the weakest links and reduces output.⁶

Going in the other direction, if $\eta \rightarrow +\infty$, output converges to the maximum of the z_i , a “superstar” kind of production function, like that studied by Rosen (1981). More generally, the higher is η , the further up the distribution is the power mean. This case is not usually emphasized in growth models — notice that it implies a *negative* elasticity of substitution — but it turns out to play an important and intuitive role in our model.

3. Setting Up the Model

We now apply this basic discussion of intermediate goods and complementarity to construct a model of economic development.

3.1. The Economic Environment

A continuum of goods indexed on the unit interval by i are produced in this economy using a Cobb-Douglas production function:

$$Q_i = A_i (K_i^\alpha H_i^{1-\alpha})^{1-\sigma} X_i^\sigma, \quad (7)$$

where α and σ are both between zero and one. K_i and H_i are the amounts of physical capital and human capital used to produce good i , and A_i is an exogenously-given productivity level. The novel term in this production specification is X_i , which

⁶Benabou (1996) studies this approach to complementarity. Interestingly, standard intertemporal preferences with a constant relative risk aversion coefficient greater than one represent a familiar example.

denotes the quantity of intermediate goods used to produce variety i .

Each of these fundamental goods in the economy can be used for one of two purposes: as a final good (c_i) or as an intermediate input (z_i). Therefore,

$$c_i + z_i = Q_i. \quad (8)$$

The next two equations show how these uses affect the economy. In principle, we could specify a utility function over the continuum of final consumption uses. Instead, it proves more convenient (for modeling capital) to follow the standard trick of aggregating these final uses into a single final good, which will represent GDP in this economy:

$$Y = \left(\int_0^1 c_i^\theta di \right)^{1/\theta}, \quad 0 < \theta < 1. \quad (9)$$

These final goods aggregate up with an elasticity of substitution greater than one. Such an aggregator is standard in the literature and there are solid estimates of this elasticity that we will appeal to when it comes time for quantitative analysis.

Whereas final goods combine with an elasticity of substitution greater than one in producing GDP, intermediate inputs combine with an elasticity of substitution less than one. This is the key place where “weak links” enter the model:

$$X = \left(\int_0^1 z_i^\rho di \right)^{1/\rho}, \quad \rho < 0. \quad (10)$$

This aggregate intermediate good is what gets used by the various sectors of the economy.⁷ To keep the model simple and tractable, we assume that the same combination of intermediate goods is used to produce each variety (though potentially in a different quantity). Hence, the resource constraint:

$$\int_0^1 X_i di \leq X. \quad (11)$$

⁷An issue of timing arises here. To keep the model simple, we make the seemingly strange assumption that intermediate goods are produced and used simultaneously. A better justification goes as follows. Imagine incorporating a lag so that today’s final good is used as tomorrow’s intermediate input. The steady state of that setup would then deliver the result we have here.

An example illustrating the final and intermediate goods may be helpful here. Varieties that are used as intermediate goods involve substantial complementarity, but when these same varieties combine to produce final consumption, there is more substitutability. For example, computer services are today nearly an essential input into semiconductor design, banking, and health care. But computers are much more substitutable when used for final consumption — for entertainment, we can play computer games or watch television or ride bikes in the park. In order to produce within a firm, there are a number of complementary steps that must be taken. In final consumption (e.g. in utility), however, there appears to be a reasonably high degree of substitution across goods.

Stepping back for a moment, note that the parameter σ measures the importance of linkages in our economy. If $\sigma = 0$, the productivity of physical and human capital in each variety depends only on A_i and is independent of the rest of the economy. To the extent that $\sigma > 0$, low productivity in one sector feeds back into the others. Transportation services may be unproductive in a poor country because of inadequate fuel supplies or repair services, and this low productivity will reduce output throughout the economy.

The remainder of the model is standard. The resource constraints for physical and human capital are

$$\int_0^1 K_i di \leq K, \quad (12)$$

and

$$\int_0^1 H_i di \leq H \equiv \bar{h}\bar{L}, \quad (13)$$

where \bar{h} is an exogenously-given amount of human capital per worker and $\bar{L} = 1$ is the exogenous number of workers in the economy, both constant. We do not endogenize human capital accumulation in this environment in order to keep the model as simple as possible; this could be added easily, however. Physical capital accumulates in the usual way, and investment consists of units of the aggregate final good:

$$\dot{K} = I - \delta K, \quad K_0 \text{ given.} \quad (14)$$

$$C + I \leq Y. \quad (15)$$

Finally, preferences are standard

$$U = \int_0^{\infty} e^{-\lambda t} u(C_t) dt, \quad (16)$$

with $u'(C) > 0$ and $u''(C) < 0$. We've dropped time subscripts from this economic environment (except in this final equation) since we will primarily be concerned with the steady state of this model.

4. A Symmetric Allocation of Resources

Before turning to a competitive equilibrium, it is useful to consider a simple “rule of thumb” allocation, analogous to Solow’s fixed saving rate. There are two advantages to this approach. First, it is simple, easy to solve for, and allows us to illustrate some of the key points of the model. Second, it serves as a useful benchmark when it comes time to understand why the competitive equilibrium looks the way it does. Our rule of thumb allocation is a symmetric allocation with a constant investment rate:

Definition The *symmetric allocation of resources* in this economy has $K_i = K$, $H_i = H$, $X_i = X$, $I = \bar{s}Y$, and $z_i = \bar{z}Q_i$, where $0 < \bar{s}, \bar{z} < 1$.

Of course, this symmetric allocation is completely unrealistic — even moreso, it will turn out, than one might have guessed. But the advantages mentioned above make this a good place to start.

Under this symmetric allocation, the solution for GDP in the economy at any point in time is given in the following proposition. (Outlines of all proofs are in the Online Appendix.)

Proposition 1 (The Symmetric Allocation, Given Capital): *Given K units of capital, GDP under the symmetric allocation is*

$$Y = \phi(\bar{z})(S_{\theta}^{1-\sigma} S_{\rho}^{\sigma})^{\frac{1}{1-\sigma}} K^{\alpha} H^{1-\alpha}, \quad (17)$$

where

$$S_\rho \equiv \left(\int_0^1 A_i^\rho di \right)^{\frac{1}{\rho}}, \quad (18)$$

$$\phi(\bar{z}) \equiv ((1 - \bar{z})^{1-\sigma} \bar{z}^\sigma)^{\frac{1}{1-\sigma}}, \quad (19)$$

and S_θ is defined in a way analogous to S_ρ .

The model delivers a simple expression for GDP. Y is the familiar Cobb-Douglas combination of aggregate physical and human capital with constant returns to scale.

Two novel results also emerge, and both are related to total factor productivity. First, consider the S_θ and S_ρ terms. Each is a CES combination of the underlying sectoral TFPs. Since θ is between zero and one, S_θ is between the geometric mean and the arithmetic mean of the TFPs. But with ρ less than zero, S_ρ ranges from the geometric mean down to the minimum of the underlying A_i , depending on the strength of complementarity. Total factor productivity for the economy as a whole depends on the geometric average of the CES terms, $S_\theta^{1-\sigma} S_\rho^\sigma$. The “substitutes” term gets a weight that equals the share of value-added in gross output, while the “complements” term S_ρ gets a weight that equals the intermediate goods share of gross output, σ . In other words, the importance of “weak links” in production depends on (i) the extent of complementarity and (ii) the relative importance of intermediate goods.

To interpret this result, it is helpful to consider the special case where $\theta = 1$, $\rho \rightarrow -\infty$, and $\sigma = 1/2$. In this case, TFP is the product of the average of the A_i and the minimum of the A_i . Aggregate TFP then depends crucially on the smallest level of TFP across the sectors of the economy — that is, on the weakest link. Firms in the United States and Kenya may not differ that much in average efficiency, but if the distribution of Kenyan firms has a substantially worse lower tail, overall economic performance will suffer because of complementarity.

The second property of this solution worth noting is the multiplier associated with intermediate goods. Total factor productivity involves a multiplier, the exponent $\frac{1}{1-\sigma} > 1$. A simple example should make the reason for this transparent. Suppose $Y_t = aX_t^\sigma$ and $X_t = sY_{t-1}$; output depends in part on intermediate goods, and

the intermediate goods are themselves produced using output from the previous period. Solving these two equations in steady state gives $Y^* = a^{\frac{1}{1-\sigma}} s^{\sigma/1-\sigma}$, which is a simplified version of what is going on in our model. Notice that if we call X “capital” instead of intermediate goods, the same formulas would apply and this looks like the neoclassical growth model with full depreciation. Intermediate goods are another source of produced inputs in a growth model.

Finally, consider the role of $\phi(\bar{z})$. Differences in the allocation of resources to intermediate uses show up as aggregate TFP differences in this environment. Moreover, this term is a hump-shaped function of \bar{z} which is maximized at $\bar{z} = \sigma$. Not surprisingly, this turns out to be the optimal amount of gross output to spend on intermediate goods. Departures from this optimal amount will reduce TFP.

5. A Competitive Equilibrium with Wedges

The symmetric allocation is useful as a quick guide to how the model works, but it is clearly farfetched. We turn now to a more interesting allocation, the competitive equilibrium in the presence of micro-level distortions.

This approach builds on work by Banerjee and Duflo (2005), Chari, Kehoe and McGrattan (2007), Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), who argue that misallocation at the micro level shows up at the macro level as a reduction in aggregate TFP. Micro-level distortions can be actual formal taxes, but they could also represent theft, product and labor market regulations, protection from competition, or other forms of expropriation. Here, we model misallocation through an expropriation rate: a fraction τ_i of the output of each variety is taken from firms without compensation. This is by no means the only way to model the micro-level distortions — an alternative would be to have the distortions vary for each input as well as each variety. But it allows the main points of the paper to be made in the clearest fashion.

5.1. Optimization Problems

Before defining the competitive equilibrium, it is convenient to specify the optimization problems in the economy. Letting the final output good be the numéraire, these problems are described below.

Final Sector Problem: Taking the prices of the consumption varieties $\{p_i\}$ as given, a representative firm in the perfectly competitive final goods market solves at each point in time

$$\max_{\{c_i\}} \left(\int_0^1 c_i^\theta di \right)^{1/\theta} - \int_0^1 p_i c_i di.$$

Intermediate Sector Problem: Taking the price of the intermediate varieties $\{p_i\}$ and the price of the aggregate intermediate good q as given, a representative firm in the perfectly competitive intermediate goods market solves at each point in time

$$\max_{\{z_i\}} q \left(\int_0^1 z_i^\rho di \right)^{1/\rho} - \int_0^1 p_i z_i di.$$

Variety i 's Problem: Taking $p_i, r, w,$ and q as given, and given a variety-specific distortion τ_i , a representative firm in the perfectly competitive variety i market solves at each point in time

$$\max_{\{X_i, K_i, H_i\}} (1 - \tau_i) p_i A_i (K_i^\alpha H_i^{1-\alpha})^{1-\sigma} X_i^\sigma - (r + \delta) K_i - w H_i - q X_i.$$

Household Problem: Taking the time path of interest rates, wages, and income from the distortions ($r_t, w_t,$ and E_t) as given, and given an initial stock of assets V_0 , the representative household solves

$$\max_{\{C_t, V_t\}} \int_0^\infty e^{-\lambda t} u(C_t) dt$$

subject to

$$\dot{V}_t = r_t V_t + w_t H + E_t - C_t,$$

and subject to a no Ponzi-scheme condition.

5.2. Defining the Competitive Equilibrium

Definition A *competitive equilibrium* in this economy consists of time paths for the quantities $Y, X, C, I, K, V, E, \{Q_i, K_i, H_i, X_i\}, \{c_i, z_i\}$ and prices $\{p_i\}, q, w, r$ such that

1. C and V solve the **Household Problem**.
2. $\{c_i\}$ solve the **Final Sector Problem**.
3. $\{z_i\}$ solve the **Intermediate Sector Problem**.
4. K_i, H_i, X_i solve the **Variety i Problem** for all $i \in [0, 1]$.
5. Markets clear:

r clears the capital market: $V = K$

w clears the labor market: $\int_0^1 H_i di = H$

p_i clears market i : $c_i + z_i = Q_i$ for all $i \in [0, 1]$

q clears the intermediate goods market: $\int_0^1 X_i di = X$.

6. Expropriated funds are rebated to households: $E = \int_0^1 \tau_i p_i Q_i di$.
7. Other aspects of the environment hold:

$$\dot{K} = I - \delta K$$

$$\int_0^1 K_i di = K$$

$$Q_i = A_i (K_i^\alpha H_i^{1-\alpha})^{1-\sigma} X_i^\sigma$$

$$Y = \left(\int_0^1 c_i^\theta di \right)^{1/\theta}$$

$$X = \left(\int_0^1 z_i^\rho di \right)^{1/\rho}.$$

Counting loosely, our competitive equilibrium involves 17 endogenous variables and specifies 17 equations to pin them down. The market for final output clears by Walras' Law (so that $C + I = Y$ is redundant).

5.3. Solving for the Competitive Equilibrium

We now discuss the solution of the model, beginning with a result characterizing the aggregate production of GDP at any point in time.

Proposition 2 (The Competitive Equilibrium, Given Capital): *Given K units of capital, GDP in the competitive equilibrium is*

$$Y = \psi(\tau) (B_\theta^{1-\sigma} B_\rho^\sigma)^{\frac{1}{1-\sigma}} K^\alpha H^{1-\alpha} \quad (20)$$

where

$$B_\rho \equiv \left(\int_0^1 (A_i(1 - \tau_i))^{\frac{\rho}{1-\rho}} di \right)^{\frac{1-\rho}{\rho}} \quad (21)$$

and

$$\psi(\tau) \equiv \frac{1 - \sigma(1 - \tau)}{1 - \tau} \cdot \sigma^{\frac{\sigma}{1-\sigma}} \quad (22)$$

where $\tau \equiv E/(Y + qX)$ is the average distortion in the economy, measured relative to gross output,⁸ and B_θ is defined in a way analogous to B_ρ .

Several insights emerge from this result. Two we can get through quickly, while the third requires more consideration. First, the multiplier associated with intermediate goods appears in exactly the same way as in the symmetric allocation, and for the same reason. This multiplier is a fundamental feature of the economy reflecting the presence of additional produced factors of production. It multiplies any distortion associated with misallocation but is not itself affected by the allocation of resources.

Second, distortions affect output through TFP. Therefore, this proposition illustrates a very important result found elsewhere: the misallocation of resources at the micro level often shows up as a reduction in TFP at the macro level. This result has been emphasized by Chari, Kehoe and McGrattan (2007), Restuccia and Rogerson (2008), and Hsieh and Klenow (2009) and also plays a key role in Banerjee and Duflo (2005), Caselli and Gennaioli (2005) and Lagos (2006). Importantly, distortions get amplified by the intermediate goods multiplier. We will discuss the effect of these wedges in more detail below.

⁸The solution for τ is given implicitly by

$$\tau = (1 - \sigma(1 - \tau))T_\theta + \sigma(1 - \tau)T_\rho$$

where $T_\rho \equiv \int_0^1 \tau_i \left(\frac{A_i(1-\tau_i)}{B_\rho} \right)^{\frac{\rho}{1-\rho}} di$. That is, T_ρ is a weighted average of the sector-specific distortions, where the weights depend on ρ ; T_θ is defined analogously.

Finally, a key difference relative to the previous result on the symmetric allocation is that the curvature parameter determining the productivity aggregates has changed. For example, $\frac{\rho}{1-\rho}$ replaces the original ρ . Notice that if the domain of ρ is $(-\infty, 0]$, the range of $\frac{\rho}{1-\rho}$ is $(-1, 0]$: there is less complementarity in determining B_ρ than S_ρ .

This result can be illustrated with an example. Suppose $\rho \rightarrow -\infty$. In this case, the symmetric allocation depends on the smallest of the A_i , the pure weak link story. In contrast, the equilibrium allocation depends on the harmonic mean of the (distortion adjusted) productivities, since $\frac{\rho}{1-\rho} \rightarrow -1$. Disasterously low productivity in a single variety is fatal in the symmetric allocation, but not in the equilibrium allocation. Why not?

The reason is that the equilibrium allocation is able to strengthen weak links by allocating more resources to activities with low productivity. If the transportation sector has especially low productivity that would otherwise be very costly to the economy, the equilibrium allocation can put extra physical and human capital in that sector to help offset its low productivity and prevent this sector from becoming a bottleneck. Of course, this must be balanced by the desire to give this sector a *low* amount of resources in an effort to substitute away from transportation on the consumption side. This can be seen in the math: the equilibrium solution for allocating capital is

$$\frac{K_i}{K} = \frac{1 - \tau_i}{1 - \tau} \left[(1 - \sigma(1 - \tau)) \left(\frac{A_i(1 - \tau_i)}{B_\theta} \right)^{\frac{\theta}{1-\theta}} + \sigma(1 - \tau) \left(\frac{A_i(1 - \tau_i)}{B_\rho} \right)^{\frac{\rho}{1-\rho}} \right].$$

Because $\rho < 0$ while $\theta > 0$, low productivity in producing variety i increases the desired capital allocated to that sector according to the second term in parentheses (the complementarity effect) but reduces capital according to the first term (the substitution effect).

Another perspective on the solution is gained by returning to a special case we considered earlier. Suppose $\theta = 1$, $\rho \rightarrow -\infty$, and $\sigma = 1/2$, and suppose $\tau_i = 0$. In this case, $B_\theta \rightarrow \max A_i$ while B_ρ becomes the harmonic mean of the A_i . Total factor productivity is the product of the two. Contrast this with the same example

for the symmetric allocation: there, TFP was the product of the arithmetic mean and the minimum. Allocating resources optimally shifts up both of these generalized means. The strengthening of weak links leads the minimum to be replaced by the harmonic mean. Similarly, if consumption goods enter as perfect substitutes, only the good with the highest productivity will be consumed: the arithmetic mean gets replaced by the “max,” a superstar effect.

This example illustrates an intuitive way that the model can lead to large income differences across countries. Suppose two countries possess identical economic environments, including the same levels of productivity for each variety. The “rich” country allocates resources as in a competitive equilibrium with no distortions, but the “poor” country distorts the allocation sufficiently that it looks like the symmetric allocation. In the special case we are considering here, relative TFP between these two countries will be the product of two terms. First is the ratio of average TFP between the two countries, a standard term. But second is the ratio of the maximum TFP in the rich country to the minimum TFP in the poor country. Even if both countries have identical TFP distributions, this misallocation can lead to a large gap driven by the max-min effects associated with superstar and weak link forces. With less extreme parameter values, these forces are still in play, of course, as we will see in the numerical examples later.

5.4. The Steady State

Next, we see that the long-run multiplier in the model depends on the overall share of produced factors — capital as well as intermediate goods. We get the $\frac{1}{1-\alpha}$ effect since capital accumulates in response to a change in productivity or the distortions.

Proposition 3 (The Competitive Equilibrium in Steady State): *Let $y \equiv Y/\bar{L}$. The competitive equilibrium exhibits a steady state in which GDP per worker is given by*

$$y^* = \psi_1(\tau) (B_\theta^{1-\sigma} B_\rho^\sigma)^{\frac{1}{1-\sigma} \frac{1}{1-\alpha}} \left(\frac{\alpha(1-\sigma)}{\lambda + \delta} \right)^{\frac{\alpha}{1-\alpha}} \bar{h}. \quad (23)$$

where $\psi_1(\tau) \equiv \frac{1-\sigma(1-\tau)}{1-\tau} \cdot \sigma^{\frac{\sigma}{1-\sigma} \frac{1}{1-\alpha}}$.

5.5. Symmetric Wedges

A number of useful insights emerge from considering the special case in which the distortions are identical across all varieties.

Proposition 4 (Symmetric Wedges): *Suppose the distortion is identical across sectors: $\tau_i = \bar{\tau}$. Let $z^* \equiv \frac{qX}{Y+qX}$ denote the equilibrium fraction of gross output spent on intermediate goods and let $m^* \equiv (1 - z^*)^{1-\sigma} (z^*)^\sigma$. Then $z^* = \sigma(1 - \bar{\tau})$, and GDP at any given point in time is*

$$Y = \left(m^* \tilde{B}_\theta^{1-\sigma} \tilde{B}_\rho^\sigma \right)^{\frac{1}{1-\sigma}} K^\alpha H^{1-\alpha}, \quad (24)$$

where

$$\tilde{B}_\rho \equiv \left(\int_0^1 A_i^{\frac{\rho}{1-\rho}} di \right)^{\frac{1-\rho}{\rho}}, \quad (25)$$

and \tilde{B}_θ is defined analogously. Moreover, GDP per worker in steady state is

$$y^* = \zeta_1 (1 - \sigma(1 - \bar{\tau})) (1 - \bar{\tau})^{\frac{1}{1-\sigma} \frac{1}{1-\alpha} - 1} \left(\tilde{B}_\theta^{1-\sigma} \tilde{B}_\rho^\sigma \right)^{\frac{1}{1-\sigma} \frac{1}{1-\alpha}} \bar{h}, \quad (26)$$

where ζ_1 is a collection of terms that do not depend on $\bar{\tau}$.

The first part of this proposition highlights a similarity between the competitive equilibrium with symmetric wedges and the symmetric allocation we studied earlier. The overall effect of the distortion is to change the allocation of resources between final use and intermediate use. Given capital, GDP is maximized at $\bar{\tau} = 0$.

The second part of the proposition shows explicitly the different effects that a symmetric distortion has on GDP per worker in the steady state. The first term is $1 - z^* = 1 - \sigma(1 - \bar{\tau})$. Notice that this term is an increasing function of the distortion and reflects the fact that the distortion leads to lower spending on intermediate goods and therefore higher spending on final uses. The second term is the distortion wedge raised to a power that depends on the overall multiplier in the model. In fact, letting β denote the overall share of produced factors in the sectoral production function (both intermediates and capital), this second term can be written as $(1 - \bar{\tau})^{\frac{\beta}{1-\beta}}$. The $\frac{1}{1-\beta}$ term captures the standard multiplier effects of the model.

The overall exponent gets reduced by the proportion β because only that fraction of the factors of production are distorted by a symmetric wedge. In particular, the allocation of human capital across sectors is not distorted.

This raises an interesting question: if distortion is symmetric, why does it change anything at all? The answer is that it is symmetric across sectors, but not symmetric over time. In particular, goods that are used for final uses suffer the distortion only once, when they are produced. However, a good devoted to intermediate uses suffers the distortion each time production occurs, and it is this that leads to the multiplier effects. This can be viewed as a simple application of the ideas in Diamond and Mirrlees (1971), Chamley (1986), and Judd (1985) regarding the taxation of intermediate goods and capital. From the long-run perspective, capital and intermediate goods are the same: both are produced factors of production. The distortion associated with $\bar{\tau}$ gets multiplied by the production structure of the economy.

This discussion also reminds us that monopoly markups can play the same role in distorting the allocation of resources through “double marginalization,” a point emphasized by Epifani and Gancia (2009). For example, suppose every variety i is produced by a firm that charges a markup over marginal cost, which we can conveniently parameterize as $1/(1 - \tau_i)$.⁹ This wedge also gets multiplied over time through the capital multiplier and the intermediate goods multiplier, just like the distortion rate, and similar formulas to those we have derived would obtain. To the extent that poor countries have higher markups than rich countries — for example because of pressures that limit competition — these same multiplier effects occur.

5.6. Random Wedges

A symmetric wedge distorts the allocation of resources in an intertemporal sense, but does not otherwise distort the allocation across the sectors of the economy. As discussed in the introduction, however, one of the key ways in which weak links can be a problem in a country is if resources are misallocated across firms or sectors:

⁹Think of this markup as being less than the unconstrained monopoly markup because of regulations, entry threats, and other competitive pressures (all of which may be heterogeneous). This is important because the inelastic demand associated with complementarities could otherwise point toward infinite markups.

electricity may be absolutely essential to production, and problems in that sector can lead to severe disruptions.

To get a sense of how misallocation across firms can matter, we suppose the distortion and productivity levels are distributed log-normally across our continuum of sectors. We then have the following result, which also proves useful when it comes time to examine the model quantitatively:

Proposition 5 (Random Productivity and Wedges): *Let $a_i \equiv \log A_i$ and $\omega_i \equiv \log(1 - \tau_i)$ be jointly normally distributed so that $a_i \sim N(\mu_a, \nu_a^2)$, $\omega_i \sim N(\mu_\omega, \nu_\omega^2)$, and $\text{Cov}(\omega_i, a_i) = \nu_{a\omega}$, and let $1 - \bar{\tau} \equiv e^{\mu_\omega + \nu_\omega^2/2}$ implicitly define the average distortion across varieties. Then the result of Proposition 3 implies that*

$$\log y^* = \underbrace{\log \left(\frac{1 - \sigma(1 - \tau)}{1 - \tau} \right)}_{\textcircled{1}} + \underbrace{\frac{1}{1 - \sigma} \frac{1}{1 - \alpha} ((1 - \sigma) \log B_\theta + \sigma \log B_\rho)}_{\textcircled{2}} + \zeta_2$$

where ζ_2 is a collection of terms that do not depend on the wedges or productivity.

Using the log-normality assumptions, terms $\textcircled{1}$ and $\textcircled{2}$ are given by

$$\textcircled{1} = \log (1 - \sigma(1 - \bar{\tau}) \exp[\eta_\rho(\nu_\omega^2 + \nu_{a\omega})]) - (\log(1 - \bar{\tau}) + \eta_\theta(\nu_\omega^2 + \nu_{a\omega}))$$

and

$$\textcircled{2} = \frac{1}{1 - \sigma} \frac{1}{1 - \alpha} \left(\mu_a + \log(1 - \bar{\tau}) + \frac{1}{2} \tilde{\eta} \nu_a^2 + \tilde{\eta} \nu_{a,\omega} - \frac{1}{2} (1 - \tilde{\eta}) \nu_\omega^2 \right)$$

where $\eta_\rho \equiv \frac{\rho}{1 - \rho}$, $\eta_\theta \equiv \frac{\theta}{1 - \theta}$, and $\tilde{\eta} \equiv (1 - \sigma)\eta_\theta + \sigma\eta_\rho$. Moreover, given capital, $\frac{\partial \log y}{\partial \nu_\omega^2} < 0$.

GDP per person in steady state depends on two main terms, $\textcircled{2}$ and $\textcircled{1}$. Term $\textcircled{2}$ involves the CES aggregators, and notice that productivities and the distortions enter symmetrically: this term depends basically on the properties of $A_i(1 - \tau_i)$, or, in logs, $a_i + \omega_i$. Both the means and the variance terms are subject to the fundamental multiplier of the model.

In fact, the most intuitive result from this proposition comes from considering the Cobb-Douglas limit, as shown in the following corollary:

Corollary 1 *Let $\rho \rightarrow 0$ and $\theta \rightarrow 0$, and reconsider the result in Proposition 5. In this*

case, $\tilde{\eta} = \eta_\rho = \eta_\theta = 0$, and we are left with

$$y^* = (1 - \sigma(1 - \bar{\tau})) (1 - \bar{\tau})^{\frac{1}{1-\sigma} \frac{1}{1-\alpha} - 1} \exp\left(-\frac{1}{2} \left(\frac{1}{1-\sigma} \cdot \frac{1}{1-\alpha}\right) \nu_\omega^2\right) \zeta_3 \quad (27)$$

where ζ_3 is a function of terms that do not depend on the distortions.

The average distortion rate $\bar{\tau}$ then enters exactly as in Proposition 4. Moreover, the variance of the distortions is multiplied by the fundamental multiplier of the model. Both the level of distortions and the variance of distortions are amplified by the input-output structure of the economy.

We can now return to discuss the more general result in Proposition 5, where the elasticity of substitution terms are involved. The variance terms now depend on the degrees of substitution and complementarity: $\tilde{\eta}$ is essentially a weighted average of the two effective curvature parameters $\frac{\theta}{1-\theta}$ and $\frac{\rho}{1-\rho}$. This recalls an important result highlighted earlier in the discussion of S_ρ versus B_ρ , for example. The equilibrium allocation depends on $\frac{\rho}{1-\rho}$ as a curvature parameter rather than just ρ . When $\rho \rightarrow -\infty$, to maximize the significance of weak links, $\frac{\rho}{1-\rho}$ falls only as far as -1. As this is the term that multiplies the variance of the distortions in the proposition, problems with weak links cannot get too large in some sense: the economic incentives to overcome a weak link problem ensure that distortions or low productivity cannot be too costly. In contrast, $\frac{\theta}{1-\theta}$ goes to infinity as θ approaches one, so that the superstar effects from distortions can be severe.

The last part of the proposition makes the important point that variation in the wedges across sectors unambiguously reduces GDP at a point in time. Efficiency, of course, requires no distortion.¹⁰

¹⁰This result can be contrasted with the effect of variation in productivity. Changes in ν_a^2 have an ambiguous effect. From the standpoint of final uses, a higher variance is a good thing. For example (loosely speaking), if goods were perfect substitutes in consumption, only the good with the highest productivity would be consumed, and a higher variance increases the highest productivity. From the standpoint of intermediate goods, however, the opposite is true.

6. Development Accounting

To what extent can this model with linkages and complementarity help us understand income differences across countries? In this section, we attempt to quantify the mechanisms at work in our theory.

In the analysis that follows, some key parameters — such as the intermediate goods share — are calibrated quite precisely, while others — such as the degree of complementarity or the precise nature of micro-level distortions — are known with much less precision. The robust result that emerges is that intermediate goods can substantially magnify income differences relative to the standard neoclassical growth model, even with conservative choices for parameter values.

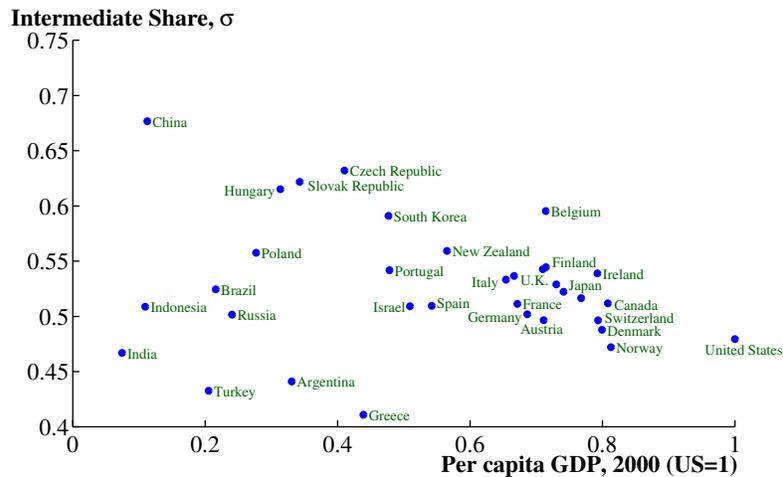
6.1. Measuring the Intermediate Goods Share

The crucial parameter of the model for explaining large income differences across countries is the intermediate goods share, σ . Fortunately, detailed empirical evidence exists regarding the magnitude of this parameter.

Basu (1995) recommends a value of 0.5 based on the numbers from Jorgenson, Gollop and Fraumeni (1987) for the U.S. economy between 1947 and 1979. Ciccone (2002), citing the extensive analysis in Chenery, Robinson and Syrquin (1986), observes that the intermediate goods share at least sometimes rises with the level of development. However, the numbers cited for South Korea, Taiwan, and Japan in the early 1970s are all substantially higher than conventional U.S. estimates, ranging from 61% to 80%.

For more systematic and recent evidence, there are rich data sets on input-output tables for many countries. For example, the OECD Input-Output Database now covers 35 countries (including 9 non-OECD countries) at the level of 48 industries for a year close to 2000; see Yamano and Ahmad (2006). Figure 1 displays the intermediate goods share of gross output using this data. For the United States and India, the share is about 47%. Japan has a share of 52%, and China has the highest share, at 68%. Across 35 countries (mostly OECD, but including Brazil, China, and India as well), the average intermediate goods share is 52.6%, with a standard deviation

Figure 1: The Intermediate Goods Share of Gross Output



Note: Computed using the OECD Input-Output Database. The numerator is spending on intermediate goods at purchaser prices; the denominator is gross output less net taxes on production.

of about 6%. Interestingly, there is no apparent correlation between this share and per capita GDP across countries. These numbers are discussed in greater detail in Jones (2010). Given this evidence, we take $\sigma = 1/2$ as a benchmark value.

6.2. A Numerical Example Based on Hsieh and Klenow

To illustrate the potential quantitative significance of intermediate goods and weak links, we consider a numerical example based on Proposition 5 above, the case where productivity and distortions are log-normally distributed. For this exercise, we need measures of the underlying productivity levels A_i and the distortions themselves. Such measurement is currently only just beginning; for example, see Hsieh and Klenow (2009) and Bartelsman, Haltiwanger and Scarpetta (2008).

Hsieh and Klenow measure both plant-level TFP and distortions within 4-digit manufacturing sectors for China, India, and the United States, treating each plant as a distinct variety. They find that the 90/10 percentile ratios of plant-level TFP in a value-added production function are about 9 for the United States, 11 for China, and 22 for India. These statistics do not correspond exactly to what we want for our

model. We'd like to see the variation across all firms and sectors in the economy. For example, the weak link story involves electricity, transportation, replacement parts, machine tools, etc. — inputs that are taken from different sectors. Also, measurement problems may lead Hsieh and Klenow to overstate TFP differences across plants.¹¹

Still, these are useful observations to get us started. In particular, the large differences that Hsieh and Klenow observe across plants producing different varieties within a 4-digit industry suggest that the cases we consider below are relatively conservative. For example, variation across all plants in the economy is almost certainly larger than the average variation across plants within a 4-digit industry. In our baseline calibration, we use their estimates of the standard deviation of log TFP of 0.84 for the United States and 1.23 for India.

For distortions, Hsieh and Klenow report the standard deviation of log “TFPR” (revenue TFP) across plants within 4-digit sectors. Once again, this is not exactly what we'd like — it would be nice to have a measure of distortions across sectors as well as within — but this is a useful starting point. In our setup, the standard deviation of log “TFPR” corresponds to the standard deviation of $\log(1 - \tau_i)$. Averaging across their years, Hsieh and Klenow report a standard deviation of 0.45 for the United States and 0.68 for China and India. It is unclear what to make of the number for the United States: this presumably represents some combination of measurement error and actual distortions. The “excess” distortion in China and India then has a standard deviation of $\sqrt{.68^2 - .45^2} = .51$, which is substantial.

Hsieh and Klenow (2007) report a correlation coefficient of $-.647$ between TFP and the distortion measure — more productive sectors face higher distortions. But this correlation is imprecisely estimated and insignificantly different from zero; hence we will consider both the estimated value and zero in our results below. Regarding the overall average level of the distortions, Hsieh and Klenow are silent. We assume

¹¹In addition, the mapping between their value-added TFP and our gross-output TFP is not entirely clear. One-good models like that discussed at the beginning of this paper can lead this difference to undo the multiplier. In the main multi-good model, however, the standard deviation of value-added TFP and the standard deviation of gross-output TFP across firms are equal; the intuition is that all the different sectors' TFPs contribute to the productivity implicit in X , which is then symmetric across varieties. This observation justifies our use of the Hsieh-Klenow TFP data.

baseline distortions for our two-country illustration of zero in the rich country and $1/4$ in the poor country.

Next, we need values for the elasticity of substitution curvature parameters, θ and ρ . Following Hsieh and Klenow (2009), we take $\theta = 2/3$, corresponding to an elasticity of substitution of 3 among final goods. For the complementarity parameter, we take $\rho = -1$, corresponding to an elasticity of substitution of $1/2$ for intermediates, halfway between Cobb-Douglas and Leontief. We will consider a wide range of robustness checks below.

Finally, we pick $\alpha = 1/3$ to match the empirical evidence on capital shares; see Gollin (2002), who shows that capital shares across countries have a mean of $1/3$ and are uncorrelated with GDP per worker. Rather than modeling differences in human capital we simply assume that it contributes a factor of 1.52 to the income ratio between the U.S. and China/India (corresponding to a 6 year educational attainment differential and a 7% return to education).

6.3. Quantitative Results

Using the Hsieh-Klenow statistics and the parameter values discussed above, we compute incomes according to Proposition 5 for two countries, a “rich” country based on U.S. values and a “poor” country based on data from China and India. Empirically, the ratios of GDP per worker in the United States to China and India in the year 2000 were 8.9 and 14.6, respectively (according to the Penn World Tables, Version 6.3), so these are the kind of numbers we hope to generate in the results.

Table 1 reports the results under a range of different assumptions motivated by the Hsieh-Klenow evidence. Focus on the two middle columns. The first reports the income ratio (between the “U.S.” and “China/India”) when the intermediate goods share is zero, shutting off the effects of both the intermediate goods multiplier and complementarity. The other shows this ratio when the intermediate goods share is $1/2$. In general, one sees that the income differences are substantially larger in the presence of intermediate goods. In fact, the last column of the table quantifies this difference, reporting the factor by which income differences increase in the presence of intermediate goods and complementarity. This factor ranges from a low of

Table 1: Output per Worker Ratios using the Hsieh-Klenow Statistics

Scenario	Description	Average	No Interme-	Baseline	<i>Ampli-</i> <i>fication</i> <i>Factor</i>
		TFP in Poor Country	diante Goods $\sigma = 0$	Case $\sigma = 1/2$	
1.	Baseline	0.860	4.5	26.7	5.9
2.	Identical TFPs	1.000	5.6	11.1	2.0
3.	$\nu_a^{rich} = \nu_a^{poor} = 0.84$	0.800	7.8	21.7	2.8
4.	$\nu_a^{rich} = \nu_a^{poor} = .5$	0.800	4.8	10.0	2.1
5.	$\nu_a^{rich} = .5, \nu_a^{poor} = .75$	0.741	4.7	15.1	3.2
6.	Same as 5, but $\nu_{aw} = 0$	0.741	3.4	12.4	3.7

Note: The two main columns of the table report income ratios between a “rich” and a “poor” (e.g. China or India) country based on Proposition 5. For comparison, the income ratios between the U.S. and China/India were 8.9 and 14.6 in 2000. The “Amplification Factor” column shows the ratio of the two previous columns — that is, the overall amplification from having $\sigma = 1/2$. Baseline parameter values are $\theta = 2/3$, $\rho = -1$, $\bar{h}^{rich}/\bar{h}^{poor} = 1.52$, $\nu_a^{rich} = 0.84$ (U.S.), $\nu_a^{poor} = 1.23$ (India), $\nu_w^{rich} = .45$, $\nu_w^{poor} = .68$, $\mu_a^{rich} = 0$, $\mu_w^{rich} = 0$, $\mu_w^{poor} = \log(1 - 1/4)$, $\nu_{aw}^{rich} = 0$ and $\nu_{aw}^{poor} = -.647 \cdot \nu_a^{poor} \nu_w^{poor}$. Most of these numbers are taken from Hsieh and Klenow (2009), Tables 1 - 2. The main exception is the value for μ_w , on which Hsieh and Klenow are silent; we assume a baseline distortion in the poor country of 1/4. The correlation coefficient 0.647 is based on Table 4 of the NBER WP version, Hsieh and Klenow (2007), not reported in the published version. In Scenarios 1, 5, and 6, μ_a^{poor} is chosen so that TFP at the 99th percentile is the same in the rich and poor countries (otherwise, the higher variance in the poor country would lead firms at the top of the distribution to be substantially more productive than those in the rich country). Scenarios 3 and 4 set the simple average of TFP across firms in the poor country at 0.8.

2.0 to a high of 5.9.

Scenario 1 is most faithful to the Hsieh-Klenow statistics and leads to the largest amplification from intermediate goods and weak links. In the absence of these forces, the calibration leads to an income ratio of 4.5. When intermediate goods and weak links are present, this income ratio rises sharply to 26.7, an amplification of 5.6 times. While the income ratio is much larger than what we see for China and India, this case nicely illustrates the amplification that is possible from intermediate goods and weak links.

The remaining scenarios consider alternatives, especially ones with smaller variations in productivity since these seem particularly large and suggestive of measurement error (for example, the ratio of productivities at the 97.5th and 2.5th percentiles in India, with a standard deviation of 1.23 in logs, is roughly $\exp(4 * 1.23) \approx 137$.) Scenario 2 shuts off all TFP differences between the countries. This case yields the smallest amplification, but intermediate goods still double the income ratio. The remaining scenarios consider smaller variation in TFP in the countries, consistent with the assumption that some of the variation found by Hsieh and Klenow is measurement error. These cases produce more plausible income ratios when $\sigma = 1/2$ and amplification factors ranging from 2.1 to 3.7.

6.4. Robustness: Complementarity and Substitution

The size of the weak link and superstar effects in this framework depends on the elasticities of substitution for intermediate goods and consumption goods. In the baseline case explored so far, we set these elasticities at $1/2$ ($\rho = -1$) and 3 ($\theta = 2/3$). Neither of these parameters is especially well pinned down in the literature; this is particularly true of the degree of complementarity among inputs. Hence, checking the robustness of the results along this dimension is important.

The first point to make in this respect is that the amplification factor — the last column of the results — is invariant to the value of θ (this can be confirmed with some simple algebra). So the basic results are completely robust to different values of θ .

Table 2 explores robustness to the extent of complementarity. Three cases are

Table 2: Robustness Results with the Hsieh-Klenow Statistics

Scenario	Description	— Amplification Factors —		
		Cobb-Douglas $\rho = 0$	Baseline $\rho = -1$	“Leontief” $\rho = -100$
1.	Baseline	5.1	5.9	6.6
2.	Identical TFPs	2.2	2.0	1.8
3.	$\nu_a^{rich} = \nu_a^{poor} = 0.84$	3.1	2.8	2.5
4.	$\nu_a^{rich} = \nu_a^{poor} = .5$	2.2	2.1	2.0
5.	$\nu_a^{rich} = .5, \nu_a^{poor} = .75$	3.1	3.2	3.2
6.	Same as 5, but $\nu_{aw} = 0$	3.1	3.7	4.4

Note: The main columns report the “amplification factor,” i.e. the proportional increase in the rich/poor income ratio that occurs when going from $\sigma = 0$ to $\sigma = 1/2$. There are no robustness results associated with varying θ because the amplification factors are invariant to this parameter. See notes to Table 1.

reported: Cobb-Douglas ($\rho = 0$), the baseline case considered before ($\rho = -1$), and a near-Leontief case ($\rho = -100$). To keep the table manageable, we just report the amplification factors in these robustness results (that is, the analog of the last column of Table 1).

Two results are clear. First, the amplification factors range from about 2 to about 6, just as they did in the first table of results. Second, the difference between the Cobb-Douglas case and the Leontief case is not nearly as large as the weak link intuition might suggest. As indicated in our simple examples earlier, the ability of the economy to substitute capital and labor for low productivity in a weak link sector seems to mitigate the worst of these problems.

7. Remarks and Reflections

A helpful example for thinking about the amplification associated with the input-output structure of the economy is based on theft. First, notice that when output is stolen from a firm, it is obviously the gross output that gets stolen, not the value added. Now imagine a simple world where 50 percent of the gross output of each

variety is lost due to theft, and to keep things simple, imagine that the thieves are foreigners who take the goods abroad, so they do not eventually show up in the economy as consumption. In this case, theft essentially reduces each A_i by one half.

In the pure neoclassical framework with no intermediate goods, this 2-fold difference amplifies the basic income difference in by $2^{1/1-\alpha} = 2^{3/2} \approx 2.8$. In the presence of intermediate goods, however, this multiplier is much stronger: $2^{\frac{1}{1-\sigma} \frac{1}{1-\alpha}} = 2^{2 \times 3/2} = 2^3 = 8$, amplifying income differences by a factor of $8/2.8 \approx 2.9$. The intuition is that the “theft tax” gets paid repeatedly when intermediate goods are involved: 1/2 of the steel is stolen from the steel plant, 1/2 of the cars are stolen from the automobile plant, and 1/2 of the pizzas get stolen from the pizza delivery service. In this sense, the steel gets stolen *three* times rather than just once, and this is the intermediate goods multiplier.

A large multiplier in growth models is a two-edged sword. On the one hand, it is extremely useful in getting realistic differences in investment rates, productivity, and distortions to explain large income differences. However, the large multiplier has a cost. In particular, theories of economic development often suffer from a “magic bullet” critique. If the multiplier is so large, then solving the development problem may be quite easy: small subsidies to the production of output or small improvements in a single (exogenous) productivity level have enormous long-run effects on per capita income in such a model. If there were a single magic bullet for solving the world’s development problems, one would expect that policy experimentation across countries would hit on it, at least eventually. The magic bullet would become well-known and the world’s development problems would be solved.

Complementarity can help resolve this issue. Despite the large multiplier, reforms that address a subset of an economy’s distortions may have relatively small effects on output. If a chain has a number of weak links, fixing one or two of them will not change the overall strength of the chain.

Of course, it should also be recognized that some reforms could affect distortions in multiple sectors simultaneously. For example, multinational firms and international trade may help to solve these problems if they are allowed to oper-

ate. Multinationals may bring with them knowledge of how to produce, access to transportation and foreign markets, and the appropriate capital equipment. Indeed many of the examples we know of where multinationals produce successfully in poor countries effectively give the multinational control on as many dimensions as possible: consider the maquiladoras of Mexico and the special economic zones in China and India. Countries may specialize in goods they can produce with high productivity and, to the extent possible, import the goods and services that suffer most from weak links.¹²

On the other hand, domestic weak links may still be a problem. A lack of contract enforcement may make intermediate inputs hard to obtain. Knowledge of which intermediate goods to buy and how to best use them in production may be missing. Weak property rights may lead to expropriation. Inadequate energy supplies and local transportation networks may reduce productivity. The right goods must be imported, and these goods must be distributed using local resources and nontradable inputs, as in Burstein, Neves and Rebelo (2003).

8. Conclusion

In an effort to understand the large income differences across countries, this paper explores two related channels for amplifying the effects of distortions. First, the presence of intermediate goods leads to a multiplier that depends on the share of intermediate goods in gross output. This amplification force echoes the familiar multiplier associated with capital accumulation and is relatively easy to quantify. By raising the effective share of produced inputs in total output from $1/3$ to $2/3$, the addition of intermediate goods delivers a substantial multiplier. Distortions to the transportation sector reduce the output of many other activities, including truck manufacturing and highway construction. This in turn further reduces output in the transportation sector and in the rest of the economy. This vicious cycle is the source of the multiplier associated with intermediate goods.

¹²Nunn (2007) provides evidence along these lines, suggesting that countries that are able to enforce contracts successfully specialize in goods where contract enforcement is critical. See also Grossman and Maggi (2000) and Waugh (2007).

The second amplification force — the “O”-ring effects of complementarity — is more difficult to quantify in practice. Simple examples suggest that this force can be quite powerful. For example, in comparing the symmetric allocation and the optimal allocation in the case in which final goods are perfect substitutes while intermediate goods are Leontief, income in the rich country depends on the maximum productivity (a superstar effect), while income in the poor country depends on the weakest link. However, in the numerical exercises explored in this paper, the effects of complementarity tended to be much smaller. The reason is that market forces are often quite effective at offsetting weak links. One way to see this formally is in the switch in the curvature parameter from ρ in the symmetric allocation to $\frac{\rho}{1-\rho}$ in the equilibrium allocation for the TFP aggregator functions (S_ρ versus B_ρ). Even in the extreme case of $\rho = -\infty$, the equilibrium allocation depends on the harmonic mean of distortion-adjusted productivities, not on the minimum. In this case, the Cobb-Douglas production structure for varieties lets other inputs like capital and labor substitute for low productivity to alleviate severe bottlenecks. The more general lesson seems to be that one must carefully consider various substitution possibilities when seeking to quantify the effects of weak links.

An important channel for future research is quantifying more precisely the role of intermediate goods. For example, it may be useful to pursue plant level studies like Hsieh and Klenow (2009) and Bartelsman, Haltiwanger and Scarpetta (2008) paying attention to gross output as well as value added. Capital and labor may be misallocated across plants, but there is every reason to think that the same is also true for the extensive range of intermediate goods that are used in plants as well.

Related to this, the present model simplifies considerably by focusing on a single intermediate input. The input-output matrix in this model is very special. This is a good place to start. However, it is possible that the rich input-output structure in modern economies delivers a multiplier smaller than $\frac{1}{1-\sigma}$ because of “zeros” in the matrix. Jones (2010) explores this issue in a Long and Plosser (1983) style model where each of N sectors uses the output of the other sectors as an intermediate good, and the preliminary results are encouraging. For example, if the total share of intermediate goods in each sector is σ but the sectoral composition of this

share varies arbitrarily, the aggregate multiplier is still $\frac{1}{1-\sigma}$. More generally, that paper uses linear algebra to study input-output tables for both OECD and developing countries and to compute the associated multipliers. Tentative results confirm the central role played by intermediate goods in amplifying distortions.

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Online Appendix for Intermediate Goods and Weak Links: A Theory of Economic Development

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This appendix contains outlines of the proofs of the propositions reported in the paper.

Proof of Proposition 1. The Symmetric Allocation, Given Capital

Follows directly from the fact that $Q_i = A_i m$, where $m = (K^\alpha H^{1-\alpha})^{1-\sigma} X^\sigma$ is constant across activities. **QED.**

Proof of Proposition 2. The Competitive Equilibrium, Given Capital

1. The first order conditions from the Variety i Problem are

$$(1 - \tau_i) p_i \alpha (1 - \sigma) \frac{Q_i}{K_i} = r + \delta$$

$$(1 - \tau_i) p_i (1 - \alpha) (1 - \sigma) \frac{Q_i}{H_i} = w$$

$$(1 - \tau_i) p_i \sigma \frac{Q_i}{X_i} = q.$$

Substituting these conditions back into the production function yields an equation that characterizes the price of good i :

$$p_i = \frac{mc}{A_i (1 - \tau_i) \epsilon}, \tag{1}$$

where $mc \equiv ((r + \delta)^\alpha w^{1-\alpha})^{1-\sigma} q^\sigma$ is a key piece of the marginal cost and $\epsilon \equiv (\alpha^\alpha (1 - \alpha)^{1-\alpha})^{1-\sigma} (1 - \sigma)^{1-\sigma} \sigma^\sigma$.

2. Integrating the Variety i first order conditions above gives

$$(r + \delta)K = \alpha(1 - \sigma) \int (1 - \tau_i)p_i Q_i di \quad (2)$$

$$wH = (1 - \alpha)(1 - \sigma) \int (1 - \tau_i)p_i Q_i di \quad (3)$$

$$qX = \sigma \int (1 - \tau_i)p_i Q_i di \quad (4)$$

where the limits of the integration are understood to be 0 to 1. Note that

$$\int p_i c_i di = Y, \quad \int p_i z_i di = qX, \quad \int p_i Q_i di = Y + qX.$$

Define $\tau \equiv \frac{E}{Y+qX}$ to be distortion revenues as a share of gross output. Then

$$\int (1 - \tau_i)p_i Q_i di = (1 - \tau)(Y + qX).$$

Substituting this expression into (2), (3), and (4) gives

$$(r + \delta)K = \alpha(1 - \sigma) \frac{1 - \tau}{1 - \sigma(1 - \tau)} Y \quad (5)$$

$$wH = (1 - \alpha)(1 - \sigma) \frac{1 - \tau}{1 - \sigma(1 - \tau)} Y \quad (6)$$

$$qX = \frac{\sigma(1 - \tau)}{1 - \sigma(1 - \tau)} Y. \quad (7)$$

These expressions allow us to solve for mc (see the definition under (1)) as

$$mc = \frac{1 - \tau}{1 - \sigma(1 - \tau)} \cdot \epsilon \cdot \frac{Y}{(K^\alpha H^{1-\alpha})^{1-\sigma} X^\sigma}. \quad (8)$$

3. Next, consider the first-order conditions from the Final Good and Intermediate Good Problems. For each of these problems, take the first order condition and then integrate it back into the firm's production function. For the final good, this gives

$$\left(\int p_i^{-\frac{\theta}{1-\theta}} di \right)^{-\frac{1-\theta}{\theta}} = 1 \quad (9)$$

and for the intermediate good

$$\left(\int p_i^{-\frac{\rho}{1-\rho}} di \right)^{-\frac{1-\rho}{\rho}} = q \quad (10)$$

Now substitute (1) into (9) to get

$$mc = \epsilon B_\theta \quad (11)$$

where

$$B_\theta \equiv \left(\int_0^1 (A_i(1-\tau_i))^{\frac{\theta}{1-\theta}} di \right)^{\frac{1-\theta}{\theta}}. \quad (12)$$

Combining (1) with this expression, we can solve (10) to find

$$q = \frac{B_\theta}{B_\rho} \quad (13)$$

where B_ρ is defined analogously to B_θ . Combining (8), (11), (7), and (13) yields the main result in the proposition.

4. Finally, we need to solve for τ . From the first-order conditions for the Final Goods Problem and the Intermediate Goods Problem we get

$$p_i Q_i = p_i c_i + p_i z_i = p_i^{-\frac{\theta}{1-\theta}} Y + (p_i/q)^{-\frac{\rho}{1-\rho}} (qX).$$

Multiplying this expression by τ_i , integrating, and then using (1), (11), and (13) leads to the solution for τ :

$$\tau = (1 - \sigma(1 - \tau))T_\theta + \sigma(1 - \tau)T_\rho \quad (14)$$

where $T_\rho \equiv \int_0^1 \tau_i \left(\frac{A_i(1-\tau_i)}{B_\rho} \right)^{\frac{\rho}{1-\rho}} di$. That is, T_ρ is a weighted average of the sector-specific distortions, where the weights depend on ρ ; T_θ is defined analogously. QED.

Proof of Proposition 3. The Competitive Equilibrium in Steady State

Straightforward using (5) and the Euler equation from the Household Problem. QED.

Proof of Proposition 4. Symmetric Wedges

Straightforward evaluation given earlier results. QED.

Proof of Proposition 5. Random Productivity and Wedges

1. Define $B(\eta) \equiv (\int (A_i(1 - \tau_i))^\eta di)^{1/\eta}$. Define $m_i \equiv \eta(a_i + \omega_i)$. Then $m_i \sim N(\eta(\mu_m + \mu_a), \eta^2 \nu^2)$, where $\nu^2 \equiv \nu_a^2 + \nu_\omega^2 + 2\nu_{a\omega}$. Therefore,

$$\begin{aligned} B(\eta) &= (E(e^{m_i}))^{1/\eta} \\ &= e^{\mu_a + \mu_\omega + \frac{1}{2}\eta\nu^2}. \end{aligned} \tag{15}$$

Let $\bar{B} \equiv [B(\frac{\theta}{1-\theta})]^{1-\sigma} [B(\frac{\rho}{1-\rho})]^\sigma$. Then

$$\log \bar{B} = \mu_a + \mu_\omega + \frac{1}{2} \cdot \left((1 - \sigma) \frac{\theta}{1 - \theta} + \sigma \frac{\rho}{1 - \rho} \right) \nu^2.$$

Making the substitution $1 - \bar{\tau} \equiv e^{\mu_\omega + \nu_\omega^2/2}$ then yields term ②.

2. To get term ①, we need to solve for τ . From equation (14), one can obtain

$$1 - \tau = \frac{1 - T_\theta}{1 - \sigma(T_\theta - T_\rho)}.$$

Evaluating the integrals in T_θ and T_ρ as above gives

$$T_\theta = 1 - \exp\left\{ \mu_\omega + \frac{1}{2} \cdot \frac{1 + \theta}{1 - \theta} \nu_\omega^2 + \frac{\theta}{1 - \theta} \nu_{a\omega} \right\}$$

and T_ρ is the analogous expression. Straightforward algebra then delivers term ①. QED.