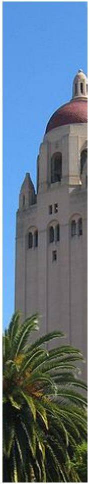


The End of Economic Growth?

Unintended Consequences of a Declining Population

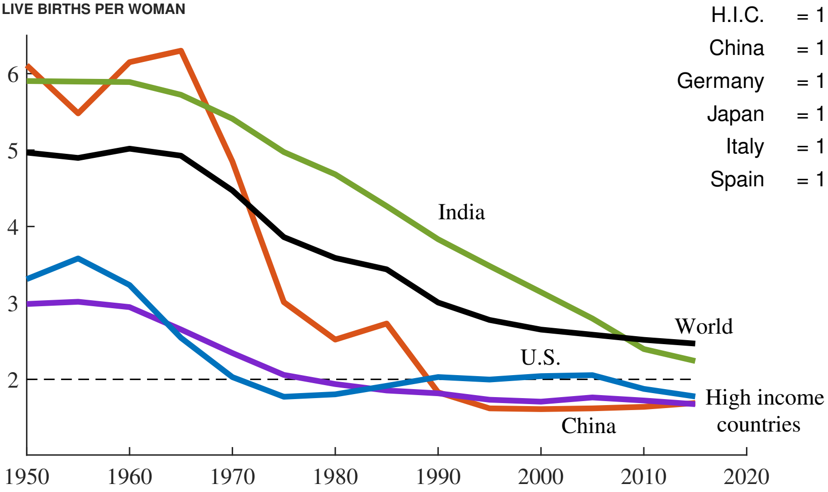
Chad Jones

September 2020

Key Role of Population

- People $\Rightarrow$ ideas $\Rightarrow$ economic growth
  - Romer (1990), Aghion-Howitt (1992), Grossman-Helpman
  - Jones (1995), Kortum (1997), Segerstrom (1998)
  - And most idea-driven growth models
- The future of global population?
  - Conventional view: stabilize at 8 or 10 billion
- Bricker and Ibbotson's *Empty Planet* (2019)
  - Maybe the future is **negative population growth**
  - High income countries already have fertility below replacement!

The Total Fertility Rate (Live Births per Woman)



U.S.	=	1.8
H.I.C.	=	1.7
China	=	1.7
Germany	=	1.6
Japan	=	1.4
Italy	=	1.3
Spain	=	1.3

What happens to economic growth if population growth is negative?

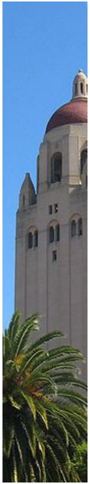
- Exogenous population decline
  - **Empty Planet Result**: Living standards stagnate as population vanishes!
  - Contrast with standard **Expanding Cosmos** result: exponential growth for an exponentially growing population
- Endogenous fertility
  - Parameterize so that the equilibrium features negative population growth
  - A planner who prefers **Expanding Cosmos** can get trapped in an **Empty Planet** – if society delays implementing the optimal allocation

Literature Review

- Many models of fertility and growth (but not $n < 0$)
  - Too many papers to fit on this slide!
- Falling population growth and declining dynamism
  - Krugman (1979) and Melitz (2003) are semi-endogenous growth models
  - Karahan-Pugsley-Sahin (2019), Hopenhayn-Neira-Singhania (2019), Engbom (2019), Peters-Walsh (2019)
- Negative population growth
  - Feyrer-Sacerdote-Stern (2008) and changing status of women
  - Christians (2011), Sasaki-Hoshida (2017), Sasaki (2019a,b) consider capital, land, and CES
  - Detroit? Or world in 25,000 BCE?

Outline

- Exogenous negative population growth
  - In Romer / Aghion-Howitt / Grossman-Helpman
  - In semi-endogenous growth framework
- Endogenous fertility
  - Competitive equilibrium with negative population growth
  - Optimal allocation



The Empty Planet Result

A Simplified Romer/AH/GH Model

Production of goods (IRS)	$Y_t = A_t^\sigma N_t$
Production of ideas	$\frac{\dot{A}_t}{A_t} = \alpha N_t$
Constant population	$N_t = N$

- Income per person: levels and growth

$$y_t \equiv Y_t / N_t = A_t^\sigma$$

$$\frac{\dot{y}_t}{y_t} = \sigma \frac{\dot{A}_t}{A_t} = \sigma \alpha N$$

- Exponential growth with a constant population
  - But population growth means exploding growth? (Semi-endogenous fix)

Negative Population Growth in Romer/AH/GH

Production of goods (IRS) $$Y_t = A_t^\sigma N_t$$

Production of ideas $$\frac{\dot{A}_t}{A_t} = \alpha N_t$$

Exogenous population decline $$N_t = N_0 e^{-\eta t}$$

- Combining the 2nd and 3rd equations (note $\eta > 0$)

$$\frac{\dot{A}_t}{A_t} = \alpha N_0 e^{-\eta t}$$

- This equation is easily integrated...

The Empty Planet Result in Romer/GH/AH

- The stock of knowledge A_t is given by

$$\log A_t = \log A_0 + \frac{g_{A0}}{\eta}\left(1 - e^{-\eta t}\right)$$

where g_{A0} is the initial growth rate of A

- A_t and $y_t \equiv Y_t/N_t$ converge to constant values A^* and y^*:

$$A^* = A_0 \exp\left(\frac{g_{A0}}{\eta}\right)$$

$$y^* = y_0 \exp\left(\frac{g_{y0}}{\eta}\right)$$

- **Empty Planet Result**: Living standards stagnate as the population vanishes!

Semi-Endogenous Growth

Production of goods (IRS)	$Y_t = A_t^{\sigma} N_t$
Production of ideas	$\frac{\dot{A}_t}{A_t} = \alpha N_t^{\lambda} A_t^{-\beta}$
Exogenous population growth	$N_t = N_0 e^{nt}, \quad n > 0$

- Income per person: levels and growth

$$y_t = A_t^{\sigma} \quad \text{and} \quad A_t^* \propto N_t^{\lambda/\beta}$$

$$g_y^* = \gamma n, \quad \text{where} \quad \gamma \equiv \lambda\sigma/\beta$$

- **Expanding Cosmos**: Exponential income growth for growing population

Negative Population Growth in the Semi-Endogenous Setting

Production of goods (IRS)	$Y_t = A_t^{\sigma} N_t$
Production of ideas	$\dfrac{\dot{A}_t}{A_t} = \alpha N_t^{\lambda} A_t^{-\beta}$
Exogenous population decline	$N_t = N_0 e^{-\eta t}$

- Combining the 2nd and 3rd equations:

$$\frac{\dot{A}_t}{A_t} = \alpha N_0^{\lambda} e^{-\lambda \eta t} A_t^{-\beta}$$

- Also easily integrated...

The Empty Planet in a Semi-Endogenous Framework

- The stock of knowledge A_t is given by

$$A_t = A_0 \left(1 + \frac{\beta g_{A0}}{\lambda\eta}\left(1 - e^{-\lambda\eta t}\right)\right)^{1/\beta}$$

- Let $\gamma \equiv \lambda\sigma/\beta$ = overall degree of increasing returns to scale.

- Both A_t and income per person $y_t \equiv Y_t/N_t$ converge to constant values A^* and y^*:

$$A^* = A_0 \left(1 + \frac{\beta g_{A0}}{\lambda\eta}\right)^{1/\beta}$$

$$y^* = y_0 \left(1 + \frac{g_{y0}}{\gamma\eta}\right)^{\gamma/\lambda}$$

Numerical Example

- Parameter values
  - $g_{y0} = 2\%, \quad \eta = 1\%$
  - $\beta = 3 \quad \Rightarrow \quad \gamma = 1/3$ (from BJVW)

- How far away is the long-run stagnation level of income?

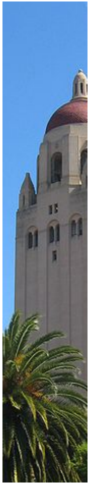
	y^*/y_0
Romer/AH/GH	7.4
Semi-endog	1.9

- The Empty Planet result occurs in both, but quantitative difference

First Key Result: The Empty Planet

- Fertility has trended down: 5, 4, 3, 2, and less in rich countries
  - For a family, nothing special about “above 2” vs “below 2”

- But macroeconomics makes this distinction critical!
  - Negative population growth may condemn us to stagnation on an **Empty Planet**
    - Stagnating living standards for a population that vanishes
  - Vs. the exponential growth in income and population of an **Expanding Cosmos**



Endogenous Fertility

The Economic Environment

ℓ = time having kids instead of producing goods

Final output	$Y_t = A_t^\sigma (1 - \ell_t) N_t$
Population growth	$\frac{\dot{N}_t}{N_t} = n_t = b(\ell_t) - \delta$
Fertility	$b(\ell_t) = \bar{b}\ell_t$
Ideas	$\frac{\dot{A}_t}{A_t} = N_t^\lambda A_t^{-\beta}$
Generation 0 utility	$U_0 = \int_0^\infty e^{-\rho t} u(c_t, \tilde{N}_t) dt, \quad \tilde{N}_t \equiv N_t / N_0$
Flow utility	$u(c_t, \tilde{N}_t) = \log c_t + \epsilon \log \tilde{N}_t$
Consumption	$c_t = Y_t / N_t$

Overview of Endogenous Fertility Setup

- All people generate ideas here
  - Learning by doing vs separate R&D
- Equilibrium: ideas are an externality (simple)
  - We have kids because we like them
  - We ignore that they might create ideas that benefit everyone
  - Planner will desire higher fertility
- This is a modeling choice — other results are possible
- Abstract from the demographic transition. Focus on where it settles

A Competitive Equilibrium with Externalities

- Representative generation takes w_t as given and solves

$$\max_{\{\ell_t\}} \int_0^\infty e^{-\rho t} u(c_t, \tilde{N}_t) dt$$

subject to

$$\dot{N}_t = (b(\ell_t) - \delta) N_t$$

$$c_t = w_t(1 - \ell_t)$$

- Equilibrium wage $w_t = \text{MP}_L = A_t^\sigma$
- Rest of economic environment closes the equilibrium

Solving for the equilibrium

- The Hamiltonian for this problem is

$$\mathcal{H} = u(c_t, \tilde{N}_t) + v_t[b(\ell_t) - \delta]N_t$$

  where v_t is the shadow value of another person.

- Let $V_t \equiv v_t N_t$ = shadow value of the population

- Equilibrium features constant fertility along transition path

$$V_t = \frac{\epsilon}{\rho} \equiv V^*_{eq}$$

$$\ell_t = 1 - \frac{1}{\bar{b}V_t} = 1 - \frac{1}{\bar{b}V^*_{eq}} = 1 - \frac{\rho}{\bar{b}\epsilon} \equiv \ell_{eq}$$

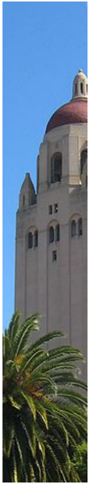
Discussion of the Equilibrium Allocation

$$n^{eq} = \bar{b} - \delta - \frac{\rho}{\epsilon}$$

- We can choose parameter values so that $n^{eq} < 0$
  - Constant, negative population growth in equilibrium

- Remaining solution replicates the exogenous fertility analysis

The Empty Planet result can arise in equilibrium



The Optimal Allocation

The Optimal Allocation

- Choose fertility to maximize the welfare of a representative generation

- Problem:

$$\max_{\{\ell_t\}} \int_0^\infty e^{-\rho t} u(c_t, \tilde{N}_t) dt$$

  subject to

$$\dot{N}_t = (b(\ell_t) - \delta) N_t$$

$$\frac{\dot{A}_t}{A_t} = N_t^\lambda A_t^{-\beta}$$

$$c_t = Y_t / N_t$$

- Optimal allocation recognizes that offspring produce ideas

Solution

- Hamiltonian:

$$\mathcal{H} = u(c_t, \tilde{N}_t) + \mu_t N_t^{\lambda} A_t^{1-\beta} + v_t(b(\ell_t) - \delta)N_t$$

μ_t is the shadow value of an idea

v_t is the shadow value of another person

- First order conditions

$$\ell_t = 1 - \frac{1}{\bar{b}V_t}, \quad \text{where} \quad V_t \equiv v_t N_t$$

$$\rho = \frac{\dot{\mu}_t}{\mu_t} + \frac{1}{\mu_t}\left(u_c \sigma \frac{y_t}{A_t} + \mu_t(1-\beta)\frac{\dot{A}_t}{A_t}\right)$$

$$\rho = \frac{\dot{v}_t}{v_t} + \frac{1}{v_t}\left(\frac{\epsilon}{N_t} + \mu_t \lambda \frac{\dot{A}_t}{N_t} + v_t n_t\right)$$

Steady State Conditions

- The social value of people in steady state is

$$V_{sp}^* = v_t^* N_t^* = \frac{\epsilon + \lambda z^*}{\rho}$$

where z denotes the social value of new ideas:

$$z^* \equiv \mu_t^* \dot{A}_t^* = \frac{\sigma g_A^*}{\rho + \beta g_A^*}$$

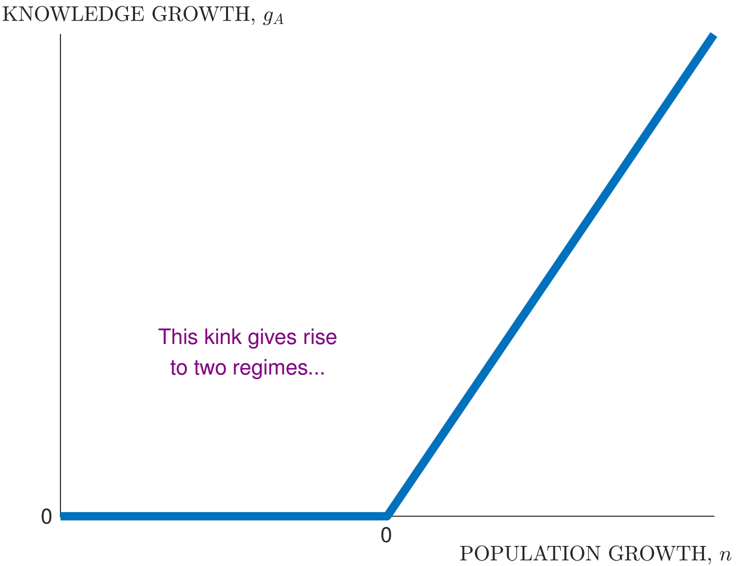
(Note: μ^* finite at finite c^* and A^*)

- If $n_{sp}^* > 0$, then we have an **Expanding Cosmos** steady state

$$g_A^* = \frac{\lambda n_{sp}^*}{\beta}$$

$$g_y^* = \gamma n_{sp}^*, \quad \text{where} \quad \gamma \equiv \frac{\lambda\sigma}{\beta}$$

Steady State Knowledge Growth



Key Features of the Equilibrium and Optimal Allocations

- Fertility in both

$$n = \bar{b}\ell - \delta$$

$$\ell = 1 - \frac{1}{\bar{b}V}$$

where V is the "utility value of people" (eqm vs optimal). Therefore

$$n(V) = \bar{b} - \delta - \frac{1}{V}$$

- Equilibrium: value kids because we love them (only): $V^{eqm} = \frac{\epsilon}{\rho}$
  - We can support $n < 0$ as an equilibrium for some parameter values
- Planner also values the ideas our kids will produce: $V^{sp} = \frac{\epsilon + \mu \dot{A}}{\rho} \quad \Rightarrow V(n)$

Optimal Steady State(s)

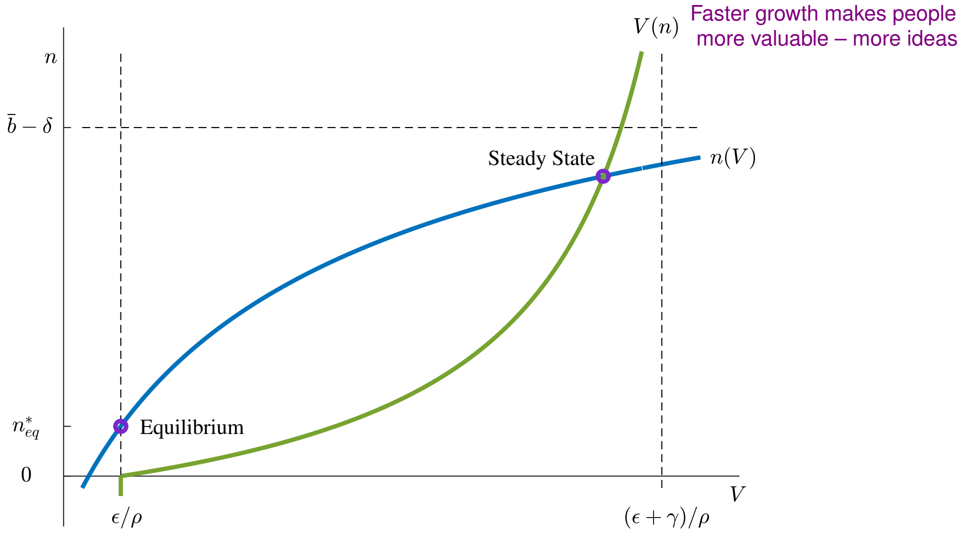
- Two equations in two unknowns (V, n)

$$V(n) = \begin{cases} \frac{1}{\rho}\left(\epsilon + \frac{\gamma}{1+\frac{\rho}{\lambda n}}\right) & \text{if } n > 0 \\ \frac{\epsilon}{\rho} & \text{if } n \leq 0 \end{cases}$$

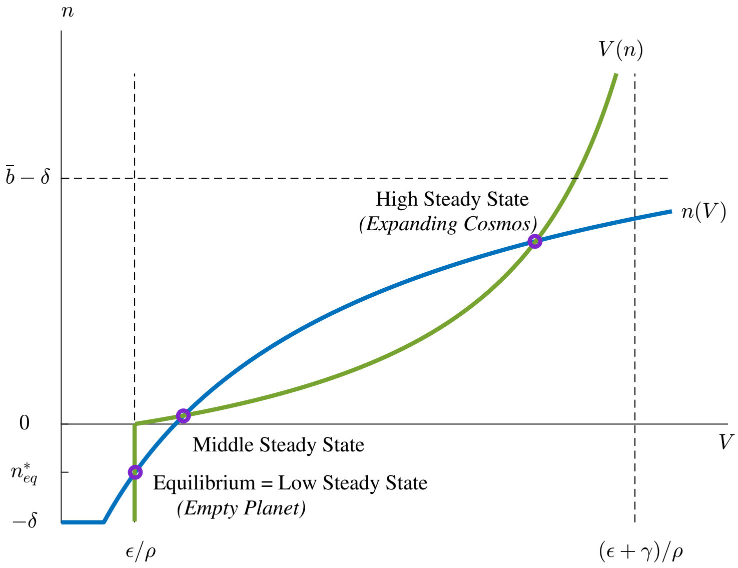
$$n(V) = \bar{b}\ell(V) - \delta = \bar{b} - \delta + \frac{1}{V}$$

- We show the solution graphically

A Unique Steady State for the Optimal Allocation when $n_{eq}^* > 0$



Multiple Steady State Solutions when $n^*_{eq} < 0$



Parameter Values for Numerical Solution

Parameter/Moment	Value	Comment
σ	1	Normalization
λ	1	Duplication effect of ideas
β	1.25	BJVW
ρ	.01	Standard value
δ	1%	Death rate
n^{eq}	-0.5%	Suggested by Europe, Japan, U.S.
ℓ^{eq}	1/8	Time spent raising children

Implied Parameter Values and "Expanding Cosmos" Steady-State Results

Result	Value	Comment
$\bar{b}$	.040	$n^{eq} = \bar{b}\ell^{eq} - \delta = -0.5\%$
ϵ	.286	From equation for ℓ^{eq}
n^{sp}	1.74%	From equations for ℓ^{sp} and n^{sp}
ℓ^{sp}	0.68	From equations for ℓ^{sp} and n^{sp}
$g_y^{sp} = g_A^{sp}$	1.39%	Equals γn^{sp} with $\sigma = 1$

Transition Dynamics

- State variables: N_t and A_t
- Redefine "state-like" variables for transition dynamics solution: N_t and

  $x_t \equiv A_t^{\beta}/N_t^{\lambda}$ = "Knowledge per person"

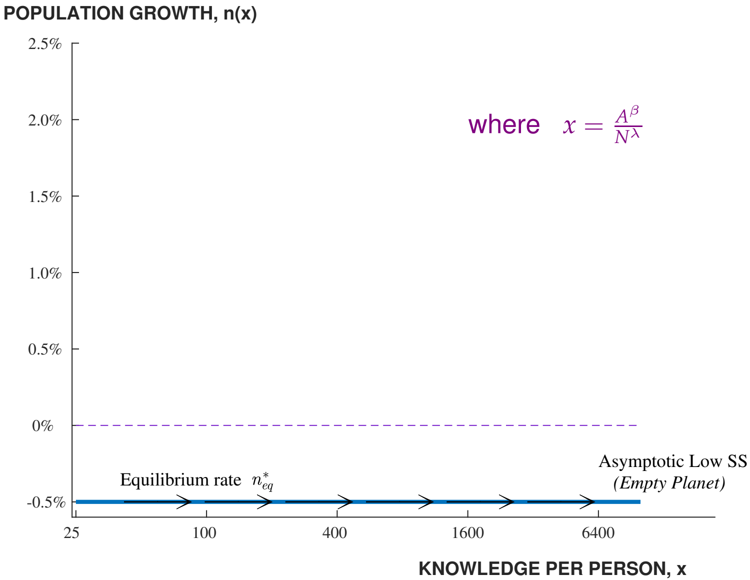
- Why?

$$\frac{\dot{A}_t}{A_t} = \frac{N_t^{\lambda}}{A_t^{\beta}} = \frac{1}{x_t}$$

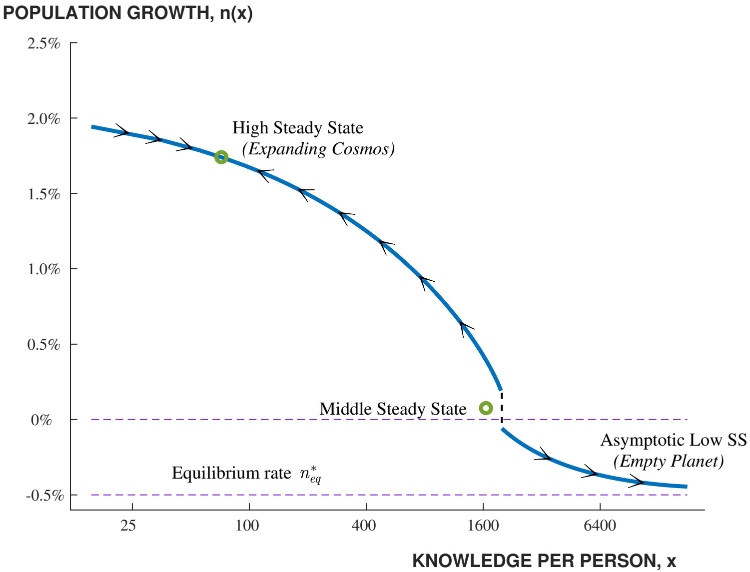
Key insight: optimal fertility only depends on x_t

- Note: x is the ratio of A and N, two stocks that are each good for welfare.
  - So a bigger x is not necessarily welfare improving.

Equilibrium Transition Dynamics



Optimal Population Growth



The Economics of Multiple SS's and Transition Dynamics

- **The High SS is saddle path stable as usual**
  - Equilbrium fertility depends on utility value of kids
  - Planner also values the ideas the kids will produce $\Rightarrow n_t^{sp} > n_t^{eq}$
- **Why is there a low SS?**
  - Diminishing returns to each input, including ideas
  - As knowledge per person, x, goes to ∞, the "idea value" of an extra kid falls to zero $\Rightarrow n_{sp}(x) \rightarrow n_{eq}$
- **Why is the low SS stable?**
  - Since $n_{eq} < 0$, we also have $n_{sp}(x) < 0$ for x sufficiently high
  - With $n_{sp}(x) < 0$, $x = A^{\beta}/N^{\lambda}$ *rises* over time

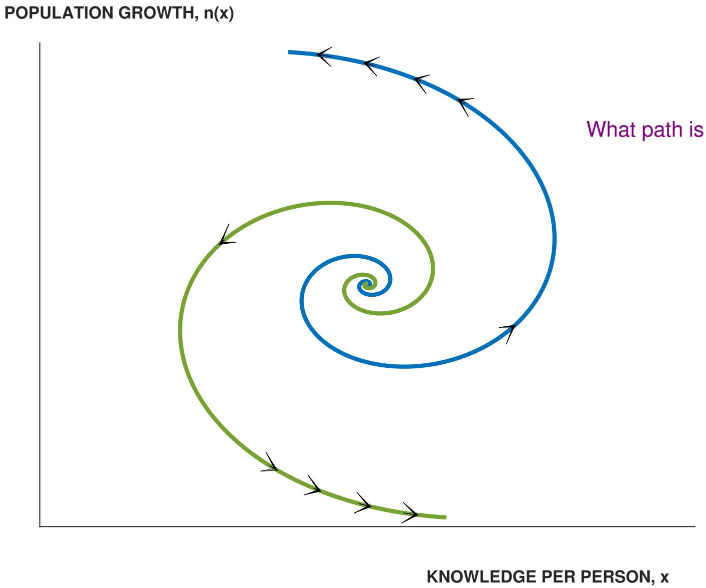
What about the middle candidate steady state?

- Linearize the FOCs. Dynamic system has
  - imaginary eigenvalues
  - with positive real parts

- So the middle SS is an unstable spiral — a “Skiba point” (Skiba 1978)

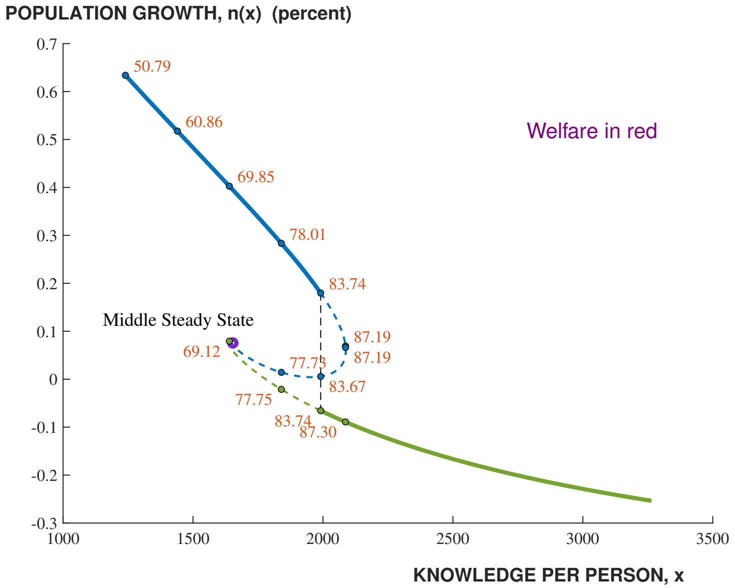
- Numerical solution reveals what is going on...

The Middle Steady State: Unstable Spiral Dynamics



What path is optimal?

Population Growth Near the Middle Steady State



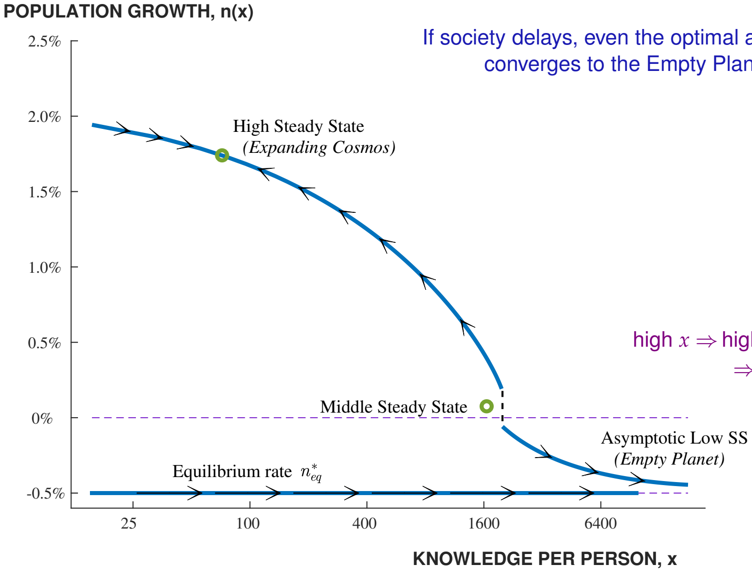
Surprising Result

- The optimal allocation features *two* very different steady states
  - One is an **Expanding Cosmos**
  - One is the **Empty Planet**

- Start the economy with low x
  - The equilibrium converges to the Empty Planet steady state
  - If society adopts optimal policy soon, it goes to the Expanding Cosmos

But if society delays, even the optimal allocation converges to the Empty Planet

Even the optimal allocation can get trapped



If society delays, even the optimal allocation converges to the Empty Planet

high $x \Rightarrow$ high ideas per person

$\Rightarrow$ low $\mu\dot{A}$

Conclusion

- Fertility has trended down: 5, 4, 3, 2, and less in rich countries
  - For a family, nothing special about “above 2” vs “below 2”
- But macroeconomics makes this distinction critical:
  - Negative population growth may condemn us to stagnation on an **Empty Planet**
  - Vs. the exponential growth in income and population of an **Expanding Cosmos**
- Surprise: Even the optimum can get trapped in the Empty Planet if society delays.

Fertility considerations may be more important than we thought!