The End of Economic Growth?
Unintended Consequences of a Declining Population

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Key Role of Population

- People $\Rightarrow$ ideas $\Rightarrow$ economic growth
  - Romer (1990), Aghion-Howitt (1992), Grossman-Helpman
  - And most idea-driven growth models

- The future of global population?
  - Conventional view: stabilize at 8 or 10 billion

- Bricker and Ibbotson’s *Empty Planet* (2019)
  - Maybe the future is **negative population growth**
  - High income countries already have fertility **below** replacement!
The Total Fertility Rate (Live Births per Woman)

<table>
<thead>
<tr>
<th>Country</th>
<th>Fertility Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>1.8</td>
</tr>
<tr>
<td>H.I.C.</td>
<td>1.7</td>
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<tr>
<td>China</td>
<td>1.7</td>
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<tr>
<td>Germany</td>
<td>1.6</td>
</tr>
<tr>
<td>Japan</td>
<td>1.4</td>
</tr>
<tr>
<td>Italy</td>
<td>1.3</td>
</tr>
<tr>
<td>Spain</td>
<td>1.3</td>
</tr>
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</table>

LIVE BIRTHS PER WOMAN

- U.S. = 1.8
- H.I.C. = 1.7
- China = 1.7
- Germany = 1.6
- Japan = 1.4
- Italy = 1.3
- Spain = 1.3
What happens to economic growth if population growth is negative?

- **Exogenous population decline**
  - **Empty Planet Result**: Living standards stagnate as population vanishes!
  - Contrast with standard *Expanding Cosmos* result: exponential growth for an exponentially growing population

- **Endogenous fertility**
  - Parameterize so that the equilibrium features negative population growth
  - A planner who prefers *Expanding Cosmos* can get trapped in an Empty Planet
    - if society delays implementing the optimal allocation
Outline

• Exogenous negative population growth
  ○ In Romer / Aghion-Howitt / Grossman-Helpman
  ○ In semi-endogenous growth framework

• Endogenous fertility
  ○ Competitive equilibrium with negative population growth
  ○ Optimal allocation
The Empty Planet Result
A Simplified Romer/AH/GH Model

Production of goods (IRS)

\[ Y_t = A_t^\sigma N_t \]

Production of ideas

\[ \frac{\dot{A}_t}{A_t} = \alpha N_t \]

Constant population

\[ N_t = N \]

- Income per person: levels and growth

\[ y_t \equiv \frac{Y_t}{N_t} = A_t^\sigma \]

\[ \frac{\dot{y}_t}{y_t} = \sigma \frac{\dot{A}_t}{A_t} = \sigma \alpha N \]

- Exponential growth with a constant population
  - But population growth means exploding growth? (Semi-endogenous fix)
Negative Population Growth in Romer/AH/GH

Production of goods (IRS)

\[ Y_t = A_t^\sigma N_t \]

Production of ideas

\[ \frac{\dot{A}_t}{A_t} = \alpha N_t \]

Exogenous population decline

\[ N_t = N_0 e^{-\eta t} \]

• Combining the 2nd and 3rd equations (note \( \eta > 0 \))

\[ \frac{\dot{A}_t}{A_t} = \alpha N_0 e^{-\eta t} \]

• This equation is easily integrated...
The Empty Planet Result in Romer/GH/AH

- The stock of knowledge $A_t$ is given by

$$\log A_t = \log A_0 + \frac{gA_0}{\eta} (1 - e^{-\eta t})$$

where $gA_0$ is the initial growth rate of $A$

- $A_t$ and $y_t \equiv Y_t/N_t$ converge to constant values $A^*$ and $y^*$:

$$A^* = A_0 \exp \left( \frac{gA_0}{\eta} \right)$$

$$y^* = y_0 \exp \left( \frac{gy_0}{\eta} \right)$$

- **Empty Planet Result**: Living standards stagnate as the population vanishes!
Semi-Endogenous Growth

Production of goods (IRS)

\[ Y_t = A_t^\sigma N_t \]

Production of ideas

\[ \frac{\dot{A}_t}{A_t} = \alpha N_t^\lambda A_t^{-\beta} \]

Exogenous population growth

\[ N_t = N_0 e^{nt}, \quad n > 0 \]

- Income per person: levels and growth

\[ y_t = A_t^\sigma \quad \text{and} \quad A_t^* \propto N_t^{\lambda/\beta} \]

\[ \frac{\dot{y}_t}{y_t} = \gamma n, \quad \text{where} \quad \gamma \equiv \lambda \sigma / \beta \]

- Expanding Cosmos: Exponential income growth for growing population
Negative Population Growth in the Semi-Endogenous Setting

Production of goods (IRS)

\[ Y_t = A_t^\sigma N_t \]

Production of ideas

\[ \dot{A}_t = \alpha N_t^\lambda A_t^{-\beta} \]

Exogenous population decline

\[ N_t = N_0 e^{-\eta t} \]

- Combining the 2nd and 3rd equations:

\[ \frac{\dot{A}_t}{A_t} = \alpha N_0^\lambda e^{-\lambda \eta t} A_t^{-\beta} \]

- Also easily integrated...
The stock of knowledge $A_t$ is given by

$$A_t = A_0 \left( 1 + \frac{\beta g A_0}{\lambda \eta} \left( 1 - e^{-\lambda \eta t} \right) \right)^{1/\beta}$$

Let $\gamma \equiv \lambda \sigma / \beta = \text{overall degree of increasing returns to scale.}$

Both $A_t$ and income per person $y_t \equiv Y_t / N_t$ converge to constant values $A^*$ and $y^*$:

$$A^* = A_0 \left( 1 + \frac{\beta g A_0}{\lambda \eta} \right)^{1/\beta}$$

$$y^* = y_0 \left( 1 + \frac{g y_0}{\gamma \eta} \right)^{\gamma / \lambda}$$
Numerical Example

- Parameter values
  - $g_{y0} = 2\%$, $\eta = 1\%$
  - $\beta = 3 \Rightarrow \gamma = 1/3$ (from BJVW)

- How far away is the long-run stagnation level of income?

  \[
  \frac{y^*/y_0}{Romer/AH/GH} = 7.4
  \]

  \[
  \frac{y^*/y_0}{Semi-endog} = 1.9
  \]

- The Empty Planet result occurs in both, but quantitative difference
Endogenous Fertility
The Economic Environment

\( \ell = \text{time having kids instead of producing goods} \)

Final output

\[ Y_t = A_t^\sigma (1 - \ell_t) N_t \]

Population growth

\[ \frac{\dot{N}_t}{N_t} = n_t = b(\ell_t) - \delta \]

Fertility

\[ b(\ell_t) = \bar{b}\ell_t \]

Ideas

\[ \frac{\dot{A}_t}{A_t} = N_t^\lambda A_t^{-\beta} \]

Generation 0 utility

\[ U_0 = \int_0^\infty e^{-\rho t} u(c_t, \tilde{N}_t) dt, \quad \tilde{N}_t \equiv N_t/N_0 \]

Flow utility

\[ u(c_t, \tilde{N}_t) = \log c_t + \epsilon \log \tilde{N}_t \]

Consumption

\[ c_t = \frac{Y_t}{N_t} \]
• All people generate ideas here
  o Learning by doing vs separate R&D

• Equilibrium fertility
  o We have kids because we like them
  o We ignore that they might create ideas that benefit everyone
  o Planner will desire higher fertility

• This is a modeling choice — other results are possible
A Competitive Equilibrium with Externalities

- Representative generation takes $w_t$ as given and solves

$$\max_{\{\ell_t\}} \int_0^\infty e^{-\rho t} u(c_t, \tilde{N}_t) dt$$

subject to

$$\dot{N}_t = (b(\ell_t) - \delta)N_t$$

$$c_t = w_t(1 - \ell_t)$$

- Equilibrium wage $w_t = \text{MP}_L = A_t^\sigma$

- Rest of economic environment closes the equilibrium
Solving for the equilibrium

• The Hamiltonian for this problem is

\[ H = u(c_t, \tilde{N}_t) + v_t[b(\ell_t) - \delta]N_t \]

where \( v_t \) is the shadow value of another person.

• Let \( V_t \equiv v_tN_t \) = shadow value of the population

• Equilibrium features constant fertility along transition path

\[ V_t = \frac{\epsilon}{\rho} \equiv V_{eq}^* \]

\[ \ell_t = 1 - \frac{1}{bV_t} = 1 - \frac{1}{bV_{eq}^*} = 1 - \frac{\rho}{b\epsilon} \equiv \ell_{eq} \]
Discussion of the Equilibrium Allocation

\[ n^{eq} = \bar{b} - \delta - \frac{\rho}{\epsilon} \]

- We can choose parameter values so that \( n^{eq} < 0 \)
  - Constant, negative population growth in equilibrium

- Remaining solution replicates the exogenous fertility analysis

*The Empty Planet result can arise in equilibrium*
The Optimal Allocation
The Optimal Allocation

- Choose fertility to maximize the welfare of a representative generation

- Problem:

\[
\max_{\{\ell_t\}} \int_0^\infty e^{-\rho t} u(c_t, \tilde{N}_t) \, dt
\]

subject to

\[
\dot{N}_t = (b(\ell_t) - \delta)N_t
\]

\[
\frac{\dot{A}_t}{A_t} = N_t^\lambda A_t^{-\beta}
\]

\[
c_t = \frac{Y_t}{N_t}
\]

- Optimal allocation recognizes that offspring produce ideas
Solution

- Hamiltonian:

\[ \mathcal{H} = u(c_t, \tilde{N}_t) + \mu_t N_t^\lambda A_t^{1-\beta} + v_t(b(\ell_t) - \delta)N_t \]

\( \mu_t \) is the shadow value of an idea
\( v_t \) is the shadow value of another person

- First order conditions

\[ \ell_t = 1 - \frac{1}{bV_t}, \text{ where } V_t \equiv v_tN_t \]

\[ \rho = \frac{\dot{\mu}_t}{\mu_t} + \frac{1}{\mu_t} \left( u_c \sigma y_t - \frac{\mu_t(1 - \beta)}{A_t} \right) \]

\[ \rho = \frac{\dot{v}_t}{v_t} + \frac{1}{v_t} \left( \epsilon - \mu_t \lambda \frac{\dot{A}_t}{N_t} + v_i n_t \right) \]
Steady State Conditions

- The social value of people in steady state is

\[ V_{sp}^* = v_t^* N_t^* = \frac{\epsilon + \lambda z^*}{\rho} \]

where

\[ z^* \equiv \mu_i^* \dot{A}_i^* = \frac{\sigma g_A^*}{\rho + \beta g_A^*} \]

- If \( n_{sp}^* > 0 \), then we have an **Expanding Cosmos** steady state

\[ g_A^* = \frac{\lambda n_{sp}^*}{\beta} \]

\[ g_y^* = \gamma n_{sp}^*, \text{ where } \gamma \equiv \frac{\lambda \sigma}{\beta} \]
Optimal Steady State(s)

- Two equations in two unknowns \((V, n)\)

\[
V(n) = \begin{cases} 
\frac{1}{\rho} \left( \epsilon + \frac{\gamma}{1 + \frac{\epsilon}{\lambda n}} \right) & \text{if } n > 0 \\
\frac{\epsilon}{\rho} & \text{if } n \leq 0
\end{cases}
\]

\[
n(V) = \bar{b} \ell(V) - \delta = \bar{b} - \delta + \frac{1}{V}
\]

- We show the solution graphically
A Unique Steady State for the Optimal Allocation when $n_{eq}^* > 0$
Multiple Steady State Solutions when $n_{eq}^* < 0$

High Steady State
(Expanding Cosmos)

Middle Steady State

Equilibrium = Low Steady State
(Empty Planet)
## Parameter Values for Numerical Solution

<table>
<thead>
<tr>
<th>Parameter/Moment</th>
<th>Value</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma )</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>1</td>
<td>Duplication effect of ideas</td>
</tr>
<tr>
<td>( \beta )</td>
<td>1.25</td>
<td>BJVW</td>
</tr>
<tr>
<td>( \rho )</td>
<td>.01</td>
<td>Standard value</td>
</tr>
<tr>
<td>( \delta )</td>
<td>1%</td>
<td>Death rate</td>
</tr>
<tr>
<td>( n^{eq} )</td>
<td>-0.5%</td>
<td>Suggested by Europe, Japan, U.S.</td>
</tr>
<tr>
<td>( \ell^{eq} )</td>
<td>1/8</td>
<td>Time spent raising children</td>
</tr>
</tbody>
</table>
### Implied Parameter Values and “Expanding Cosmos” Steady-State Results

<table>
<thead>
<tr>
<th>Result</th>
<th>Value</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{b}$</td>
<td>.040</td>
<td>$n^{eq} = \bar{b}\ell^{eq} - \delta = -0.5%$</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>.286</td>
<td>From equation for $\ell^{eq}$</td>
</tr>
<tr>
<td>$n^{sp}$</td>
<td>1.74%</td>
<td>From equations for $\ell^{sp}$ and $n^{sp}$</td>
</tr>
<tr>
<td>$\ell^{sp}$</td>
<td>0.68</td>
<td>From equations for $\ell^{sp}$ and $n^{sp}$</td>
</tr>
<tr>
<td>$\delta^{sp} = \delta^{sp}_A$</td>
<td>1.39%</td>
<td>Equals $\gamma n^{sp}$ with $\sigma = 1$</td>
</tr>
</tbody>
</table>
Transition Dynamics

- State variables: $N_t$ and $A_t$

- Redefine “state-like” variables for transition dynamics solution: $N_t$ and

  \[ x_t \equiv \frac{A_t^\beta}{N_t^\lambda} = \text{“Knowledge per person”} \]

- Why?

  \[ \frac{\dot{A}_t}{A_t} = \frac{N_t^\lambda}{A_t^\beta} = \frac{1}{x_t} \]

- Key insight: optimal fertility only depends on $x_t$
Optimal Population Growth

POPLATION GROWTH, $n(x)$

- High Steady State (Expanding Cosmos)
- Middle Steady State
- Asymptotic Low SS (Empty Planet)

KNOWLEDGE PER PERSON, $x$
The Economics of Multiple SS’s and Transition Dynamics

- **The High SS is saddle path stable as usual**
  - Equilibrium fertility depends on utility value of kids
  - Planner also values the ideas the kids will produce $\Rightarrow n_{sp} > n_{eq}$

- **Why is there a low SS?**
  - Diminishing returns to each input, including ideas
  - As knowledge per person, $x$, goes to $\infty$, the “idea value” of an extra kid falls to zero $\Rightarrow n_{sp}(x) \rightarrow n_{eq}$

- **Why is the low SS stable?**
  - Since $n_{eq} < 0$, we also have $n_{sp}(x) < 0$ for $x$ sufficiently high
  - With $n_{sp}(x) < 0$, $x = A^\beta / N^\lambda$ rises over time
What about the middle candidate steady state?

- Linearize the FOCs. Dynamic system has
  - imaginary eigenvalues
  - with positive real parts

- So the middle SS is an unstable spiral

- Numerical solution reveals what is going on...
The Middle Steady State: Unstable Spiral Dynamics

What path is optimal?

POPULATION GROWTH, n(x)

KNOWLEDGE PER PERSON, x
Population Growth Near the Middle Steady State

Welfare in red

 KNOWLEDGE PER PERSON, x

POPULATION GROWTH, n(x) (percent)
Surprising Result

- The optimal allocation features *two* very different steady states
  - One is an **Expanding Cosmos**
  - One is the **Empty Planet**

- Start the economy with low $x$
  - The equilibrium converges to the Empty Planet steady state
  - If society adopts optimal policy soon, it goes to the Expanding Cosmos

*But if society delays, even the optimal allocation converges to the Empty Planet*
Even the optimal allocation can get trapped

If society delays, even the optimal allocation converges to the Empty Planet
Conclusions from this model

- Negative population growth may condemn us to an **Empty Planet**
  - Stagnating living standards for a population that vanishes
- In contrast, the optimal allocation may be an **Expanding Cosmos**
  - Exponential growth in living standards and population.
- **Surprise:** Even the optimum can get trapped in the Empty Planet if society delays.

*Fertility considerations may be more important than we thought!*