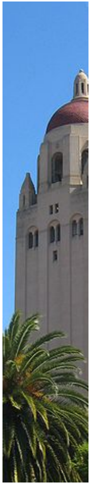

Past Automation and Future AI: How Weak Links Tame the Growth Explosion

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Stanford-Berkeley Macro Day

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How much of past economic growth is due to automation?

- Automation has being going on since the Industrial Revolution
  - Human with hoes, to horses with plows, to tractors, to GPS automation
  - Robots replacing welders and painters in auto factories
  - AI is the latest form of automation, e.g., writing software

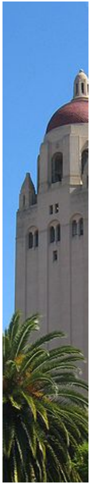
Growth accounting for past and use what we learn
to speculate on the future with A.I.

How we answer these questions

- Task-based growth model — weak links ($\sigma < 1$)
- One key assumption
  - The new tasks we automate on average are those with the highest labor costs
  - Equivalently: low labor productivity (Moravec's paradox)
- One heroic measurement — the automation rate
  - Historically, around 2% of not-yet-automated tasks get automated each year
  - Measure using ChatGPT Deep Research — plan to do better on this soon

Main Results: Quantify the Impact of Automation

- Key gain from automation:
  - Switch from slowly-improving labor to rapidly-improving machines on $\uparrow$ tasks
  - Machines get better rapidly: 5pp faster than the rate at which humans get better
  - Majority of historical growth is from rapid improvement in machine productivity
- The future of A.I.
  - Calibrate an idea-based growth model to our historical facts
  - Growth explodes over the next 75 years, and into the future in our baseline
  - The explosion is slow: 2040 – output is 4% higher. 2060 – 19% higher
  - The path looks very similar over the next 75 years regardless of whether the capital share ends at 100% or 0% (LR future is very different)



The Model

Economic Environment: The Canonical Task Model

$$Y_t = Z_t \left(\int_0^1 \alpha_i Y_{it}^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

$$Y_{it} = \tilde{\psi}_{kit} K_{it} + \tilde{\psi}_{lit} L_{it} \tag{2}$$

$$K_t = \int_0^1 K_{it} \, di \quad \text{and} \quad L_t = \int_0^1 L_{it} \, di \quad \text{where} \quad Z_t, \tilde{\psi}_{kit}, \tilde{\psi}_{\ell it}, K_t, L_t \quad \text{exogenous} \tag{3}$$

Economic Environment: The Canonical Task Model

Complementarity $$Y_t = Z_t \left(\int_0^1 \alpha_i Y_{it}^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}} \quad \text{where } \sigma < 1 \tag{1}$$

Substitution $$Y_{it} = \tilde{\psi}_{kit} K_{it} + \tilde{\psi}_{lit} L_{it} \tag{2}$$

$$K_t = \int_0^1 K_{it} \, di \quad \text{and} \quad L_t = \int_0^1 L_{it} \, di \quad \text{where } Z_t, \tilde{\psi}_{kit}, \tilde{\psi}_{\ell it}, K_t, L_t \text{ exogenous} \tag{3}$$

Economic Environment: The Canonical Task Model

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- New "procedures" as a way of incorporating new tasks

$$Y_{it} = \tilde{\psi}_{kit}^1 K_{it}^1 + \cdots + \tilde{\psi}_{kit}^{N_{kt}} K_{it}^{N_{kt}} + \tilde{\psi}_{\ell it}^1 L_{it}^1 + \cdots + \tilde{\psi}_{\ell it}^{N_{\ell t}} L_{it}^{N_{\ell t}}$$

- Notation: $\psi_{kit} = \alpha_i^{\frac{\sigma}{\sigma-1}} \tilde{\psi}_{kit}$ and $\psi_{\ell it} = \alpha_i^{\frac{\sigma}{\sigma-1}} \tilde{\psi}_{\ell it}$

Representative Firm in a Perfectly Competitive Sector

$$\max_{\{K_{it},L_{it}\}} P_t Y_t - w_t \int_0^1 L_{it}\, di - r_t \int_0^1 K_{it}\, di \quad \text{s.t. (1) + (2) given} \quad P_t, w_t, r_t$$

- Use capital to produce task i whenever

$$\frac{\psi_{kit}}{r_t} \geq \frac{\psi_{\ell it}}{w_t}$$

- Define the set of tasks using capital and labor as

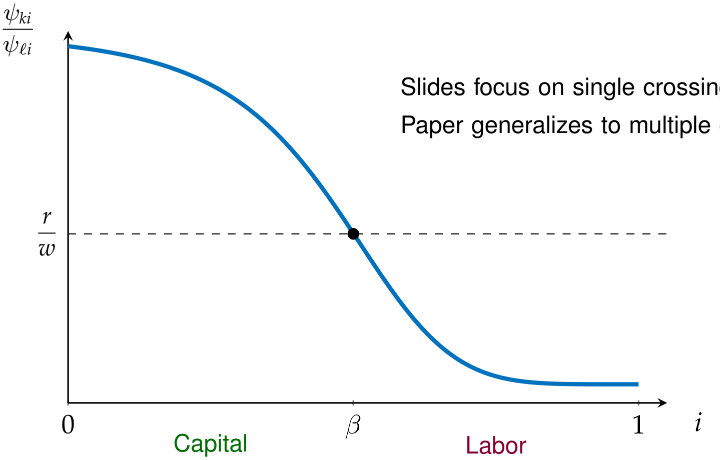
$$\Omega_{kt} = \{i \mid \psi_{kit}/\psi_{\ell it} \geq r_t/w_t\} \text{ and } \Omega_{\ell t} = \{i \mid \psi_{kit}/\psi_{\ell it} < r_t/w_t\}$$

$$\beta_t \equiv \|\Omega_{kt}\| \text{ and } 1 - \beta_t \equiv \|\Omega_{\ell t}\|$$

- Any task β that is just at the margin of being automated:

$$\frac{\psi_{k\beta t}}{\psi_{\ell\beta t}} = \frac{r_t}{w_t}$$

Automation and Comparative Advantage: Example



Slides focus on single crossing case.

Paper generalizes to multiple crossings.

Proposition 1 (*Reduced-form production function*)

Equilibrium output Y_t can be represented as a CES-like production function:

$$
\begin{aligned}
Y_t &= F(B_t K_t, A_t L_t) \\
&= \left((B_t K_t)^{\frac{\sigma-1}{\sigma}} + (A_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}
\end{aligned}
$$

where

$$
\begin{aligned}
B_t &= Z_t \left(\int_{\Omega_{kt}} \psi_{kit}^{\sigma-1} \, di \right)^{\frac{1}{\sigma-1}} \\
A_t &= Z_t \left(\int_{\Omega_{\ell t}} \psi_{\ell it}^{\sigma-1} \, di \right)^{\frac{1}{\sigma-1}}
\end{aligned}
$$

Proposition 1 (continued)

Factor shares are given by

$$s_{Kit} \equiv \frac{r_t K_{it}}{P_t Y_t} = \left(\frac{\psi_{kit} Z_t K_{it}}{Y_t}\right)^{\frac{\sigma-1}{\sigma}}$$

$$s_{Lit} \equiv \frac{w_t L_{it}}{P_t Y_t} = \left(\frac{\psi_{\ell it} Z_t L_{it}}{Y_t}\right)^{\frac{\sigma-1}{\sigma}}$$

$$s_{Kt} \equiv \frac{r_t K_t}{P_t Y_t} = \left(\frac{B_t K_t}{Y_t}\right)^{\frac{\sigma-1}{\sigma}} = \int_{\Omega_{kt}} s_{Kit}\, di \qquad \uparrow B \Rightarrow \downarrow s_K$$

$$s_{Lt} \equiv \frac{w_t L_t}{P_t Y_t} = \left(\frac{A_t L_t}{Y_t}\right)^{\frac{\sigma-1}{\sigma}} = \int_{\Omega_{\ell t}} s_{Lit}\, di$$

Task model has a familiar CES-like representation.

Not truly CES in that the A_t and B_t depend on w_t/r_t via Ω_{kt} and $\Omega_{\ell t}$.

The Weights, ω_{kit} and $\omega_{\ell it}$

- Weights ω_{kit} and $\omega_{\ell it}$ are the shares of capital and labor costs for the tasks:

$$\frac{L_{it}}{L_t} = \frac{w_t L_{it}}{w_t L_t} = \frac{\psi_{\ell it}^{\sigma-1}}{\int_{\Omega_{\ell t}} \psi_{\ell it}^{\sigma-1}\, di} \equiv \omega_{\ell it}$$

$$\frac{K_{it}}{K_t} = \frac{r_t K_{it}}{r_t K_t} = \frac{\psi_{kit}^{\sigma-1}}{\int_{\Omega_{kt}} \psi_{kit}^{\sigma-1}\, di} \equiv \omega_{kit}$$

Proposition 2 (*Growth rates of B_t and A_t*)

Assuming tasks only get automated and never "de-automated,"

$$\hat{B}_t = \hat{Z}_t + \hat{\psi}_{kt} \underbrace{- \frac{1}{1-\sigma}\omega_{k\beta t}\dot{\beta}_t}_{\text{Capital depleting}} \quad \text{where} \quad \hat{\psi}_{kt} \equiv \int_{\Omega_{kt}} \hat{\psi}_{kit}\omega_{kit}\, di$$

$$\hat{A}_t = \hat{Z}_t + \hat{\psi}_{\ell t} \underbrace{+ \frac{1}{1-\sigma}\omega_{\ell\beta t}\dot{\beta}_t}_{\text{Labor augmenting}} \quad \text{where} \quad \hat{\psi}_{\ell t} \equiv \int_{\Omega_{\ell t}} \hat{\psi}_{\ell it}\omega_{\ell it}\, di$$

- Effect of $\uparrow \beta_t$:
  - Capital depleting ($\downarrow B$): K spread over more tasks — weakens each link
  - Labor augmenting ($\uparrow A$): L concentrated on fewer tasks — strengthens links

(see Aghion, Jones, and Jones 2019 for homogenous case)

Data and Sources: Y/L, TFP, and s_K

NAICS	Sector	Years
—	Private business sector	1950–2023
—	Agriculture	1950–2021
334	Computer and electronic products	1987–2017
3361–63	Motor vehicles, bodies and trailers, and parts	1987–2017
44–45	Retail trade	1987–2017
511, 516	Publishing industries (includes software)	1987–2017

- Private business: BLS Multifactor Productivity database (1950-2023)
- Agriculture: USDA Economic Research Service (1950-2021)
- BEA-BLS KLEMS Integrated Industry Production Accounts (1987-2021)

Identification of B_t and A_t

- Baseline: $\sigma = 0.5$ (e.g. Acemoglu-Restrepo 2022, Young 2025)
  - Task elasticity σ would be the same as the EoS between K and L if the set of tasks being automated is held fixed.
  - When automation adjusts, this suggests that σ is less than the EofS(K,L) estimated in the literature

- Using model:
  - $y_t = Y_t / L_t$
  - $A_t = s_{Lt}^{\frac{\sigma}{\sigma-1}} y_t$
  - $\hat{Y}_t = s_{Kt}\left(\hat{B}_t + \hat{K}_t\right) + s_{Lt}\left(\hat{A}_t + \hat{L}_t\right)$
  - $T\hat{F}P_t \equiv \hat{Y}_t - s_{Kt}\hat{K}_t - s_{Lt}\hat{L}_t = s_{Kt}\hat{B}_t + s_{Lt}\hat{A}_t$ implies B_t

TFP Growth: Basic Data

Sector	TFP growth	Growth in B_t	Growth in A_t	Factor share of capital	Factor share of labor
Private business	1.2	-1.2	2.4	0.35	0.65
Agriculture	3.3	2.4	4.6	0.57	0.43
Computers	12.8	8.6	15.6	0.41	0.59
Motor vehicles	1.7	-0.8	3.5	0.43	0.57
Retail trade	1.7	-2.9	2.8	0.20	0.80
Software	1.8	-1.4	4.8	0.47	0.53

- Private business: standard TFP growth and factor shares.
- $\hat{A}_t = 2.4\%$ while $\hat{B}_t = -1.2\%$ — negative!
- $1.2 = .35 \cdot (-1.2) + .65 \cdot 2.4$. Invariant to σ if factor shares are stable.
- $\hat{B}_t$ is negative in 4 of the 6 sectors. Supports $\sigma < 1$ and $\uparrow \beta_t$

TFP Growth: ψ_{kit}, $\psi_{\ell it}$, and β_t

- Using Proposition 2:

$$\begin{aligned}\widehat{TFP}_t &= s_{Kt}\hat{B}_t + s_{Lt}\hat{A}_t \\ &= \underbrace{s_{Kt}\hat{\psi}_{kt}}_{\text{Better capital}} + \underbrace{s_{Lt}\hat{\psi}_{\ell t}}_{\text{Better labor}} + \underbrace{\frac{\dot{\beta}_t}{1-\sigma}\left(s_{Lt}\bar{\omega}_{\ell\beta t} - s_{Kt}\bar{\omega}_{k\beta t}\right)}_{\text{Automation effect}} + \hat{Z}_t \qquad (4)\end{aligned}$$

- TFP growth comes from
  - Improvements in productivity on tasks that don't switch automation status
  - Automation effect (switching tasks from labor to capital)
  - Other TFP growth ($\hat{Z}_t$)

Automation Indifference Condition

- Setting
  - Continuity: Let ψ_{kit}, $\psi_{\ell it}$, w_t, and r_t only change smoothly
  - Any task β that is just at the margin of being automated: $\frac{\psi_{k\beta t}}{\psi_{\ell\beta t}} = \frac{r_t}{w_t}$
- Therefore, at marginal task β:

$$\begin{aligned} s_{Lt}\omega_{\ell\beta t} - s_{Kt}\omega_{k\beta t} &= \frac{w_t L_t}{P_t Y_t} \cdot \frac{w_t L_{\beta t}}{w_t L_t} - \frac{r_t K_t}{P_t Y_t} \cdot \frac{r_t K_{\beta t}}{r_t K_t} \\ &= \frac{w_t L_{\beta t}}{P_t Y_t} - \frac{r_t K_{\beta t}}{P_t Y_t} \\ &= 0 \end{aligned}$$

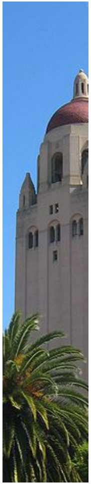
- Automation effect is zero!
  - Automation occurs precisely where costs of using labor and capital are equal

Proposition 3 (*Zero TFP growth from automation*)

The automation effect in the TFP growth decomposition (4) is zero. Therefore, TFP growth equals

$$\widehat{TFP}_t = s_{Kt}\hat{B}_t + s_{Lt}\hat{A}_t$$

$$= \underbrace{s_{Kt}\hat{\psi}_{kt}}_{\text{Capital productivity}} + \underbrace{s_{Lt}\hat{\psi}_{\ell t}}_{\text{Labor productivity}} + \underbrace{\hat{Z}_t}_{\text{Other TFP growth}}$$



How much of TFP growth

is due to $\hat{\psi}_{kt}$?

Key Insight to Answering this Question

- Labor augmenting productivity growth (from Proposition 2):

$$\hat{A}_t = \hat{Z}_t + \hat{\psi}_{\ell t} + \frac{1}{1-\sigma}\omega_{\ell\beta t}\dot{\beta}_t \tag{5}$$

- Recall weight $\omega_{\ell\beta t}$:

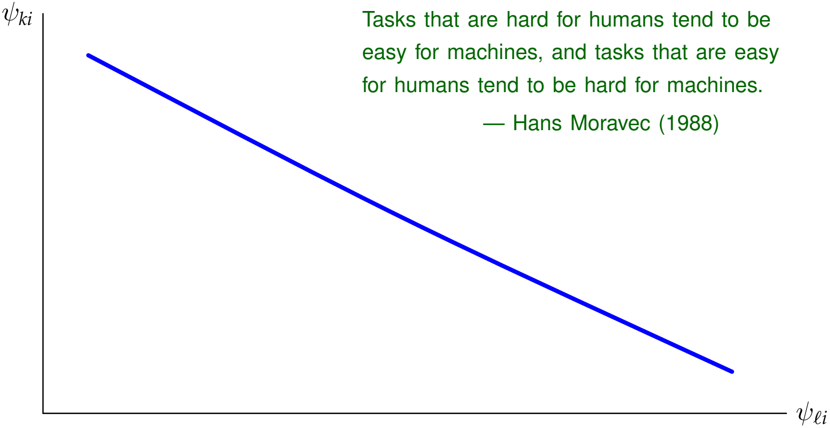
$$\omega_{\ell\beta t} \equiv \frac{\psi_{\ell\beta t}^{\sigma-1}}{\int_{\Omega_{\ell t}} \psi_{\ell i t}^{\sigma-1}\, di} = \frac{L_{\beta t}}{L_t}$$

- **Assumption**: $L_{\beta t} \geq \frac{L_t}{1-\beta_t}$
  - The newly automated task uses more labor than average
  - With $\sigma < 1 \iff$ the newly automated task has lower ψ_ℓ than the average non-automated task.

Newly automated tasks had high labor costs and low labor productivity $\longleftarrow$ testable

And "same" would be great — equality

Moravec's Paradox



Tasks that are hard for humans tend to be easy for machines, and tasks that are easy for humans tend to be hard for machines.

— Hans Moravec (1988)

Using Moravec's Paradox

- Moravec $\Rightarrow \omega_{\ell\beta} = \frac{L_\beta}{L} \geq \frac{1}{1-\beta}$. Now substitute into (5):

$$\begin{aligned}\hat{A}_t &= \hat{Z}_t + \hat{\psi}_{\ell t} + \frac{1}{1-\sigma}\omega_{\ell\beta}\dot{\beta} \\ &\geq \hat{Z}_t + \hat{\psi}_{\ell t} + \frac{1}{1-\sigma} \cdot \frac{1}{1-\beta_t}\dot{\beta}_t\end{aligned}$$

- Let $x_t \equiv \frac{\dot{\beta}_t}{1-\beta_t} = -\frac{d\log(1-\beta_t)}{dt}$, so $x_t \equiv$ the automation rate:

  $x_t =$ What fraction of non-automated tasks get automated in year t?

$$\hat{A}_t - \frac{1}{1-\sigma}x_t \geq \hat{Z}_t + \hat{\psi}_{\ell t} \qquad (6)$$

Key: Equation (6) *provides an upper bound on* $\hat{Z}_t + \hat{\psi}_{\ell t}$

Getting useful empirical bounds

- TFP growth from Proposition 3:

$$\widehat{TFP}_t = s_{Kt}\hat{\psi}_{kt} + (1 - s_{Kt})\hat{\psi}_{\ell t} + \hat{Z}_t$$

$$= \hat{Z}_t + \hat{\psi}_{\ell t} + s_{Kt}\left(\hat{\psi}_{kt} - \hat{\psi}_{\ell t}\right)$$

Baseline TFP growth ($\hat{Z}_t + \hat{\psi}_{\ell t}$)

Automation effect: boost from machines getting better ($s_{Kt}\left(\hat{\psi}_{kt} - \hat{\psi}_{\ell t}\right)$)

Getting useful empirical bounds

- TFP growth from Proposition 3:

$$\widehat{TFP}_t = s_{Kt}\hat{\psi}_{kt} + (1 - s_{Kt})\hat{\psi}_{\ell t} + \hat{Z}_t$$

$$= \underbrace{\hat{Z}_t + \hat{\psi}_{\ell t}}_{\text{Small!}} + \underbrace{s_{Kt}\left(\hat{\psi}_{kt} - \hat{\psi}_{\ell t}\right)}_{\text{Automation effect: boost from machines getting better}}$$

- Substituting the bound from equation (6) gives the next proposition.

Proposition 4 (*Key bounds on $\hat{\psi}_{kt}$ and $\hat{\psi}_{\ell t}$*)

Upper bound on neutral + labor productivity growth:

$$\hat{A}_t - \frac{1}{1-\sigma} x_t \ \geq \ \hat{Z}_t + \hat{\psi}_{\ell t}$$

How much faster is capital getting better than labor?

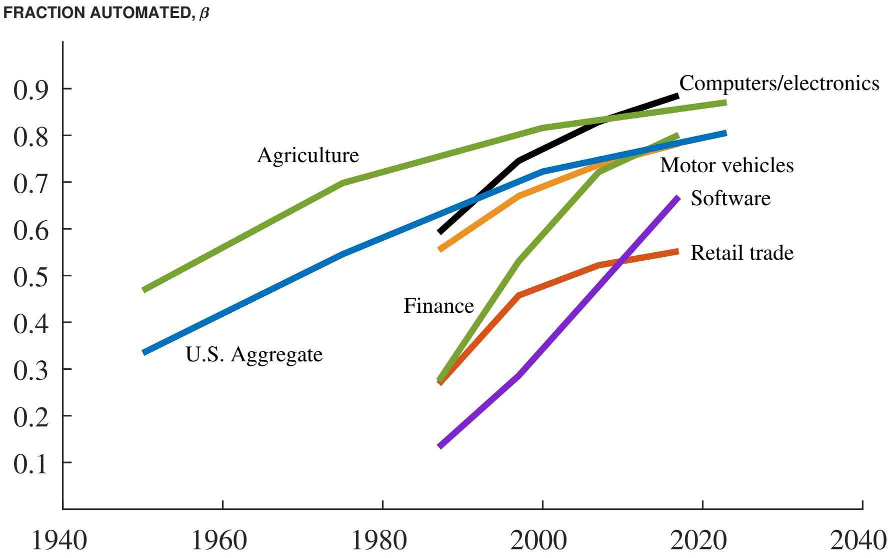
$$\hat{\psi}_{kt} - \hat{\psi}_{\ell t} \geq \frac{1}{s_{Kt}} \left(\widehat{TFP}_t - \left[\hat{A}_t - \frac{1}{1-\sigma} x_t \right] \right)$$

With measures of the automation rate x_t, we can calculate these bounds.

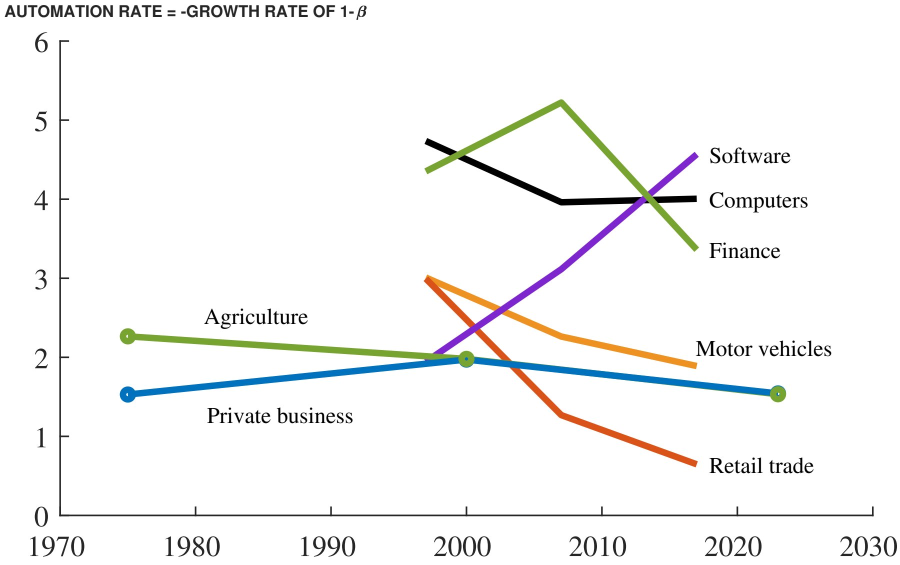
Identification of β_t through Narrative Historical Analysis

- For each industry, ask world-class specialist historian to list 150 specific tasks that are necessary for production of that industry
  - Consider years 1987, 1997, 2007, and 2017 (longer for Agriculture and Pvt Bus)
  - For each task-year, label each task as automated or not
    - For partially automated tasks (subtasks) list percent automated
  - Provide a narrative summary of the path of automation
  - Provide a spreadsheet with automation percentages
- Calculate β_t as equally weighted share of tasks that were automated in a given year
- Historian is ChatGPT Deep Research (and Google Gemini for robustness)

Share of Tasks that are Automated, β_t



Automation Rates, $x_t \equiv \frac{\dot{\beta}_t}{1-\beta_t} = -\frac{d\log(1-\beta_t)}{dt}$



Bounds on $\hat{Z}_t + \hat{\psi}_{\ell t}$

$$\hat{A}_t - \frac{1}{1-\sigma} x_t \;\geq\; \hat{Z}_t + \hat{\psi}_{\ell t}$$

Sector	Growth rate of A_t	Automation rate, x_t	Upperbound on $\hat{Z}_t + \hat{\psi}_{\ell t}$
Private business	2.4	1.7	-0.9
Agriculture	4.6	2.0	0.6
Computers	15.6	4.2	7.1
Motor vehicles	3.5	2.4	-1.2
Retail trade	2.8	1.6	-0.5
Software	4.8	3.2	-1.7

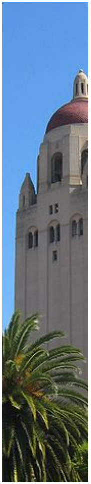
$\hat{Z}_t + \hat{\psi}_{\ell t}$ *is small and often negative*

Bounds on $\hat{\psi}_{kt} - \hat{\psi}_{\ell t}$

$$\hat{\psi}_{kt} - \hat{\psi}_{\ell t} \geq \frac{1}{s_{Kt}} \left(\widehat{TFP}_t - \left[\hat{A}_t - \frac{1}{1-\sigma} x_t \right] \right)$$

Sector	Growth rate of A_t	Automation rate, x_t	Upperbound on $\hat{Z}_t + \hat{\psi}_{\ell t}$	TFP Growth	Capital share, s_{Kt}	Lowerbound on $\hat{\psi}_{kt} - \hat{\psi}_{\ell t}$
Private business	2.4	1.7	-0.9	1.2	0.35	6.2
Agriculture	4.6	2.0	0.6	3.3	0.57	4.9
Computers	15.6	4.2	7.1	12.8	0.41	14.1
Motor vehicles	3.5	2.4	-1.2	1.7	0.43	7.2
Retail trade	2.8	1.6	-0.5	1.7	0.20	12.1
Software	4.8	3.2	-1.7	1.8	0.47	7.3

$\hat{\psi}_{kt} - \hat{\psi}_{\ell t}$ is at least 5% per year



How much TFP growth would be lost
if we “froze” automation in 1950 (or 1987)?

“Freeze” Automation in 1950 or 1987

- Counterfactual accounting exercise, using historical data
- Consider the set of automated tasks from some initial year
  - Hold automation set fixed at that initial value: $\Omega_{kt} = \Omega_{k0}$
  - Assumption 3: $\hat{\psi}_{kt}$, $\hat{\psi}_{\ell t}$, and $\hat{Z}_t$ are unchanged.
- Recall TFP growth:

$$\widehat{TFP}_t = s_{Kt}(\hat{\psi}_{kt} - \hat{\psi}_{\ell t}) + \hat{Z}_t + \hat{\psi}_{\ell t}$$

$\longrightarrow$ only affects TFP growth via s_{Kt}

(continued)

- Hold the set of tasks fixed while machines continue to get better:
  - Recall: $\uparrow \beta_t \Rightarrow \uparrow s_{Kt}$ and $\uparrow \psi_{kt} \Rightarrow \downarrow s_{Kt}$
  - Holding β fixed $\Rightarrow$ counterfactual capital share will be lower:

$$\frac{s_{Kt}^{cf}}{1 - s_{Kt}^{cf}} \leq \frac{s_{Kt}}{1 - s_{Kt}} \exp\left(- \int_0^t \frac{1}{s_{K\tau}} x_\tau d\tau \right) \quad (7)$$

- Lower share of costs s_{Kt} benefits from $\hat{\psi}_{kt} - \hat{\psi}_{\ell t} \Rightarrow$ "lost" TFP growth

Proposition 5 (*Counterfactual contribution of automation*)

Under Assumptions 1 to 3,

$$\begin{aligned}\widehat{TFP}_t - \widehat{TFP}_t^{cf} &= \left(s_{Kt} - s_{Kt}^{cf}\right)\left(\hat{\psi}_{kt} - \hat{\psi}_{\ell t}\right) \\ &\geq \left(s_{Kt} - \text{upperbound on } s_{Kt}^{cf})\right) \times \text{lower bound on } (\hat{\psi}_{kt} - \hat{\psi}_{\ell t})\end{aligned}$$

where the upper bound on s_{Kt}^{cf} is given by equation (7) and the lower bound on $\hat{\psi}_{kt} - \hat{\psi}_{\ell t}$ is given by Proposition 4.

Counterfactual Lost Growth from Freezing Automation

Sector	Capital share $s_{K,T}$	$s^{cf}_{K,T}$	$\hat{\psi}_{kt} - \hat{\psi}_{\ell t}$	Lost Growth $\widehat{TFP}_t - \widehat{TFP}^{cf}_t$	TFP Growth	Lost Growth Share of $\widehat{TFP}_t$
Automation set frozen in 1950:						
Private business	0.420	0.004	6.2	1.5	1.2	134%
Agriculture	0.655	0.127	4.9	1.3	3.3	39%
Automation set frozen in 1987:						
Computers	0.459	0.033	14.1	3.9	12.8	30%
Motor vehicles	0.524	0.161	7.2	1.3	1.7	74%
Retail trade	0.259	0.022	12.1	1.2	1.7	73%
Software	0.463	0.102	7.3	1.7	1.8	95%

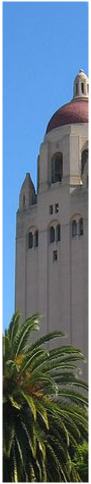
40% - 70% of TFP growth would be lost in the typical sector

Robustness

Sector	$\sigma = 0$	$\sigma = 0.25$	$\sigma = 0.5$	$\sigma = 0.75$	x_t halved
			Upper bound on $\hat{Z}_t + \hat{\psi}_{\ell t}$		
Private business	0.7	0.2	-0.9	-4.3	0.7
Agriculture	2.6	1.9	0.6	-3.4	2.6
Computers	11.3	9.9	7.1	-1.4	11.3
Motor vehicles	1.1	0.4	-1.2	-6.0	1.1
Retail trade	1.2	0.6	-0.5	-3.7	1.2
Software	1.5	0.5	-1.7	-8.1	1.5
			Lower bound on $\hat{\psi}_{kt} - \hat{\psi}_{\ell t}$		
Private business	1.2	2.8	6.1	15.8	1.2
Agriculture	1.3	2.4	4.8	11.8	1.3
Computers	3.7	7.1	14.0	34.7	3.7
Motor vehicles	1.4	3.2	7.0	18.2	1.4
Retail trade	2.4	5.1	10.4	26.4	2.4
Software	0.6	2.9	7.5	21.2	0.6

Robustness

Sector	$\sigma = 0$	$\sigma = 0.25$	$\sigma = 0.5$	$\sigma = 0.75$	x_t halved
Lost TFP growth – freeze automation (percent share; lower bound)					
Private business	45	79	134	284	45
Agriculture	19	26	39	77	19
Computers	12	18	30	66	12
Motor vehicles	44	54	74	133	44
Retail trade	32	46	73	154	32
Software	16	42	95	254	16



The Future Consequences of A.I.?

What has happened to the factor income share of computers / IT?

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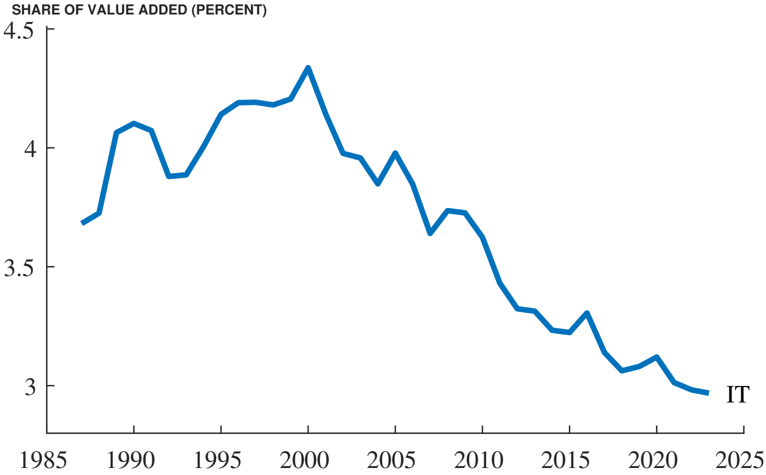


Computers are everywhere, but rapid price declines dominate. $\sigma < 1$

The Role of Weak Links ($\sigma < 1$)

- Firm production requires the successful completion of a number of tasks
  - Failing at sourcing inputs or quality control or timely delivery or other tasks can be very detrimental
  - Examples: the space shuttle Challenger's O-ring or Covid-19 supply chain issues
- Automation $\Rightarrow$ fast computers / machines perform tasks instead of people
  - Large cost savings in long run — machines get better rapidly
  - Talented people are the scarce input

Weak links remain the limit to production and hence earn high returns

What are the consequences of fully automating software?

- A simple calculation is revealing
  - Assume ψ_{kit} on the fully-automated tasks becomes infinite
  - Assume the set of tasks that benefit is frozen, and no effect on idea production
  - We relax these assumptions later
- Ω_{∞} = set of infinitely automated tasks, $\Omega_{\varnothing}$ = other tasks (unaffected)
- Before infinite automation,

$$\begin{aligned} Y_t^{\frac{\sigma-1}{\sigma}} &= \int_0^1 \alpha_i Y_{it}^{\frac{\sigma-1}{\sigma}} di \\ &= \int_{\Omega_\infty} \alpha_i Y_{it}^{\frac{\sigma-1}{\sigma}} di + \int_{\Omega_\varnothing} \alpha_i Y_{it}^{\frac{\sigma-1}{\sigma}} di \\ &= \alpha_\infty Y_{\infty t}^{\frac{\sigma-1}{\sigma}} + \alpha_\varnothing Y_{\varnothing t}^{\frac{\sigma-1}{\sigma}} \end{aligned}$$

The Future of Automation: Perfectly Automated Software

- Today's pre-automation factor shares:

$$s_{jt} \equiv \frac{P_{jt} Y_{jt}}{P_t Y_t} = \alpha_j \left(\frac{Y_{jt}}{Y_t} \right)^{\frac{\sigma - 1}{\sigma}}$$

- With infinite ψ_k, Y_∞ to infinity, so $Y_\infty^{\frac{\sigma-1}{\sigma}}$ to zero — because $\sigma < 1$

$$Y_{cf} = \alpha_{\varnothing}^{\frac{\sigma}{\sigma - 1}} Y_{\varnothing t}$$

  - We essentially removed some of the weak links / bottlenecks

The Future of Automation: Perfectly Automated Software

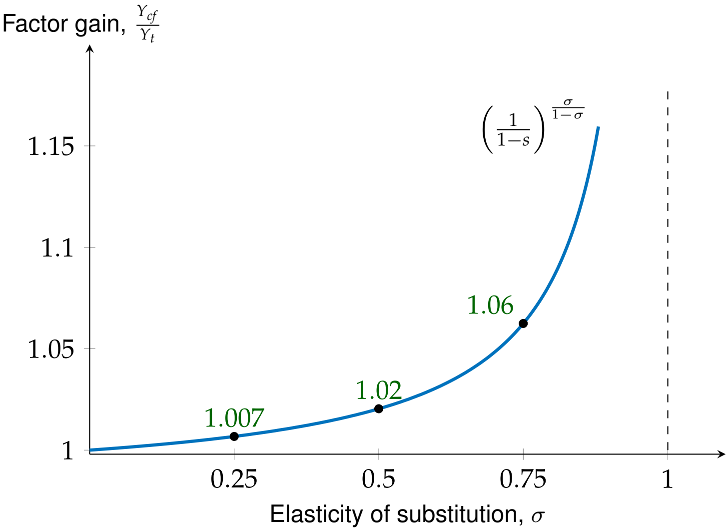
- Dividing by Y_t,

$$\frac{Y_{cf}}{Y_t} = \alpha_{\varnothing}^{\frac{\sigma}{\sigma-1}} \frac{Y_{\varnothing t}}{Y_t}$$
$$= s_{\varnothing t}^{\frac{\sigma}{\sigma-1}}$$

$$\frac{Y_{cf}}{Y_t} = \left(\frac{1}{1-s_\infty}\right)^{\frac{\sigma}{1-\sigma}} \approx 1 + \frac{\sigma}{1-\sigma} s_\infty \quad \text{for } s \text{ small} \qquad (8)$$

- Key insight:
  - $s \approx 2\%$ in software $\Rightarrow$ about 2% increase in GDP from complete-automation of the software industry with infinite productivity!
  - Surprisingly small (for $\sigma = 1/2$)

Infinite software ($s = 2\%$) with different values of σ



What if A.I. automates all cognitive tasks?

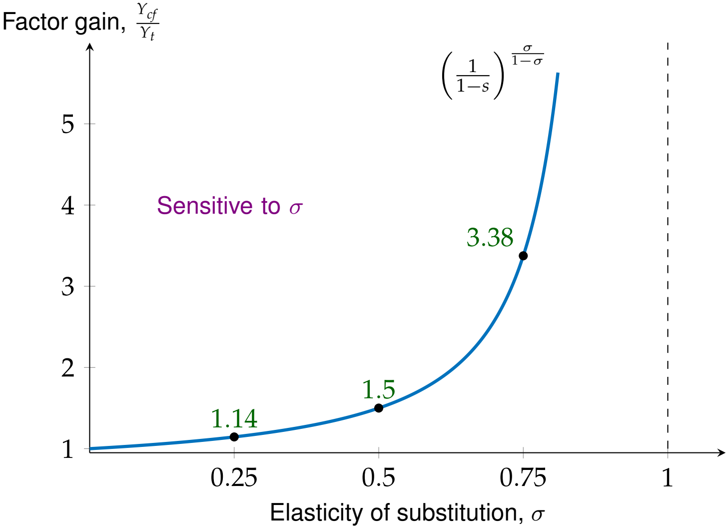
- For now, assume $s = 1/3$: half of labor compensation is for cognitive tasks
  - In future, we can measure share of college and above, for example

- With $\sigma = 1/2$, equation (8) gives

$$\frac{1}{1 - 1/3} = 1.50$$

- Infinitely automating 1/3 of the cost share of GDP would increase GDP by 50%
  - The weak links / bottleneck forces of $\sigma < 1$ are powerful
  - If completed in a decade, could raise growth by ~5pp / year
    — but still only one-time level effect

Infinitely automating all cognitive tasks, $s = 1/3$



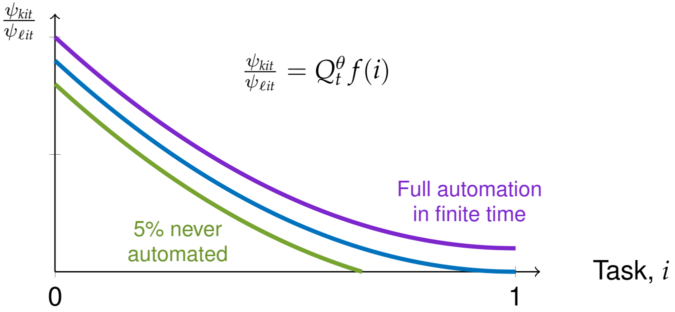
The Dynamic Model including Automating Idea Production

- The model so far continues to apply $\Rightarrow Y_t$ and Ω_{kt}, etc.
- Dynamics via the production of new ideas
  - Ideas $\Rightarrow \psi_{kit}$ and $\psi_{\ell it}$
  - Lab equipment setup: automating goods also automates ideas
- Calibrate model to match the facts we've already documented
- Use the model to project the future
  - What happens to growth and factor shares as automation continues?
  - Very preliminary! Numbers will definitely change...

The Dynamic Model: Semi-endogenous growth plus automation

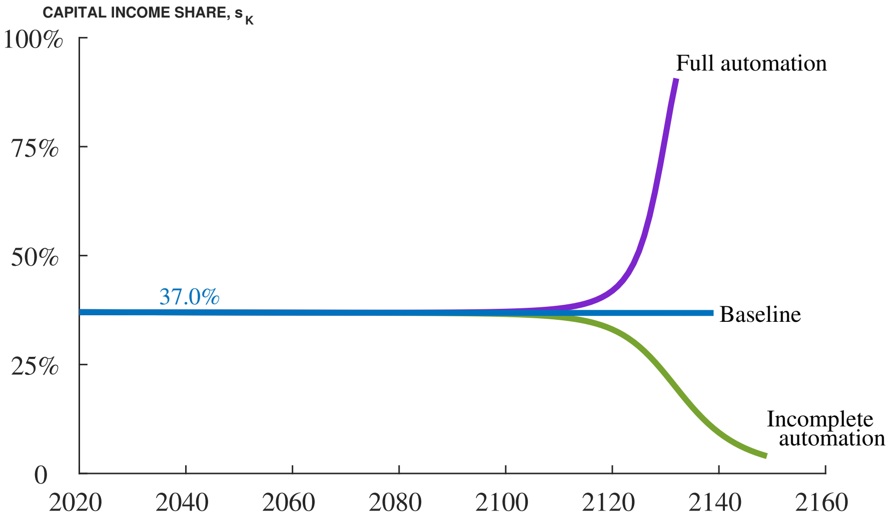
CES task model	Same as before $\Rightarrow Y_t$ and Ω_{kt}
Idea PF	$\dot{Q}_t = \bar{q} R_t^{\lambda} Q_t^{\phi}$
Ideas $\Rightarrow \psi_{kit}$	$\psi_{kit} = Q_t^{\theta_k} f(i)$
Ideas $\Rightarrow \psi_{\ell it}$	$\psi_{\ell it} = Q_t^{\theta_\ell}$ (homogeneous)
Heterogeneity	$f(i) = \frac{(1-i)^{\mu}}{1+\mu_0(1-i)^{\mu}} + \bar{f}$
Resource constraint	$C_t + I_t + R_t = Y_t$
Capital accumulation	$\dot{K}_t = I_t - \delta K_t$
Population growth	$L_t = L_0 e^{nt}$
Allocations	$R_t = \bar{\iota}_R Y_t$ and $I_t = \bar{\iota}_K Y_t$

Nature of Automation

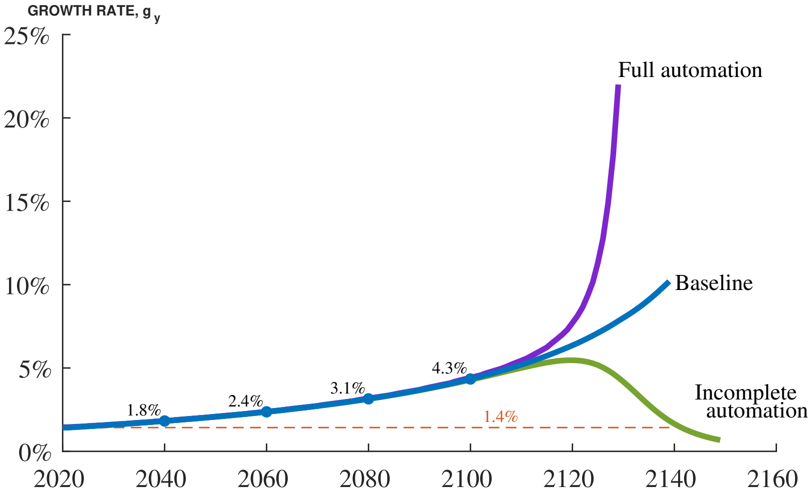


- Some tasks never automated if $f(\beta) = 0$ for $\beta > \beta^*$
- All tasks automated in finite time if $f(1) > 0$
- What happens if $f(1) = 0$?
    - Labor always used, but on a vanishing range of tasks

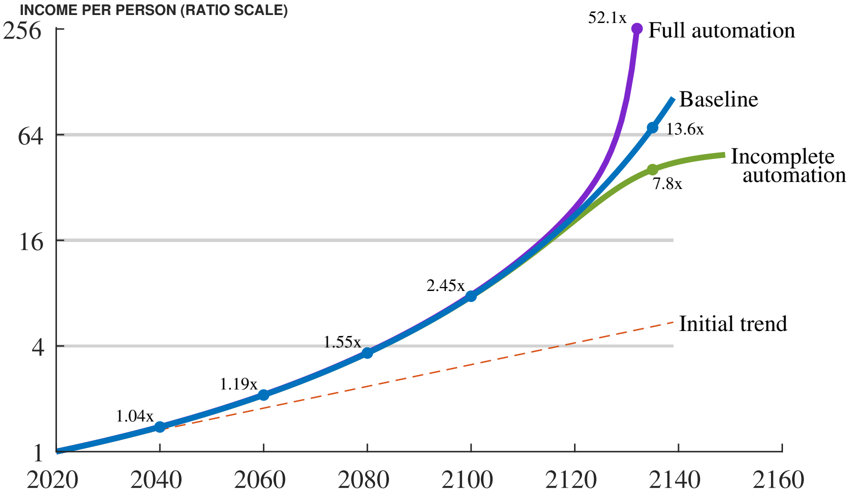
Simulating the Future: Capital Share



Simulating the Future: Economic Growth



Simulating the Future: GDP per Person



Intuition for Slowly Accelerating Growth

- With $\bar{f} = 0$, the capital share stabilizes at $s_K^* = \frac{1}{\mu(1-\sigma)}$ (weak links)
- Key measure of dynamic increasing returns:

$$\Phi = \frac{\lambda}{1-\phi}\left(\theta_\ell + \frac{s_K^*}{1-s_K^*}\theta_k\right)$$

- When $\Phi < 1 \Rightarrow$ semi-endogenous growth:

$$g_y = \frac{\Phi n}{1-\Phi}$$

  - $\Phi = 1 \Rightarrow$ knife edge of fully endogenous growth
  - $\Phi > 1 \Rightarrow$ explosive growth

Intuition (continued)

- With our baseline parameter values: $\Phi = 1.40 \;\Rightarrow$ explodes
  - Even for $\bar{f} = 0$ — even when labor is always used but vanishingly
  - But shouldn't the explosion then occur quickly? E.g. 40% above cutoff
- Consider a differential equation $\dot{X}_t = \bar{g} X_t^{\Phi}$. The growth rate is $\hat{X}_t = \bar{g} X_t^{\Phi - 1}$
  $\Rightarrow$ explodes for $\Phi > 1$
- Integrating: Infinite income occurs in finite time at date

$$t_\infty = \frac{1}{(\Phi - 1)\bar{g}} = \frac{1}{0.40 \times 0.014} = \text{178 years} \tag{9}$$

Conclusion

- How much of historical growth is due to automation?
  - More than 50%
  - The gain from automation is switching from slowly-improving humans (e.g. 0.5% per year) to rapidly-improving machines (e.g. 5% per year)
- The future of A.I.?
  - Weak links are crucial
  - Having infinite computing power on a subset of tasks is still limited by weak links
  - Automation could certainly accelerate growth

    ... but the effects may take decades to be substantial