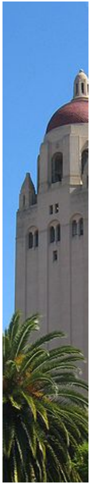

When GDP Misleads:
Inferring Living Standards from the *VSL*

Phil Trammell and Chad Jones
Stanford

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Preliminary and incomplete

Issues with GDP and Welfare Measurement

- Standard concerns: new goods, quality improvements, nonmarket goods
  - Even more important than many of us have appreciated...
- Motivating examples: bounded utility from a fixed set of goods
  - Nathan Rothschild (1836): enormous wealth, dies of infection
  - Bill Gates / Jeff Bezos: marginal utility of spending may be near zero

  $\Rightarrow$ new goods and quality improvements are central to rising utility
- Deeper issue: nonhomotheticity $\Rightarrow$ GDP growth can be hugely misleading
  - Understood since the 1970s but commonly ignored
  - A poor measure better than none at all?

Inferring welfare from the *VSL*

- VSL: People trade off wages for mortality risk
  - Puts a dollar value on the continuation lifetime utility
  - Includes new goods, quality improvements, climate change, freedom
- VSL = lifetime utility / marginal utility of consumption
  - If we knew *muc*, we could measure lifetime utility directly
  - ... but growth rate of *muc* from Euler equation is $\rho - i_t$
- Key result:

$$g_t^{utility} = g_t^{\mathsf{VSL}} + \rho - i_t$$

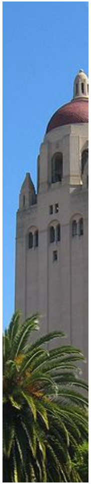
Simple way to measure very-inclusive lifetime utility improvements

Empirical Results: U.S. since 1940

- Standard approach:
  - Pass 2% consumption growth through a bounded utility function
  - Welfare rises by $\approx$ 2x since 1940
  - Even explosive consumption growth not that valuable!
- VSL-based lifetime utlity
  - Baseline calculation rises by 5x or 7x
  - A natural interpretation: new/better goods lift the upper bound on utility
- Sensitive to g_t^{VSL}, ρ, i_t
  - One percentage point difference $\iff$ one pp growth in welfare

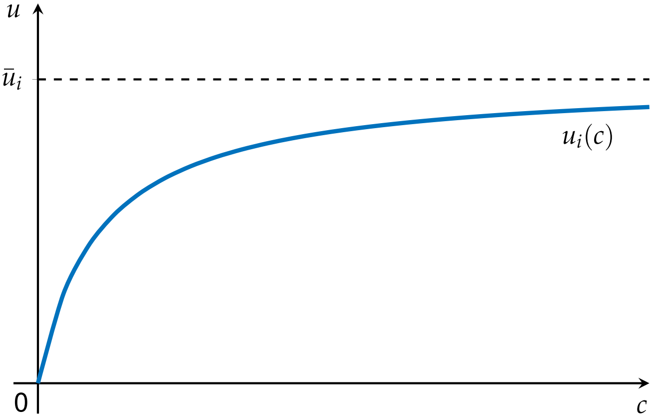
Related literature

- GDP, index numbers, and welfare measurement
  - Hulten (1973), Samuelson and Swamy (1974), Diewert (1978), Reinsdorf (1998), Oulton (2008)
  - Bresnahan and Gordon (1996), Hausman (1996), Brynjolfsson et al. (2025)
- Improved welfare measures with nonhomotheticity, new goods, etc.
  - Baqaee and Burstein (2023), Jaravel and Lashkari (2024), Davila and Schaab (2025), Oberfield (2023)
  - Feenstra and Reinsdorf (2000), Blundell, Browning, and Crawford (2003), Redding and Weinstein (2020), Matsuyama (2023)
- Value of a statistical life
  - Costa and Kahn (2004), Aldy and Viscusi (2003), Knieser and Viscusi (2024)



A Simple Example where GDP Misleads

Utility from consuming a given good



Infinite consumption of apples or iphones on a given day does not lead to infinite utility

A simple way GDP fails: bounded utility + new goods

- Flow utility sums across the set of goods available at time t:

$$U_t = \sum_{i=1}^{N_t} u_i(c_{it}), \qquad u_i(0) = 0, \;\; u_i(\infty) = \bar{u}_i$$

- Long-run progress comes mainly from new goods / quality improvements
  - Captures key features of growth since 1800 — not just more wheat!
- Violates nonhomotheticity $\Rightarrow$ “real” GDP can mislead

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Real GDP can be arbitrarily higher if new goods are *not* invented, even though utility is higher when they are.

Proof by Example ($N = 2$): Baumol cost disease

- Two goods produced with labor: food f and string quartets s

$$c_t^f = A_t^f \ell_t, \qquad c_t^s = A_t^s (1 - \ell_t)$$

- Bounded per-good utility (Stone–Geary shift ensures $u_i(0) = 0$):

$$u_i(c) = \bar{u}_i + \frac{(c + \bar{c}_i)^{1-\theta}}{1-\theta}, \qquad \theta > 1$$

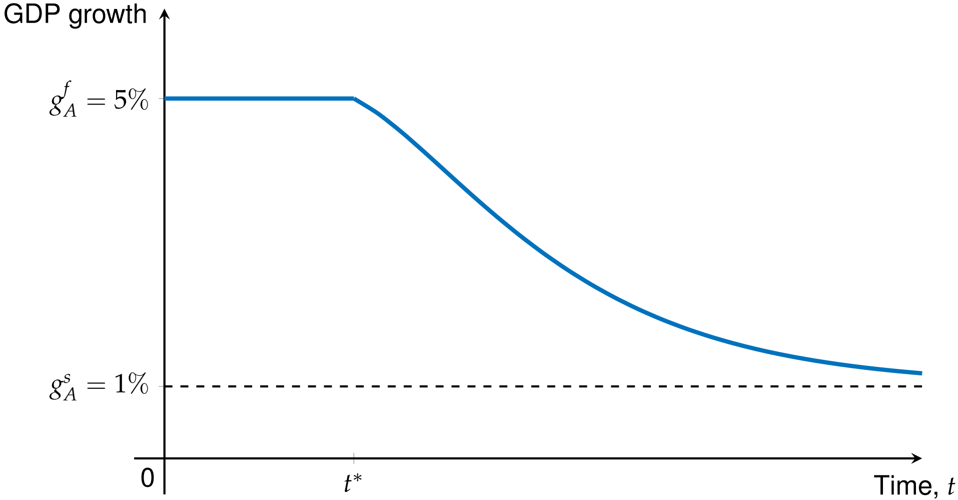
- Baumol cost disease
  - String quartets invented at time t^*
  - Food has faster productivity growth: $g_A^f > g_A^s$ (e.g., 5% vs 1%)

Classic Baumol Results

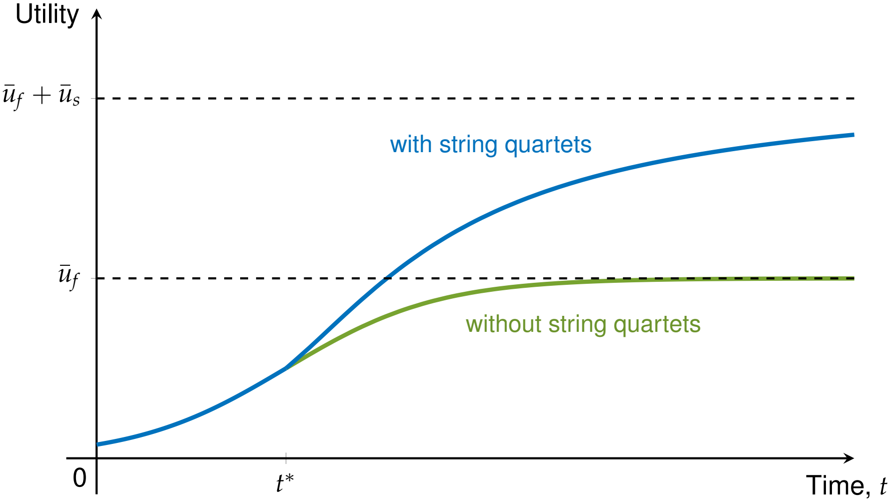
- Spending share shifts over time toward the slow growing good
  - Consumption of food grows rapidly but runs into diminishing returns
  - Initially 100% on food, eventually 100% on string quartets
- Chain-weighted GDP
  - GDP growth = weighted average of 5% and 1%
  - Weights are the spending shares

GDP growth starts at 5% and falls toward 1%

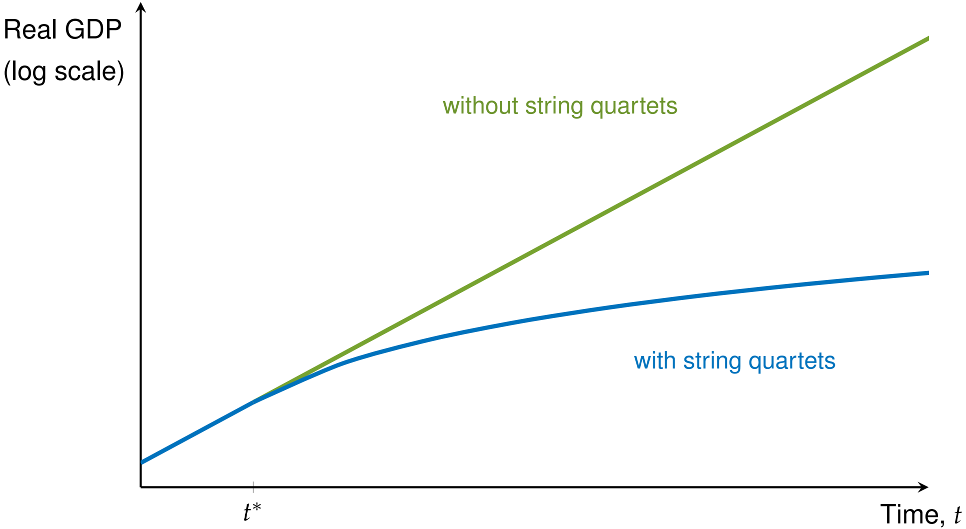
Baumol example: GDP growth slows after t^*



Baumol example: utility is higher with the new good

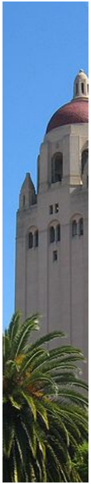


Baumol example: real GDP is higher *without* the new good



Takeaway from the Baumol example

- GDP arbitrarily higher (eventually) in the world where string quartets never invented
  - ... even though clearly welfare is higher when they are!
- Viewing GDP as a measure of production does not help
  - A world with infinitely more GDP is not better
- And of course GDP has other problems
  - Quality improvements
  - Nonmarket goods: climate, pollution, crime, freedom, etc.



Using the Value of a Stastical Life

A choice problem with very general flow utility

- Given $\{p_{it}\}_{i=1}^{\infty}$, the wage w_t, the interest rate $\tilde{i}_t$, and mortality rates δ_t,

$$\max_{\{c_{it}\}} \quad W_0 = \int_0^\infty e^{-\rho t} S_t U(c_{1t}, c_{2t}, \ldots, c_{nt}, \ldots; t)\, dt \quad \text{subject to}$$

$$c_t^e = \sum_{i=1}^{\infty} p_{it} c_{it}$$

$$\dot{a}_t = \tilde{i}_t a_t + w_t - c_t^e, \quad a_0 \text{ given}$$

$$\dot{S}_t = -\delta_t S_t, \quad S_0 = 1$$

  - $S_t =$ probability of surviving to time t
  - $c_t^e =$ total consumption spending on a changing set of goods
  - Flow utility allowed to depend on t (quality change, nonmarket goods)

Static Problem

- Allocate a given amount of total spending c_t^e at date t:

$$U_t(c_t^e; \{p_{it}\}) \;=\; \max_{\{c_{it}\}} \; U(c_{1t}, c_{2t}, \ldots, c_{nt}, \ldots; t) \quad \text{subject to}$$

$$c_t^e = \sum_{i=1}^{\infty} p_{it} c_{it}$$

- Overload notation:
  - $U_t(c_t^e) \equiv U_t(c_t^e; \{p_{it}\})$ = indirect utility

    = the optimal utility from spending c_t^e dollars at prices $\{p_{it}\}$

Dynamic Problem

- Given the time path of wages, interest rates, and prices

$$\max_{\{c_t^e\}} \quad W_0 = \int_0^\infty e^{-\rho t} S_t U_t(c_t^e)\, dt \quad \text{subject to}$$

$$\dot{a}_t = \tilde{i}_t a_t + w_t - c_t^e, \quad a_0 \ \text{given}$$

$$\dot{S}_t = -\delta_t S_t, \quad S_0 = 1$$

- The first order conditions $\Rightarrow$ a standard Euler equation:

$$g_{muc,t} = \rho + \delta_t - \tilde{i}_t$$

  - *"muc"* = marginal utility of consumption spending $U_t'(c_t^e)$
  - $g_{muc,t} \equiv \frac{d \log U_t'(c_t^e)}{dt}$ is growth rate of *muc*

Blanchard (1985) Annuity Markets

- What happens to wealth when agent dies?
- Assume actuarially fair annuity markets
  - Extra return on assets equal to mortality rate: $\tilde{i}_t = i_t + \delta_t$
- Updated Euler equation:

$$g_{muc,t} = \rho - i_t$$

where i_t is risk free rate (e.g. on T-bills)

The Value of a Statistical Life (Arthur 1981, Rosen 1988)

- Cumulative hazard $\Delta_t \equiv \Delta_0 + \int_0^t \delta_s \, ds$ where $\Delta_0 = 0$ in the baseline problem and $S_t = \exp(-\Delta_t)$
- Let $W_0(a_0, \Delta_0, \{p_{it}, w_t, i_t, \delta_t\})$ = optimal lifetime utility
- Consider a change in initial assets a_0 and mortality Δ_0 that keeps optimal lifetime utility unchanged ($dW_0 = 0$):

$$\frac{\partial W_0}{\partial a_0} \cdot da_0 + \frac{\partial W_0}{\partial \Delta_0} \cdot d\Delta_0 = 0$$

$$\Rightarrow \quad \left.\frac{da_0}{d\Delta_0}\right|_{W_0} = -\frac{\partial W_0 / \partial \Delta_0}{\partial W_0 / \partial a_0}.$$

This is the VSL: willingness to pay to avoid a small mortality risk

(continued)

- A change in mortality is valued in proportion to the loss in lifetime utility W_0:

$$\frac{\partial W_0}{\partial \Delta_0} = -W_0.$$

- The value of a change in assets is the marginal utility of consumption spending:

$$\frac{\partial W_0}{\partial a_0} = S_0 U_0'(c_0^e) = U_0'(c_0^e)$$

- The *VSL* is the willingness to pay per unit reduction in hazard

$$VSL_t \equiv \left.\frac{da_t}{d\Delta_t}\right|_{W_t} = \frac{W_t}{U_t'(c_t^e)}$$

The Value of a Statistical Life

$$\mathit{VSL}_t \equiv \left.\frac{da_t}{d\Delta_t}\right|_{W_t} = \frac{W_t}{U_t'(c_t^e)}$$

- Dying means you lose W_t (in utils). Dividing by $U_t'(c_t^e)$ converts to dollars
- Rewriting gives an expression for Lifetime Utility

$$W_t = \mathit{VSL}_t \cdot U_t'(c_t^e)$$

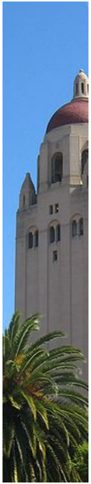
- If we observed the marginal utility of consumption spending, we'd have the level of lifetime utility
- But we can take growth rates and use the Euler equation!

Key Result: Growth in Lifetime Utility

Proposition. The growth rate of lifetime utility is the growth rate of the VSL adjusted for the rate at which the marginal utility of consumption spending declines:

$$\begin{aligned} g_t^W &= g_t^{VSL} + g_{muc,t} \\ &= g_t^{VSL} + \rho - i_t \end{aligned}$$

- Measures welfare growth robustly to new goods, quality, and nonmarket factors
- Seemingly simple: only involves three objects!



Measuring Utility: Empirical Application

Measuring the VSL

- Large literature (e.g. for safety regulation and legal proceedings)
  - Compensating wage differentials for jobs with different mortality risk
  - We need a time series — growth rate of VSL
- Two approaches
  - Costa and Kahn (2004) uses Census to get results by decade 1940 to 1980. Combine with latest VSL from Department of Transportation ($13.7 million)
  - Meta analyses suggest income elasticity of VSL equal in range of 0.7 to 1.5.

Other Measurement Issues

- Time preference: $\rho = 1\%$
- Interest rate
  - T-bill rate plus 1% (offsetting convenience yield)
  - Robustness to other rates
- Nominal vs "real"
  - Key: problems with "real" GDP mean problems with "real" – no deflator
  - Advantage of VSL approach: nominal data is all that is required!
    (will report some "real" numbers for comparative purposes)

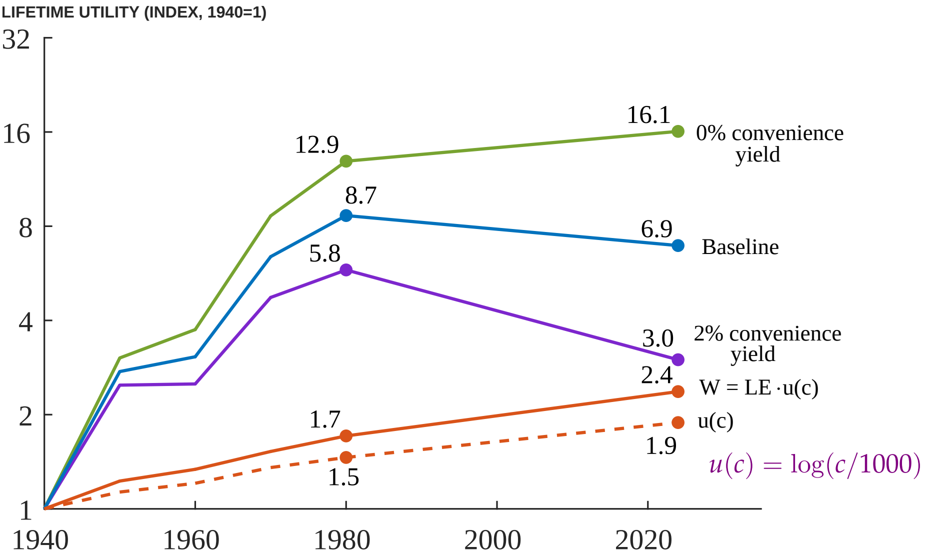
Average annual growth rates of Lifetime Utility (nominal VSL and interest rate)

Period	g_t^W	g_t^{VSL}	$g_{muc,t}$	Discount Rate	Interest Rate
Nominal Data (Current Dollars)					
1940–1950	10.1	10.6	-0.5	1	1.5
1950–1960	1.1	3.2	-2.1	1	3.1
1960–1970	7.4	11.5	-4.2	1	5.2
1970–1980	3.0	9.6	-6.5	1	7.5
1980–2024	-0.5	3.4	-3.9	1	4.9
1940–2024	**2.3**	**6.0**	**-3.7**	**1**	**4.7**

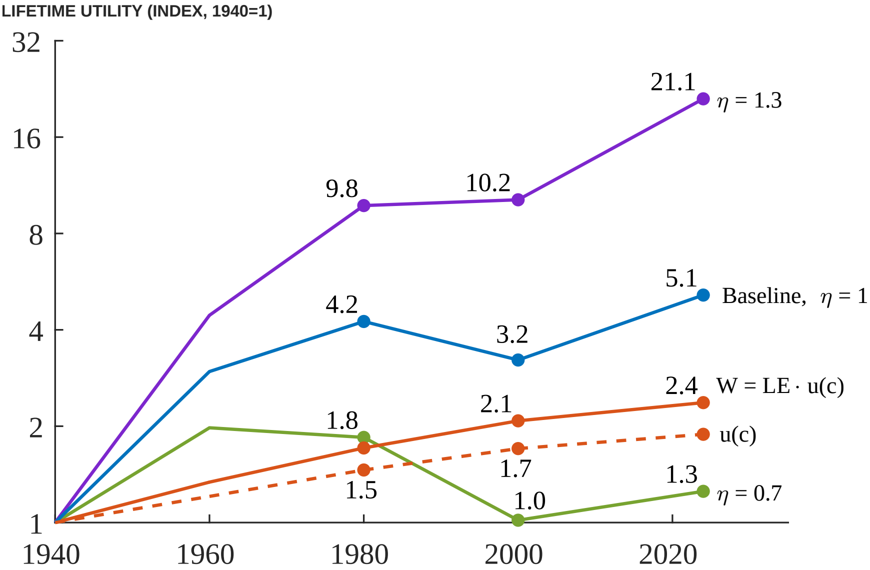
Average annual growth rates of Lifetime Utility: “Real” VSL and interest rate

Period	g_t^W	g_t^{VSL}	$g_{muc,t}$	Discount Rate	Interest Rate
“Real” Data (2024 Prices)					
1940–1950	10.1	5.2	4.9	1	-3.9
1950–1960	1.1	1.1	0.0	1	1.0
1960–1970	7.4	8.8	-1.4	1	2.4
1970–1980	3.0	2.1	1.0	1	0.0
1980–2024	-0.5	0.4	-0.9	1	1.9
1940–2024	**2.3**	**2.2**	**0.1**	**1**	**0.9**

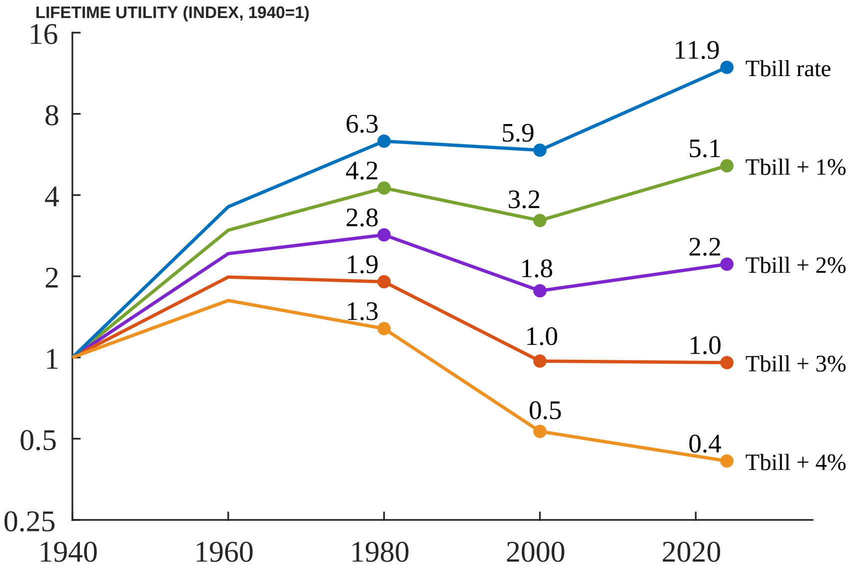
Utility levels: Costa–Kahn (1940-1980) + DOT 2024 VSL



Utility levels: VSL = income$^{\eta}$ where $\eta \in \{0.7, 1.0, 1.3\}$



Utility levels: sensitivity to the interest rate



Other Robustness Exercises

- What if no annuity markets — adds mortality rate to time preference
  - Adds $\approx$ 1% to the growth rate of utility
  - Factor increase since 1940 becomes 17.3 (Costa-Kahn) or 12.8 ($\eta = 1$)
- Risk — tricky!
  - Tension between macro and finance (equity premium puzzle and very high estimates of σ_{muc}) can make utility growth *negative*
  - Covariance between VSL and muc
  - Still working on this...

Interpreting the gap with traditional measures

- Traditional approach: pass consumption growth through a stable bounded $u(c)$
  - Strong diminishing returns, utility gains are mechanically small
  - Even explosive growth does not raise welfare much
- Potentially much larger gains from the *VSL* approach
  - Back to Rothschild / Gates: new goods and quality improvements can shift the upper bound and deliver large gains
  - Even when utility from a given set of goods is bounded

Concluding Thoughts

- GDP growth can be a profoundly misleading welfare statistic
- Simple alternative: infer welfare growth from *VSL* growth
  - $g_t^W = g_t^{VSL} + \rho - i_t$
  - Captures new goods, quality improvements, Δtastes, nonmarket factors
- Baseline U.S. evidence suggests large welfare gains since 1940
  - But sensitive! Need better measures of VSL_t, ρ, and i_t

Why violates nonhomotheticity

- Homothetic: for any two consumption bundles x and y,

$$x \succsim y \implies \alpha x \succsim \alpha y \quad \forall \alpha > 0.$$

- Consider symmetric example $\bar{u}_i = \bar{u}$
- Bundles: $x =$ tiny amounts of two goods; $y =$ 1 unit of one good.
  - Initially $y \succ x$ (tiny)
  - But if we make α very large, $x \succ y$ because of bounded u
- Intuition: Neither good is necessary + satiation

(back)