

# Household Portfolios, Monetary Policy and Asset Prices\*

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## Abstract

In this paper we study the role of household portfolio rebalancing channel for the aggregate and redistributive effects of monetary policy. The transmission of monetary policy works not only through the usual income and substitution motives, but also through an endogenous portfolio rebalancing effect which generates changes in equilibrium asset prices and a subsequent wealth effect on consumption. In order to jointly study these effects, we introduce an heterogeneous household life-cycle model with multiple assets and combine it with an incomplete markets asset pricing framework. We model monetary policy shocks as a reduction in expected return on safe assets. In equilibrium the reduction in bonds investment prompts a portfolio rebalancing toward riskier assets with a consequent increase in their asset prices and an increase in wealth. According to our model, the positive wealth effect on consumption is offset by an increase in the saving margin induced by the overall reduction in expected return on household portfolio. However, the strength of these two forces notably varies depending on households age. We find that, absent wealth effects, older cohorts reduce consumption while younger cohorts increase their consumption only slightly. The positive wealth-effect increases the consumption response for all cohorts: it strengthens the positive consumption response of the young and more than offsets the reduction in consumption of the old. Nevertheless, the heterogeneity in responses remain the same: the young raise consumption by more than the old.

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# I. Introduction

Households' portfolio generally consists of tradable (equity, housing, bonds) and non-tradable (labour income) assets used to make consumption plans. In making these plans, households face both aggregate risk, such as tradable asset returns, and uninsurable idiosyncratic risk like labour income shocks. Given these risks, variation in age and asset holdings, amongst other factors, can potentially generate differences in responses across households to common aggregate shocks.

In this paper, we analyse Quantitative Easing (QE) shocks and their effect on household asset allocations and consumption plans. Household heterogeneity, in terms of age, wealth and *ex-ante* asset allocation, will notably affect the pass-through of monetary policy into consumption. In the paper we combine a life-cycle model with an asset pricing framework with heterogeneous agents and incomplete markets. This allows us to study the effect of monetary policy not only through the usual income and substitution effect, but also through an endogenous portfolio rebalancing effect and the subsequent wealth effect which results from the change in equilibrium prices of equity and housing.

We calibrate the model to Euro area countries, using microeconomic data from the Household Finance and Consumption Survey (HFCS) in order to shed light on how monetary policy shocks affect different countries in different ways. Empirical evidence shows significant variation across Euro area countries in terms of income profile, distribution of *ex-ante* asset allocations as well as demographics. These factors - as we are going to show in the paper - notably interact with the pass-through of monetary policy, leading to substantial heterogeneity in the aggregate response of different countries to the same monetary policy shocks.

We first develop a life-cycle model to compute the households' optimal portfolios. The model is a sequence of 4-year trading periods. In the model, the income process combines a deterministic age profile with aggregate shocks and a combination of idiosyncratic permanent and transitory shocks. Households of different ages enter the period with a different holdings of nominal bonds, equity and housing. They observe their income realization, consume and trade in the asset market. Households can invest in bonds, equity or housing and can borrow subject to a collateral constraint with a borrowing limit proportional to the value of their housing. All assets are risky (in real terms), and we model their expected return and variance based on historical moments. Assets can be bought and sold without any frictions or transaction costs.

We then derive equilibrium prices for equity and housing as in [Piazzesi and Schneider \(2009\)](#) and [Landoigt et al. \(2015\)](#). Based on the optimal policy of households, we can compute aggregate demand and finally compute the equilibrium prices for equity and housing that equate aggregate supply and hence clear the markets in the current period. We compute the initial price level for the assets. Expectations of future prices are exogenous (consistent with the

returns data generating process) and therefore market clearing prices establish a 'temporary equilibrium' for the current period.

Household-level endowments and the quantity of aggregate supply are derived from the data, using the Household Finance and Consumption Survey (HFCS) and the Euro Area Economic Account (EEA). The HFCS is available for 2010 and 2014 for Euro area countries. We derive household asset allocations in 2010, using directly observed figures from the HFCS and indirect exposures measured through aggregate financial data from the EEA using the methodology as in [Doepke and Schneider \(2006\)](#) and [Adam and Zhu \(2015\)](#).

Combining our model with the data, we derive, for each cohort, the asset allocation and consumption decision. We will then compare our model-implied allocation with empirical moments from the 2014 survey to assess the ability of the model to match actual household decisions. Finally, we will evaluate the impact of QE shocks. Within our framework, this is modeled as a reduction in the supply of bonds. In order to clear the bond market, this will, all else equal, necessitate a reduction in the expected return on bonds, consequently changing the demand for risky assets and so their market-clearing prices. In order to discipline the comparative statics of our model, we take from [Lenza and Slacalek \(2018\)](#) empirical estimates of movements in (i) the term premium (-0.3%), (ii) house prices (+1.5%) and (iii) equity prices (+4.0%) in response to the ECB QE shock. We then target these three estimates when modeling the shock, which generates model-implied one-time movements in expected returns on housing (-0.07%) and equity (-0.13%). The reduction in term-premium directly translates, within our model, to a one-time 0.3% lower expected return on bonds.

According to our model, the positive wealth effect on consumption caused by movements in asset prices is offset by an increase in the saving margin induced by the overall reduction in expected return on a household's portfolio. However, the strength of these two forces notably varies depending on a household's age. We find that, absent wealth effects, older cohorts reduce consumption while younger cohorts increase their consumption only slightly. Cohorts experience a negative income effect from declining expected returns, and so consume less. This can be offset by the benefits of cheaper borrowing rates, which only young cohorts benefit from in equilibrium.

Once we incorporate the positive wealth effects, consumption responses turn positive for all cohorts. In response to the QE shock, there is a reallocation away from bonds and towards risky assets. Young cohorts raise borrowing in order to increase leveraged investments in housing, while older rich cohorts reallocate away from bond savings. This increases the demand for risky assets, causing market-clearing asset prices to rise. As older cohorts hold a disproportionate share of risky assets ex ante, they experience a stronger positive wealth effect on consumption. This wealth effect dominates such that, overall, consumption rises for all cohorts. We

hence conclude that considering wealth effects is very important for both the sign and scale. Nevertheless the heterogeneity in responses is unaffected by the wealth effect.

### *A. Literature review*

The paper is related to different strands of literature. First, we relate to the literature on the interaction between monetary policy (and in particular Quantitative Easing) and asset prices. [Krishnamurthy and Vissing-Jorgensen \(2011\)](#) and [Gagnon et al. \(2011\)](#) estimate the effect of quantitative easing on asset prices. A large number of papers also estimate the effect of monetary policy on asset prices, starting from the seminal paper by [Bernanke and Kuttner \(2005\)](#), to [Rigobon and Sack \(2004\)](#) and [Gurkaynak et al. \(2004\)](#). Our model is closely related to [Piazzesi and Schneider \(2009\)](#). [Piazzesi and Schneider \(2009\)](#) studies the effect of inflation expectations on household asset allocation and asset prices.

Second, we relate to the literature of monetary policy with heterogeneous agents and the redistributive effect of monetary policy. [Lenza and Slacalek \(2018\)](#) is closely related to our work from an empirical standpoint. It evaluates the impact of quantitative easing on income and wealth of individual euro area households. [Cloyne et al. \(2018\)](#) shows the importance consumption response to monetary policy shocks of homeowners with mortgages. From a theoretical point of view, [Auclert \(2017\)](#), [Kaplan et al. \(2018\)](#) and [McKay et al. \(2016\)](#) all assess the role of household heterogeneity in the transmission of monetary policy. In our paper we develop a life-cycle model highlighting the role of demographics, as in [Wong et al. \(2016\)](#). [Wong et al. \(2016\)](#), as well as [Greenwald \(2017\)](#), focus on the role of the mortgage market and housing in generating heterogeneous responses to monetary policy shocks.

The paper is also related to the literature which empirically estimates household-level exposure to shocks, as in [Doepke and Schneider \(2006\)](#), [Adam and Tzamourani \(2016\)](#) and [Adam and Zhu \(2015\)](#). [Carroll et al. \(2014\)](#) estimate marginal propensities to consume for households, taking into account the distribution of wealth, also using the HFCS survey.

Our model, generating asset pricing appreciations following a monetary policy shock, is able to assess in a coherent framework the role of wealth effects on consumption. A recent paper by [Berger et al. \(2017\)](#) analyzes the effect of house prices on consumption in a heterogeneous agents framework. The wealth effects on consumption have also been largely explored in previous work, such as [Case et al. \(2005\)](#) and [Campbell and Cocco \(2007\)](#).

Finally, one of the main focuses of the paper is studying the interaction between monetary policy and asset allocation. Related papers are [Adrian and Shin \(2010\)](#), [Borio and Zhu \(2012\)](#), and [Hau and Lai \(2016\)](#). Moreover, a strand of literature analyzes the effect of monetary policy on leverage and risk taking. Examples are [Gambacorta \(2009\)](#) and [Altunbas et al. \(2010\)](#).

## II. Data

We estimate how households of different ages and wealth levels allocate their portfolios across different asset classes. For this purpose, we use microeconomic information from the Household Finance and Consumption Survey (HFCS) in combination with aggregate data from the Euro Area Economic Accounts (EEA).

When estimating the portfolio allocation, we consider three asset classes: equity, housing and bonds. We define the investment in bonds as the total investment in fixed income assets (i.e. deposits and bond securities) minus debt (i.e. mortgage and non-mortgage debt). Therefore, according to our definition, a household which is a net borrower has a negative share in the bond's asset class.

### *A. Euro Area Account*

The European sector accounts provide a comprehensive and comparable overview of the European economy. They record transactions between economic agents grouped by sector. Stocks of assets and liabilities are recorded in balance sheets. The institutional sectors include, amongst others, households, non-financial corporations, financial institutions, insurance corporations and pension funds.

### *B. HFCS*

The HFCS collects household-level data on a wide range of variables, with a particular focus on the composition of household savings/borrowings. It contains 20 EU member states with a sample of more than 84,000 households. The primary advantage of this survey is that data collection is harmonized across countries, allowing direct comparison of results across countries. Thus far, there have been two waves, in 2010 and 2014, which are repeated cross-sections of similar size<sup>1</sup>.

The HFCS provides us with data on household-level consumption and income as well as portfolio allocations. We extract data on holdings of equity, housing and net bonds, as well as investments in pension and mutual funds.

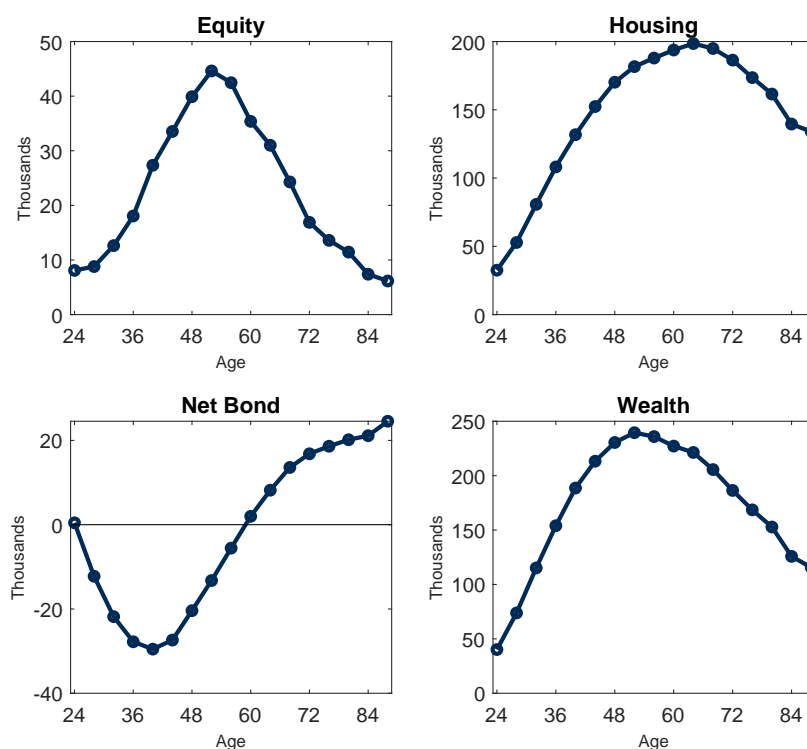
Equity investment includes the value of self-employment businesses, shares of private companies as well as publicly traded shares. We define bond investment as the net exposure to fixed income assets: deposit and bond securities minus mortgage and non-mortgage debt. Housing investment includes the value of a household's main residence as well as other real estate properties. Investment in intermediaries is the sum of investment in pension funds and mutual

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<sup>1</sup>Many countries also have panel data available, allowing for longitudinal studies

funds.<sup>2</sup> Appendix F details the variables we use to construct these series.

Figure 1 shows the portfolio allocation for Italian households. We divide households by age into 4-year cohorts. The younger cohort is composed of households ranging from 20 to 24 years old. The oldest cohort we consider includes households aged 84 to 88 years old. Investment in equity, housing and intermediaries are hump-shaped. Investment in equity ranges from around 10,000 euros for the younger and older cohorts to 40,000 for the middle aged cohorts. Housing investment is clearly larger and ranges from 40,000 to 200,000 euros. Net bond is defined as fixed income investment minus debt. We notice that younger cohorts tend to be net borrowers (borrowing up to 30,000 euros) while older cohorts tend to be net investors, investing up to 20,000 euros. In nominal terms, households of age 44 - 48 hold the largest amount of debt.



**Figure 1. Allocation in Nominal Terms**

EA: average allocation of households by cohort in 2014 EUR thousands

Although the survey includes data on household-level investment in intermediaries, it does not obviously provide any information on how these funds are invested in different asset classes. For this reason, we combine microeconomic data from the survey with aggregate data from the

<sup>2</sup>We only consider investment in mutual funds and non-defined benefits pension funds. We exclude defined benefit pension funds and insurance funds because the pay-out from these investments do not depend on the portfolio allocation, given that they are fixed payments.

EEA. In this way we are able to *look-through* the investment in intermediaries and have a complete pictures of households’ exposure to different assets.

Before detailing the *look-through* exercise, we provide information on the comparability of the HFCS and EEA data by matching household data from these two data sources. In this way, we are able to shed light on the differences and similarities between the two data sets.

*C. Aggregate: EEA vs HFCS*

We expect aggregate information provided by the EEA and HFCS to be comparable. However, we notice large discrepancies once we aggregate HFCS and compare them to the EEA figures. Table I compares the HFCS and EEA aggregates. The table shows that the numbers reported by the HFCS are significantly smaller than the EEA’s. Kavonius et al. (2013) already pointed out that there are major differences between the HFCS aggregated data and the EEA data, and discuss possible explanations for this gap. Our numbers are broadly in line with what they found in their aggregation exercise. While we acknowledge discrepancies are not negligible, we believe the information from the EEA is valuable in the assessment of indirect portfolios of households. We therefore proceed with the consolidation exercise, where we combine the two data sets and compute a comprehensive measure of household portfolios.

	HFCS	EEA	HFCS/EEA
Deposits	2928.81	7074.23	0.41
Bonds	304.42	1070.82	0.28
Listed Equity	462.22	871.90	0.53
Unlisted Equity	4061.42	3134.61	1.30
Loans	4232.04	6115.71	0.69

**Table I. EEA vs HFCS**

Aggregate data from the HFCS and EEA for the household sector as of 2014. Data are in EURbn for column 1-2.

*D. Consolidation*

As discussed, the HFCS provides information on household-level investment in pension funds and mutual funds. We then use data from the EEA to understand how these funds allocate their portfolio. The EEA does not provide any information on investment in housing and therefore we only consider investments in equity and bonds.

We firstly calculate portfolio allocations for the mutual funds (defined in EEA as Non-MMF Investment Funds). Portfolio shares are reported in Table II. Data from the EEA shows that mutual funds invest 57% of their net wealth in bonds and 44% in equity. They borrow only 1% of their net wealth.

Moving to pension funds (defined in EEA as Insurance Corporations and Pension Funds), we notice that they not only invest in equity and bonds but also in Investment Fund Shares. The Investment Fund Shares entails an indirect exposure whose allocation we assume to be equal to the portfolio shares estimated for the mutual funds. We then also consider the indirect exposure. Pension funds invest 76% of their net wealth in equity and 30% in bonds. They borrow 7% of their net wealth.

	Mutual Funds	Pension
Equity	0.44	0.31
Bond	0.57	0.77
Debt	0.01	0.07

**Table II. Intermediaries Allocation**

Mutual Funds and Pension allocation as of 2014.

With this data to hand, we are able to compute total household exposures.

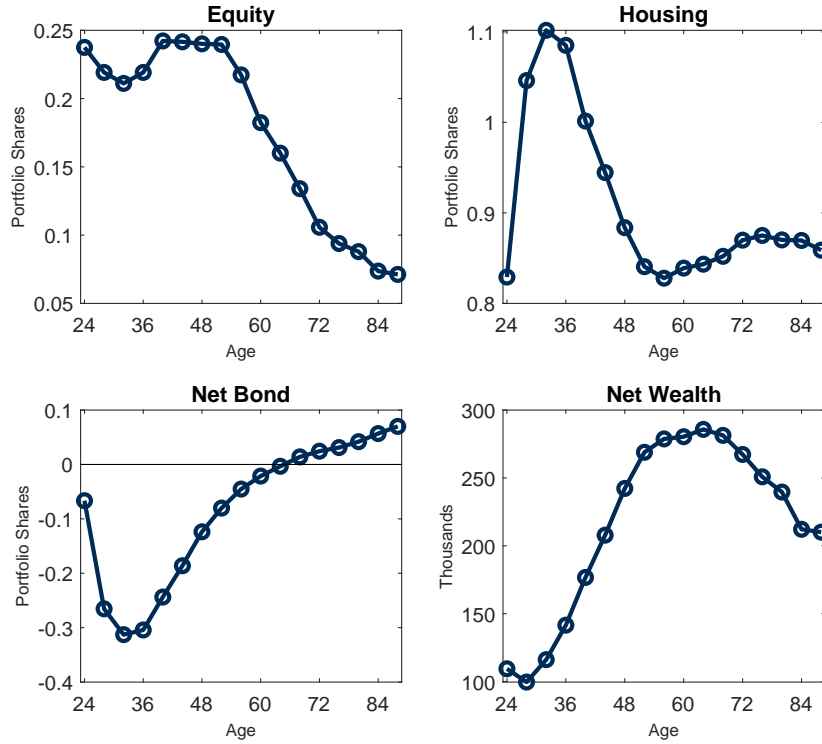
### *E. Household Portfolio Allocation*

We then consolidate the HFCS data with the EEA data. As mentioned earlier, we observe from the HFCS the investment in pension funds and mutual funds. We assign that investment pro-rata to the three asset classes of interest according to the share we calculated from the EEA. For example, if an Italian household in 2014 holds 10,000 euros in mutual funds, we assume that his indirect exposure to equity is of 4,400 euros (44% of 10,000) and we add this amount to the direct investment in equity we observe from the HFCS.

We have now calculated the total exposure for each household in our survey. We then calculate the portfolio shares allocated to the three asset classes. Figure 2 shows the average shares of household portfolio invested in each asset by households in different cohorts. For equity, housing and net bond we show the share as a percentage of net wealth (housing+equity+net bond). The bottom-right panel shows the evolution of net-wealth across the life-time.

Net wealth has the well known hump shape, increasing until the age of 52-60 and decreasing thereafter. Housing investment notably dominates all other asset classes. Equity investment ranges from 20% for the younger cohorts and decrease to 10% for the older cohort. Housing investments ranges from 80% for young households, then reaching a peak of 110% for households of age 36. Finally, we notice a large difference in the net bond shares. Young households tend to be net borrower, borrowing up to 30% of their net wealth. Older households instead tend to decrease the riskiness of their portfolio, holding a positive portfolio shares in bonds and lower equity investment.





**Figure 2. Portfolio Shares**

Average portfolio shares and net wealth of households by cohort in 2014.

We observe that portfolio allocations not only change across cohorts but also across different wealth levels. For this reason we split the cohort into three sub-cohorts according to their wealth level. We subdivide cohorts into the bottom 40, the medium 40 and the top 20 percentile. For each of these age-wealth level combinations we calculate portfolio shares. These are shown in Appendix A in Figure 10- 12.

### *F. Rest of the Economy*

As we will discuss in the model section, we also have a rest of the economy (ROE) sector. We use the EEA and national tables data in order to estimate the supply of assets provided by the ROE.

### III. Model

#### A. Environment

##### A.1. Preferences

Time  $t$  is discrete, where each period corresponds to 4 years. Households begin at the age of 24, and live up to a maximum of 18 (T) periods. The probability that a household aged  $a$  is alive at time  $(t + 1)$  conditional on being alive at time  $t$  is  $p_a$ .

Households have Epstein-Zin utility functions defined over the single homogeneous consumption good,  $C_t$ . Let  $W_t$  denote wealth at time  $t$ . Household preferences can then be written as:

$$V_{a,t} = \left[ C_t^{1-\frac{1}{\sigma}} + \beta E_t [p_a V_{a+1,t+1}^{1-\gamma} + (1-p_a) \phi_B^{1-\gamma} W_{t+1}^{1-\gamma}]^{\frac{1-\frac{1}{\sigma}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\sigma}}} \quad (1)$$

where  $\gamma$  is the coefficient of relative risk aversion,  $\sigma$  is the elasticity of intertemporal substitution, and  $\phi_B$  controls the strength of the bequest motive. Given the presence of a bequest motive, the terminal condition is:

$$V_{T,t} \equiv \phi_B W_t \quad (2)$$

##### A.2. Labour Income Process

Following what is standard in the literature, the labour income process for household  $i$  is given by:

$$Y_{i,t} = G_t P_{i,t} U_{i,t} \quad (3)$$

$$P_{i,t} = \exp(f(a)) P_{i,t-1} N_{i,t} \quad (4)$$

$$G_t = \exp(\mu^G) \exp(\epsilon_t^G) G_{t-1} \quad (5)$$

where  $f(a)$  is a deterministic function of household age  $a$ ,  $G_t$  is GDP per capita,  $P_{i,t}$  is a permanent component with innovation  $N_{i,t}$ , and  $U_{i,t}$  is a transitory component. We assume that  $\ln U_{i,t}$  and  $\ln N_{i,t}$  are iid with mean  $\{-0.5 * \sigma_u^2, -0.5 * \sigma_n^2\}$  and variances  $\sigma_u^2$  and  $\sigma_n^2$ , respectively.  $\epsilon_t^G$  is an iid normal variable with mean  $-0.5 * \sigma_G^2$  and variance  $\sigma_G^2$ . Thus,  $G_t$  is a random walk with deterministic drift  $\exp(\mu^G)$ .

### A.3. Financial Assets

There exists three financial assets each period: bonds, equity and housing. Bonds pay a real return:

$$R_{t+1}^B = \frac{1 + i_t}{\Pi_{t+1}} \quad (6)$$

where  $i_t$  is the nominal return and  $\Pi_{t+1}$  is the inflation rate. Households can short bonds (i.e. get a mortgage) by paying a spread  $\iota$  in addition to the real return on bonds.

Equity and housing are single-period lived trees with aggregate dividends at time  $t + 1$  of  $D_{t+1}^E$  and  $D_{t+1}^H$  and have prices at time  $t$  of  $p_t^E$  and  $p_t^H$ .

The gross returns on these assets are therefore defined as:

$$R_{t+1}^E = D_{t+1}^E / p_t^E \quad (7)$$

$$R_{t+1}^H = D_{t+1}^H / p_t^H \quad (8)$$

### A.4. Wealth Accumulation

We define wealth (W) as the resources that are available for consumption and saving. Household  $i$ 's next period wealth is given by:

$$W_{i,t+1} = \theta_{i,t}^E D_{t+1}^E + \theta_{i,t}^H D_{t+1}^H + B_{i,t}^+ R_{t+1}^B - B_{i,t}^- (R_{t+1}^B + \iota) + Y_{i,t+1} \quad (9)$$

where  $\theta_{i,t}^E$  and  $\theta_{i,t}^H$  denote equity and housing shares, respectively, at time  $t$ ,  $B_{i,t}^+$  is the holdings of one period bonds, and  $B_{i,t}^-$  is holdings of one-period debt.  $\iota$  is the spread faced by households who want to borrow.

Since the households must allocate wealth between consumption and savings, the budget constraint is:

$$W_{i,t} = C_{i,t} + p_t^E \theta_{i,t}^E + p_t^H \theta_{i,t}^H + B_{i,t}^+ - B_{i,t}^- \quad (10)$$

Holdings of assets are restricted to being non-negative:

$$\theta_{i,t}^E \geq 0 \quad (11)$$

$$\theta_{i,t}^H \geq 0 \quad (12)$$

$$B_{i,t}^+ \geq 0 \quad (13)$$

$$B_{i,t}^- \geq 0 \quad (14)$$

Finally, household-level borrowing is subject to a collateral constraint:

$$B_{i,t}^- \leq \phi p_t^H \theta_{i,t}^H \quad (15)$$

where housing holdings is the sole source of collateral, and  $\phi$  is the associated Loan-to-Value (LTV) limit.

#### A.5. Optimization Problem

The complete optimization problem is then:

$$\max_{\{\theta_{i,t}^E, \theta_{i,t}^H, B_{i,t}^+, B_{i,t}^-, C_{i,t}\}_{t=0}^{T-a}} E[V_{a,0}]$$

where  $a_i$  is the age of household  $i$  and  $V_{a,0}$  is given by equations 1 and 2 and is subject to the constraints given by equations 9-15 and to the stochastic labour income process given by 3-5.

#### A.6. Central Bank

The central bank chooses the nominal interest rate  $i_t$ . The central bank provides  $B_t^{CB}$  units of bonds to accommodate household sector demand, under the set nominal rate  $i_t$ :

$$B_t^{CB} = \sum_i B_{i,t}^+ - \sum_i B_{i,t}^- \quad (16)$$

This is consistent with current ECB policies whereby the central bank fixes the policy rate and provides liquidity through tender operations based on the principle of full allotment.

#### A.7. Rest of Economy (RoE)

We introduce another sector, ROE, that exogenously supplies assets to the household sector. The RoE is not explicitly modelled, but instead exogenously supplies new assets and consumes

or provides goods to households. At time  $t$ , the ROE supplies one unit of housing and one unit of equity. The ROE can consume or provide goods to the household sector. We define the net consumption as  $C^{ROE}$ ; its value is negative in case the ROE is a net supplier of goods. In order to determine  $C_t^{ROE}$ , we assume that the ROE consumes or provides goods to the household sector in order to accommodate any discrepancy between households' aggregate income and aggregate consumption. This will allow the household sector to increase (or decrease) their savings in a way that is usually ignored in an endowment economy.

### B. Temporary Equilibrium at Time 0

The economy starts at time 0. Under our equilibrium concept, asset prices in period 0 are set such that asset markets clear in period 0, taking as given household expectations on future income and future asset dividends  $\{D_{t+1}^E, D_{t+1}^H\}_{t=0}^\infty$  and prices  $\{p_{t+1}^E, p_{t+1}^H\}_{t=0}^\infty$  as well as future nominal interest rates and inflation:  $\{i_{t+1}, \Pi_{t+1}\}_{t=0}^\infty$ .

An **equilibrium** for time 0 is a vector of prices  $(p_0^H, p_0^E)$ , a supply of bonds  $B_0^{CB}$ , and period 0 choices  $A_{i,0} = \{\theta_{i,0}^E, \theta_{i,0}^H, B_{i,0}^+, B_{i,0}^-, C_{i,0}\}_i$  s.t.

1.  $A_{i,0}$  is part of their optimal plan given age, endowment, beliefs and the nominal rate  $i_0$ .
2. All markets clear i.e.

$$\sum_i p_0^E \theta_{i,0}^E (a_0, W_{i,0}, P_{i,0}, p_0^E, p_0^H, i_0) = p_0^E \quad \text{(equity)} \quad (17)$$

$$\sum_i p_0^H \theta_{i,0}^H (a_0, W_{i,0}, P_{i,0}, p_0^E, p_0^H, i_0) = p_0^H \quad \text{(housing)} \quad (18)$$

$$\sum_i B_{i,0}^+ (a_0, W_{i,0}, P_{i,0}, p_0^E, p_0^H, i_0) - \sum_i B_{i,0}^- (a_0, W_{i,0}, P_{i,0}, i_0) = B_0^{CB} \quad \text{(bonds)} \quad (19)$$

$$\sum_i C_{i,0} (a_0, W_{i,0}, P_{i,0}, p_0^E, p_0^H, i_0) + C_0^{ROE} = \sum_i Y_{i,0} \quad \text{(goods)} \quad (20)$$

### C. Equivalent Formulation

Lets define  $E_{i,t} = p_t^E \theta_{i,t}^E$  and  $H_{i,t} = p_t^H \theta_{i,t}^H$  as the amount of wealth household  $i$  invested in equity and housing, respectively. We can re-define Equation 9 as:

$$W_{i,t+1} = E_{i,t} R_{t+1}^E + H_{i,t} R_{t+1}^H + B_{i,t}^+ R_{t+1}^B - B_{i,t}^- (R_{t+1}^B + \iota) + Y_{i,t+1} \quad (21)$$

where  $R_{t+1}^B, R_{t+1}^E, R_{t+1}^H$  are the gross returns on the three assets. The budget constraint is then:

$$W_{i,t} = C_{i,t} + E_{i,t} + H_{i,t} + B_{i,t}^+ - B_{i,t}^- \quad (22)$$

Non-negativity constraints on holdings are now defined as:

$$E_{i,t} \geq 0 \quad (23)$$

$$H_{i,t} \geq 0 \quad (24)$$

$$B_{i,t}^+ \geq 0 \quad (25)$$

$$B_{i,t}^- \geq 0 \quad (26)$$

Finally, the collateral constraint is now defined as:

$$B_{i,t}^- \leq \phi H_{i,t} \quad (27)$$

Moreover, as households take initial prices as given, we can solve the equivalent problem:

$$\max_{\{E_{i,t}, H_{i,t}, B_{i,t}^+, B_{i,t}^-, C_{i,t}\}_{t=0}^{T-a}} E[V_{a,0}]$$

where  $V_{a,0}$  is given by equations 1 and 2 and is subject to the constraints given by the new equations 21-27 and to the stochastic labour income process given by 3-5. The new formulation is consistent with usual portfolio problems where households take into account the distribution of expected returns.

#### D. Beliefs

We assume that households have expectations about future returns for housing and equity  $R^E, R^H$  as given by:

$$\log R_{t+1}^E = \mu_t^E + \epsilon_{t+1}^E \quad (28)$$

$$\log R_{t+1}^H = \mu_t^H + \epsilon_{t+1}^H \quad (29)$$

where  $\epsilon_{t+1}^E, \epsilon_{t+1}^H$  are iid normal variables with mean zero and variances  $\sigma_E^2, \sigma_H^2$ , respectively. Inflation expectations are given by

$$\log \Pi_{t+1} = \mu_t^\Pi + \epsilon_{t+1}^\Pi \quad (30)$$

Finally, using Equation 6, bonds return expectations are hence given by:

$$\log R_{t+1}^B = \log(1 + i_t) - \log(\Pi_{t+1}) = \mu_t^B + \epsilon_{t+1}^B \quad (31)$$

where  $\mu_t^B = \log(1 + i_t) - \mu_t^\Pi$  and  $\epsilon_{t+1}^B = -\epsilon_{t+1}^\Pi$  are iid normal variables with mean zero and variances  $\sigma_B^2 = \sigma_\Pi^2$ . The central bank can therefore directly affect the expected return on bonds by choosing  $i_t$ .

### E. Monetary Policy Shock

At time  $0^+$ , after households have chosen their allocation and made their consumption decision, the central bank can decide to change the nominal interest rate  $i_0$  to a new rate:

$$\bar{i}_{0+} = i_0 + \bar{v}_{0+} \quad (32)$$

where  $\bar{v}_{0+}$  is the monetary policy shock.

The shock is fully transitory and we assume that i) expected labor income is unchanged, ii) expectations about future dividends  $\{D_{t+1}^E, D_{t+1}^H\}_{t=0}^\infty$ , prices  $\{p_{t+1}^E, p_{t+1}^H\}_{t=0}^\infty$  as well as future nominal interest rates and inflation:  $\{i_{t+1}, \Pi_{t+1}\}_{t=0}^\infty$  are unchanged. This set of assumptions makes sure that the problem from period  $t = 1$  onward is unchanged and hence the value functions at time  $t = 1$ ,  $V_{a,1}$ , for each cohort  $a$  are still valid.

After the shocks, households update their optimal plan  $\bar{A}_{i,0+} = \{\bar{\theta}_{i,0+}^E, \bar{\theta}_{i,0+}^H, \bar{B}_{i,0+}^+, \bar{B}_{i,0+}^-, \bar{C}_{i,0+}\}_i$  given the new interest rate  $\bar{i}_{0+}$ . The asset market opens and households can trade housing, equity and bonds. The new equilibrium after the shocks is a new vector of prices  $(\bar{p}_{0+}^H, \bar{p}_{0+}^E)$  given the new optimal plans  $\bar{A}_{i,0+}$  and such that all markets clear.

### F. Wealth-Effect

Note that, after the monetary policy shock - and the subsequent asset price change - households experience a revaluation of their wealth. Household  $i$  at time 0 holds  $\theta_0^E$  shares of equity and  $\theta_0^H$  shares of housing. After the shock, at time  $0^+$ , equilibrium prices of equity and housing have changed from  $p_0^E$  to  $\bar{p}_{0+}^E$  and from  $p_0^H$  to  $\bar{p}_{0+}^H$ , respectively. This implies a change in wealth of:

$$\Delta \bar{W}_{i,0+} = (\bar{p}_{0+}^E - p_0^E) \theta_0^E + (\bar{p}_{0+}^H - p_0^H) \theta_0^H \quad (33)$$

We define that as the *wealth effect* of monetary policy. This also means that household  $i$ 's new plan  $\bar{A}_{i,0+}$  has to be optimal given his new wealth  $\bar{W}_{i,0+}$ .

### G. Solving the Problem

Analytical solutions to this problem do not exist. Thus, we use a numerical solution method. The details are given in Appendix C, but here we provide the main idea. Firstly, we exploit how the value function can be normalized by  $P_{i,t}$ , which transforms the value function to one

containing just a single state variable  $w_{i,t} \equiv \frac{W_{i,t}}{P_{i,t}}$ . We then solve for optimal policies, starting with the terminal period  $T$ , and then iterating backwards. Computational speed for deriving optimal policy is aided by evaluating first-order conditions at each candidate policy. This will then generate the following policy functions:

$$E_{i,t}(a_i, W_{i,t}, P_{i,t}) \tag{34}$$

$$H_{i,t}(a_i, W_{i,t}, P_{i,t}) \tag{35}$$

$$B_{i,t}^+(a_i, W_{i,t}, P_{i,t}) \tag{36}$$

$$B_{i,t}^-(a_i, W_{i,t}, P_{i,t}) \tag{37}$$

$$C_{i,t}(a_i, W_{i,t}, P_{i,t}) \tag{38}$$

where  $a_i$  is the age of household  $i$ . Finally, we then define the level of savings,  $S_{i,t}$ , as:

$$S_{i,t}(a_i, W_{i,t}, P_{i,t}) = E_{i,t}(a_i, W_{i,t}, P_{i,t}) + H_{i,t}(a_i, W_{i,t}, P_{i,t}) + B_{i,t}^+(a_i, W_{i,t}, P_{i,t}) - B_{i,t}^-(a_i, W_{i,t}, P_{i,t}) \tag{39}$$

## IV. Calibration

### A. Asset Returns and Covariances

Our baseline calibration assumes that the mean and variance of returns are constant across time. We calibrate the mean ( $\mu$ ) and standard deviation  $\sigma$  in equations 28-29 to match empirical moments estimated on historical real returns for bonds, equity and housing. Appendix E details the series we used for the estimation. The resulting estimates are detailed in Table III below.

	$\mu$	$\sigma$
Bond	-0.04	0.89
Housing	5.08	1.88
Equity	4.89	21.39

**Table III.** Mean and standard deviation of real annualized returns

Our choice on  $i_0$  is the historical mean nominal return on bonds.

We include an idiosyncratic shock to the housing return, represented as a scaling factor,  $\sigma_{\text{idio}}^H$ , to aggregate housing variance, as is done in Piazzesi et al. (2007). We also scale equity return variances by  $\sigma_{\text{idio}}^E$  to



### B. Labour Income Process

The deterministic labour income profile  $f(a)$  exhibits the familiar hump-shape of earnings over the life cycle. This is estimated from the HFCS by averaging household-level income at the cohort level <sup>3</sup>. Idiosyncratic income shock variances  $(\sigma_u^2, \sigma_n^2)$  are chosen to be (7%, 2%), as estimated in Krueger et al. (2010).

### C. Initial Wealth

We take initial wealth directly from the data. To evaluate initial wealth levels we aggregate households to the cohort level, where each cohort is of a 4-year range. The representative household of each cohort holds wealth equalling the average of what is observed in the data in the survey in 2014, equalling asset holdings plus consumption. This is in accordance with 10.

We then assume that each household  $i$  in cohort  $a$  holds shares equal to the representative household of that cohort.

### D. Other Non-Calibrated Parameters

Death probabilities are extracted from the European population survey data. We decide on an LTV ratio limit of  $\phi = 0.9$  as there exists a very distinct bunching of households with a portfolio at this ratio. Finally, we choose  $\beta = \exp(-0.01 * 4)$  i.e. a 1% annual discount rate.

### E. Calibrated Parameters

We calibrate the remaining model parameters:  $(\gamma, \sigma, \phi_B, sp, s_{\text{idio}}^H, s_{\text{idio}}^E)$ . Recall that initial wealth is drawn from the 2014 wave of the HFCS. Given a parameter selection, it will generate a variety of model-implied outcomes for asset holdings that can be compared to the 2014 wave of the HFCS. We decide to choose the calibrated parameters to match the following aggregate outcomes:

1. Wealth/GDP
2. Housing/GDP
3. Equity/GDP
4. Gross Lending/GDP
5. Gross Borrowing/GDP

Firstly, we want to ensure wealth levels in the model are consistent with the data. Secondly, we want to match aggregate portfolio shares, and the composition of net lending between gross

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<sup>3</sup>For further details, see Appendix B

lending and gross borrowing. The final two are important because a key form of heterogeneity across households relevant for responses to MPSs is being a borrower vs. a saver.

Note that, given exogenous expected returns, once we have market-clearing  $(p_0^E, p_0^H)$  (equivalent to aggregate value of equity and housing), we then back out implied dividend expectations.

The empirical outcomes are shown in the table below:

	Wealth/GDP	Housing/GDP	Equity/GDP	Lending/GDP	Borrowing/GDP
data	1.72	1.09	0.18	0.01	-0.12

**Table IV.** Empirical Outcomes

Each parameter plays a distinct role in matching these targets. The degree of risk aversion,  $\gamma$ , controls the split between bonds and the relatively riskier housing and equity assets.  $\sigma$  controls aggregate wealth via intertemporal substitution.  $(\sigma_{idio}^H, \sigma_{idio}^E)$  manage the level and portfolio split between housing and equity.  $sp$  helps target the split between savings and borrowings on aggregate. Finally,  $\phi_B$ , helps to match the portfolio decision of older cohorts.

We constructed a grid of possible values for the parameters listed above. We then ran thousands of simulations (one for each combination of parameters) and selected the combination that best matched the targeted values. The parameters we choose are given in Table V below.

	$\gamma$	$\sigma$	$\sigma_{idio}^H$	$\sigma_{idio}^E$	sp	$\phi_{beq}$
parameters	9.00	0.40	6.50	1.75	0.03	1.00

**Table V.** Calibrated Parameters

	Wealth/GDP	Housing/GDP	Equity/GDP	Lending/GDP	Borrowing/GDP
data	1.72	1.09	0.18	0.01	-0.12
model	1.72	1.06	0.17	0.02	-0.12

**Table VI.** Aggregate Outcomes: Model vs. Data

Table VI compares model-implied aggregates to the data. As you can see, it does very well. However, as we have more parameters than targets, we aspire to target an additional moment, in particular borrowing of the younger cohorts

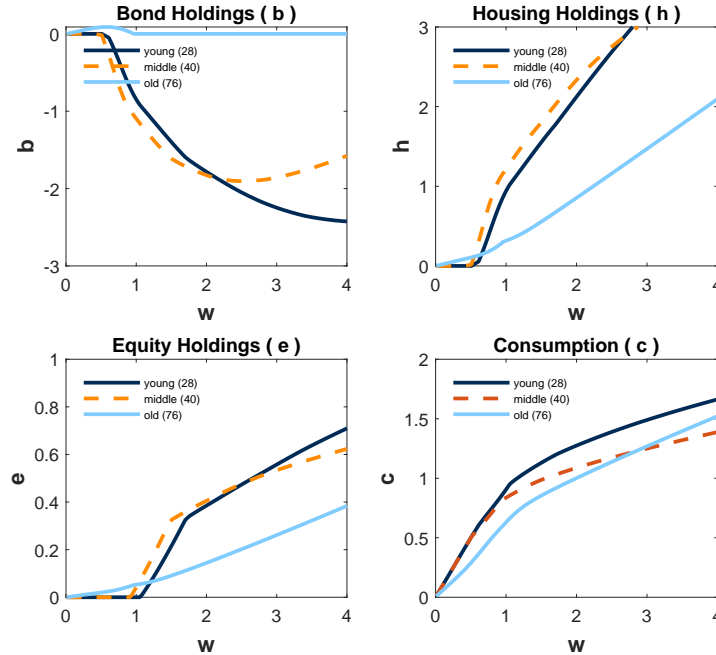
## V. Baseline Results

With the calibrated parameters at hand, we firstly present and discuss the policy functions of households. Secondly, we evaluate the success of the model by comparing model-implied wealth

and portfolio shares across cohorts to the data.

### A. Policy Functions

We now lay out the policy functions of each household conditional on their age and wealth. Specifically, we analyze both their portfolio choice and consumption levels. Figure 3 graphs such choices for households aged 28, 52 and 76 as a function of the state variable  $w_{i,t}$  i.e. normalized initial wealth.



**Figure 3.** Baseline Policy Functions

We begin with asset holdings. Even at very low wealth levels, agents prefer not to consume all of their wealth but instead decide to borrow in order to take on leveraged risky asset investments. Despite high risk aversion and marginal propensities to consume, lucrative expected returns induce early investment.

Beyond this observation, bond holdings are notably heterogeneous across cohorts. For the young with low wealth, most of their future wealth is in the form of labour income, which is uncorrelated with asset returns. Consequently, they exhibit high risk tolerance and so allocate savings to leveraged housing (i.e. they borrow to finance investment in the housing asset), the riskiest type of investment. Then, as wealth rises, risk tolerance declines. Current asset holdings become increasingly important for future wealth, causing them to deleverage and diversify their portfolio.

Middle-aged cohorts follow a similar path to the young, but start deleveraging at lower wealth levels. Such lower risk tolerance occurs because i) future income is less important simply due to lower remaining life expectancy, and ii) they anticipate a declining age profile of income.

For the old, risk tolerance is even lower, for the same reasons as middle-aged cohorts. Moreover, as conditional life expectancy is short, they no longer have long investment horizons that can smooth out negative asset return shocks. Thus, bond holdings are non-negative for almost all wealth levels.

Notice also that, for each age, there exists a region of wealth in which bond holdings are zero. This is generated by the borrowing spread ( $sp$ ) as then it is feasible to strictly prefer to neither save in bonds nor borrow. The larger is  $sp$ , the larger is this region of wealth.

Finally, we turn to consumption choices. Consumption policy functions are similar for the young and middle-aged households. Older households they tend to have higher consumption levels as well as higher propensities to consume (given the greater slope of the policy function).

### B. Comparing Model to the Data

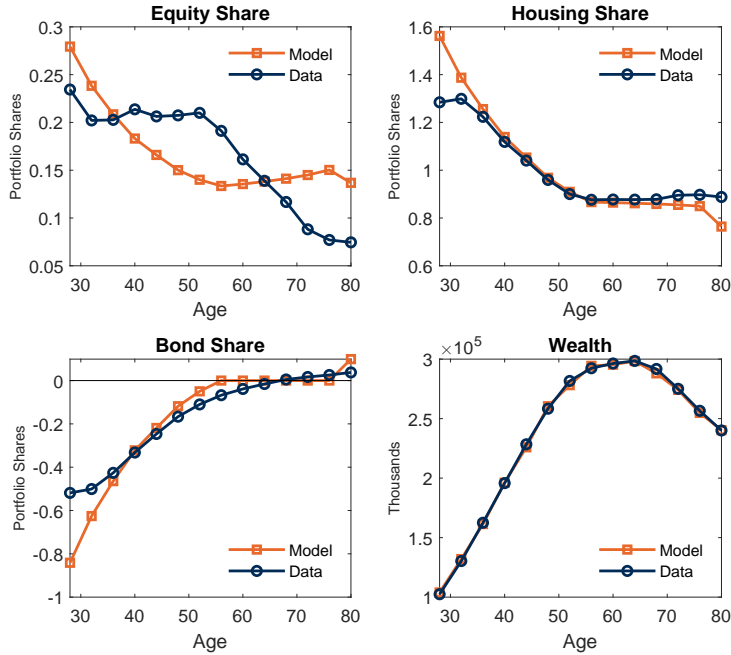
By combining market-clearing asset prices with household-level policy functions, we then derive period  $t = 0$  model-implied portfolio shares and wealth across cohorts. To evaluate the success of the model, we compare this to the data. Figure 4 below shows the portfolio shares,  $\left(\frac{E_{i,t}}{S_{i,t}}, \frac{H_{i,t}}{S_{i,t}}, \frac{B_{i,t}}{S_{i,t}}\right)$ , for different cohorts together with the wealth levels.

The model matches wealth dynamics and bond shares very well. Moreover, it gets the level and trend housing and equity shares reasonably right. The only real shortfall is that the youngest agents borrow a bit too much. However, as they are a small fraction of overall households, it matters little for responses of asset prices to shocks.

## VI. Monetary Policy Shock

### A. Assumptions

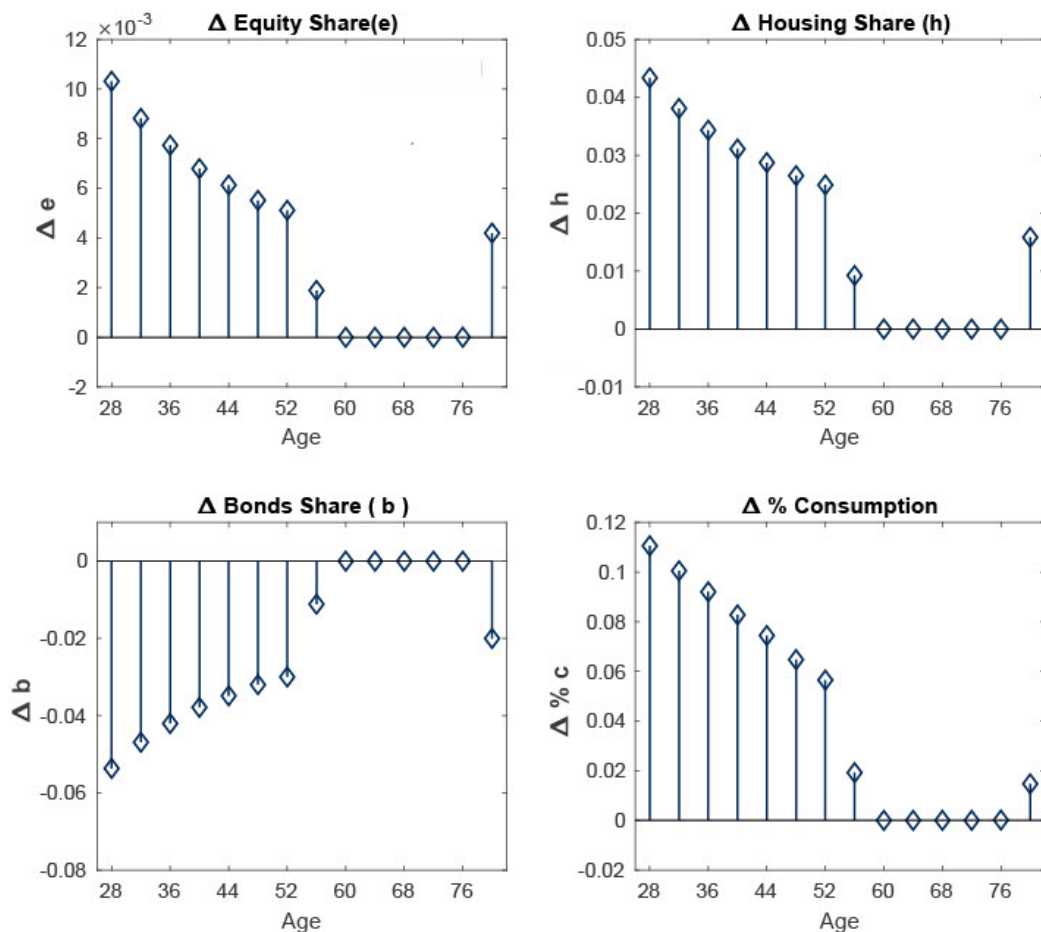
After we calibrated the model, we estimate the effects of a monetary policy shock. Within our framework, a monetary policy shock is a 21bps change in the nominal interest rate, as described in Equation 32. As explained in the Model section, expectations about future dividends,  $\{D_{t+1}^E, D_{t+1}^H\}_{t=0}^\infty$ , prices  $\{p_{t+1}^E, p_{t+1}^H\}_{t=0}^\infty$ , nominal interest rates  $\{i_{t+1}\}_{t=0}^\infty$  and inflation  $\{\Pi_{t+1}\}_{t=0}^\infty$  are unchanged. Given the equity and housing prices  $(p_0^E, p_0^H)$ , we are then able to derive  $t = 1$  dividends expectations  $E_0 [D_1^E], E_0 [D_1^H]$  for time  $t = 1$ . We assume that these expectations are unaffected by the monetary policy shock.



**Figure 4.** Portfolio Shares and Wealth: Model vs. Data

### B. Portfolio Rebalancing

The change in the nominal interest rate, and the consequent shift in expectations about time  $t = 1$  bond returns, will prompt households to rebalance their portfolio. In this section, we detail the portfolio rebalancing of different households following the change in the interest rate, holding constant the price of other assets (hence keeping expected returns on housing and equity unchanged). Figure 5 shows the change in portfolio shares for each asset class together with the change in consumption. The reduction in the nominal interest rate leads younger households to increase their leveraged investment in riskier assets. For very young households, we notice the largest change in debt, increasing their shares in debt (with a peak of 5% for households of age 28) and use the borrowing to buy more housing shares and equity shares. The eldest households, who tend to be net investors in bonds, also reduce their bond shares and rebalance toward riskier assets. For households of age 60 to 76, though, they do not change in any way their portfolio. They are neither borrowing nor investing in bonds, and so the change in the interest rate is not enough to make them willing to take on leverage. Finally a 21 bps change in the nominal rate induces a consumption response of 3% for younger households. This response decreases in age, to the point where older households barely adjust consumption.



**Figure 5.** Households portfolio rebalancing

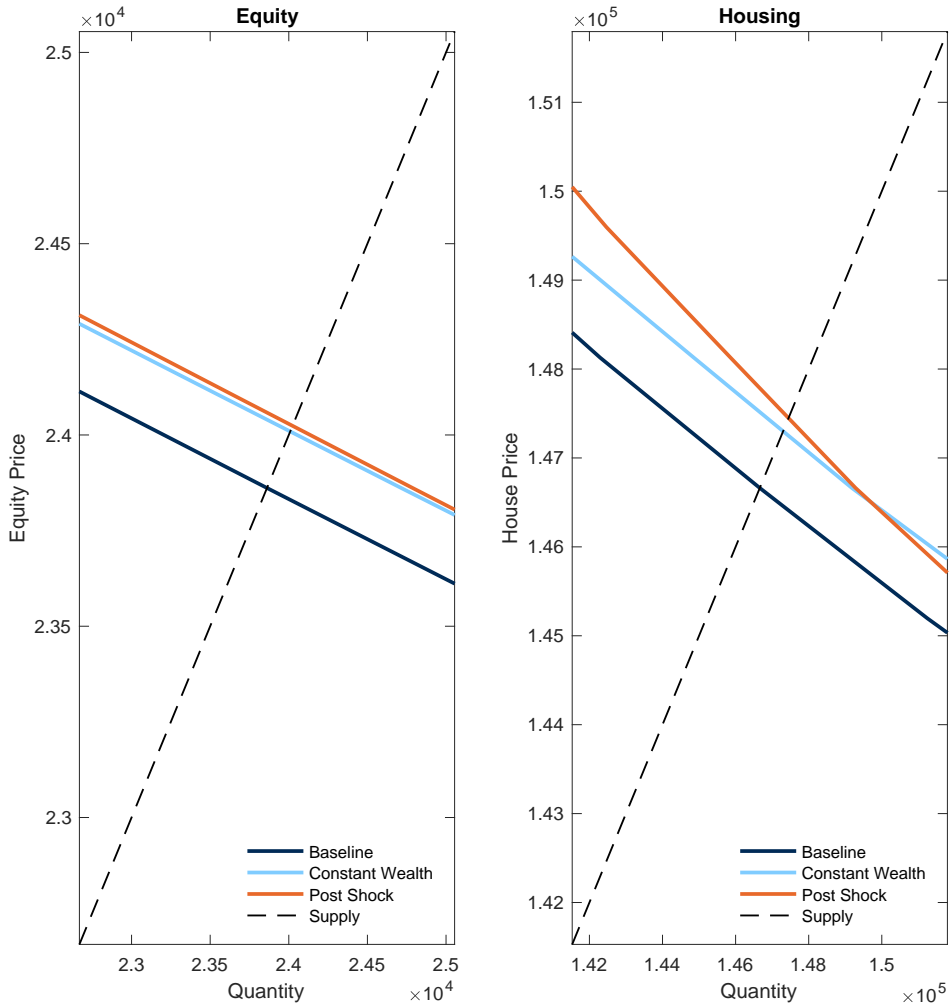
### C. Asset Prices

The portfolio rebalancing described above increases aggregate demand for riskier assets. In order to clear markets, asset prices need to adjust.

Figure 6 plots the demand and supply for risky assets. The demand curve is always downward sloping. Intuitively, a higher asset price reduces the expected return on the asset, reducing demand as households rebalance their portfolio in response. As the quantity supplied is fixed at 1, the value of supply is its asset price, hence the 45-degree line for the supply curve.

The Baseline demand curve crosses the supply at the pre-shock equilibrium price. The monetary policy shock prompts an outward shift in the demand curve, holding wealth constant, from Baseline to Constant Wealth as households increase demand for riskier assets.<sup>4</sup> This drives

<sup>4</sup>Note that each curve in Figure 6 takes into account the ultimate equilibrium effect on the expected return of the other asset.



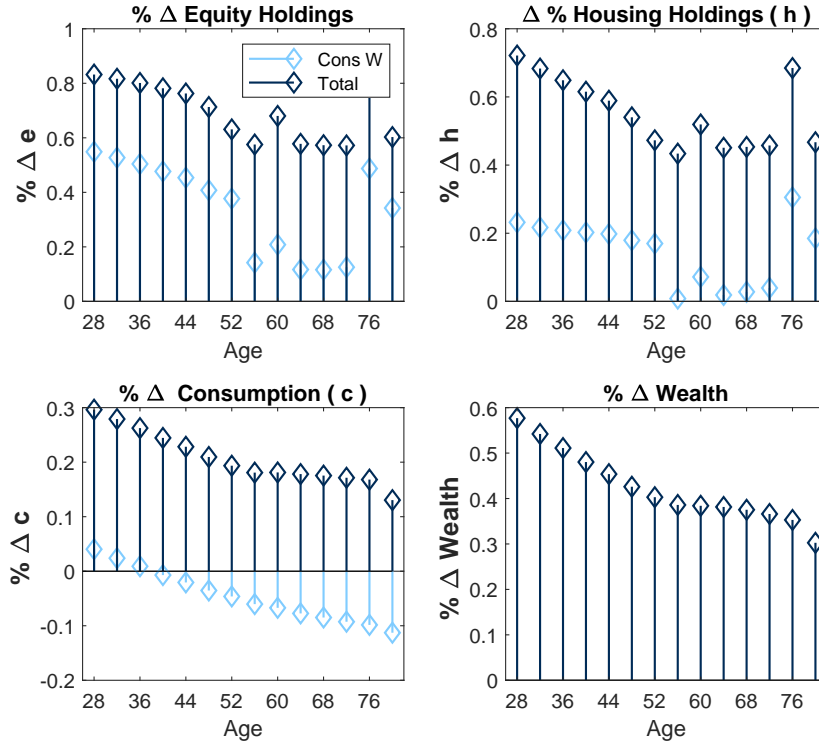
**Figure 6.** Demand and Supply of Risky Assets

up the increase in asset prices and so wealth. As a result, it induces a positive wealth effect on demand, which is reflected in the outward shift in the demand curve from Constant Wealth to Post Shock in Figure 6. The intersection of the supply and Post Shock demand curve determines the new equilibrium asset prices.

#### *D. Overall Effect on Consumption and Portfolio Choice*

Now that we have understood the effect on asset prices, we finally turn to the general equilibrium effects on portfolio choice and consumption. Results are shown in Figure 7 above.

Given the change in asset prices induced by the monetary policy shock, we find that, in equilibrium, the risk premia on risky assets rise. In particular, for a 21bp shock to the return



**Figure 7.** Household Total Portfolio Rebalancing

on bonds, equity and housing risk premia increase by 1.2bps and 0.2bps, respectively.

Intuitively, in response to a lower targeted real return on bonds, households rebalance toward the risky assets, equity and housing, and away from bonds. Consequently, they now hold more risk in their portfolio. However, households need to be compensated for bearing more risk in equilibrium, hence the risk premium rises.

The top two quadrants of Figure 7 demonstrate the percentage change in risky asset holdings holding wealth constant (in light blue) and incorporating wealth effects (in dark blue). Because of the increase in the risk premia on risky assets, households rebalance in favour of housing and equity across all cohorts. This effect is somewhat more pronounced for the young as their willingness to take risk is higher. However, once we incorporate the wealth effects induced by higher asset prices, reflected in the bottom right quadrant, holdings rise significantly for all cohorts, although the heterogeneity in asset holding responses remains.

Finally, and most importantly, we turn to consumption responses demonstrated in the bottom left quadrant of Figure 7. Absent wealth effects (in light blue), older cohorts reduce consumption while younger cohorts raise consumption slightly. Intuitively, all cohorts experience a negative income effect from declining expected returns, and so consume less. This can



be offset by the benefits of cheaper borrowing rates, which only young cohorts benefit from in equilibrium. But once we incorporate positive wealth effects, consumption rises for all cohorts (in dark blue), while the heterogeneity remains.

It is quite clear, then, that the percentage change in consumption is higher for the younger cohorts partly because they benefit relatively more from higher risk premia. This suggests that the marginal propensity to bear risk is heterogeneous across age. This then begs an additional question: is age heterogeneity important for movements in risk premia in response to MPSs?

To answer this question, we consider an alternative framework that differs from our baseline by abstracting from age heterogeneity. We then analyse the extent to which the movement in the risk premium differs from our Baseline, which will shed light on the importance of age heterogeneity.

## VII. Monetary Policy and Risk Premia

We wish to understand the role of heterogeneity by age for the transmission of monetary policy shocks on asset prices and risk premia. To do so, we compare our results to a representative agent model, featuring an infinitely lived household with a survival probability  $p$ <sup>5</sup>.

Households exhibit Epstein-Zin utility functions defined over the single homogeneous consumption good,  $C_t$ . Let  $W_t$  denote wealth at time  $t$ . Household preferences can then be written as:

$$V_t = \left[ C_t^{1-\frac{1}{\sigma}} + \beta E_t [pV_{t+1}^{1-\gamma} + (1-p)\phi_B^{1-\gamma}W_{t+1}^{1-\gamma}]^{\frac{1-\frac{1}{\sigma}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\sigma}}} \quad (40)$$

As in the heterogeneous case, we have a bequest motive, whose strength is controlled by  $\phi_B$ . When the agent dies, the flow utility is:

$$V_t \equiv \phi_B W_t \quad (41)$$

The labor income process is described by Equation 3-5, but we now set  $f(a) = 1$  (i.e. we drop the deterministic age profile). The representative household still experiences idiosyncratic income shocks.

The complete optimization problem is then:

$$\max_{\{\theta_{i,t}^E, \theta_{i,t}^H, B_{i,t}^+, B_{i,t}^-, C_{i,t}\}_{t=0}^{\infty}} E[V_0]$$

---

<sup>5</sup> $p$  is chosen such that life expectancy conditional on being aged 24 is the same as that in the data

$V_0$  is given by equations 40 and 41 and is subject to the constraints given by equations 9-15, the modified stochastic labour income process given by 3-5, and the same structure of beliefs on future asset returns as the heterogeneous case.

### A. Calibration

We re-calibrate the representative agent problem to target the same aggregate moments as in the heterogeneous agents case. We use the same non-calibrated parameters, and calibrate the remaining model parameters  $(\gamma, \sigma, \phi_B, sp, s_{idio}^H, s_{idio}^E)$  to match the following aggregate outcomes:

1. Wealth/GDP
2. Housing/GDP
3. Equity/GDP
4. Net Borrowing/GDP

Notice that, compared to the heterogeneous agents case, we cannot separately target gross lending and gross borrowing, as we only have one agent that either borrows or lends. For this reason, we only target the Net Borrowing/GDP moment. The estimated parameters are:

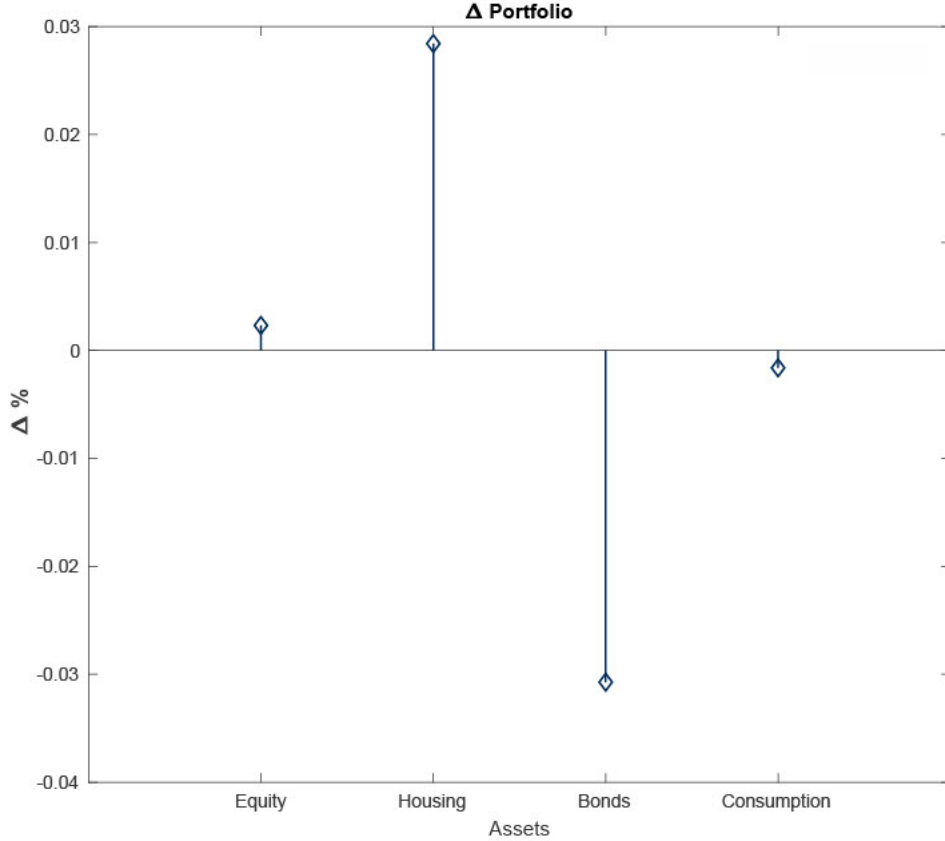
	$\gamma$	$\sigma$	$\sigma_{idio}^H$	$\sigma_{idio}^E$	sp	$\phi_{beq}$
parameters	7.00	0.30	9.00	4.00	0.03	1.00

**Table VII.** Calibrated Parameters Rep

	Wealth/GDP	Housing/GDP	Equity/GDP	Bonds/GDP
data	1.72	1.09	0.18	-0.11
model	1.70	1.08	0.18	-0.10

**Table VIII.** Aggregate Outcomes: Model vs. Data Rep

As you can see, we choose the parameters to match the following empirical outcomes closely. The main difference vs. the heterogeneous agent calibration is that risk aversion,  $\gamma$ , declines, coupled with a rise in the idiosyncratic variance of the risky assets. Details of the calibrations are listed in Appendix D.



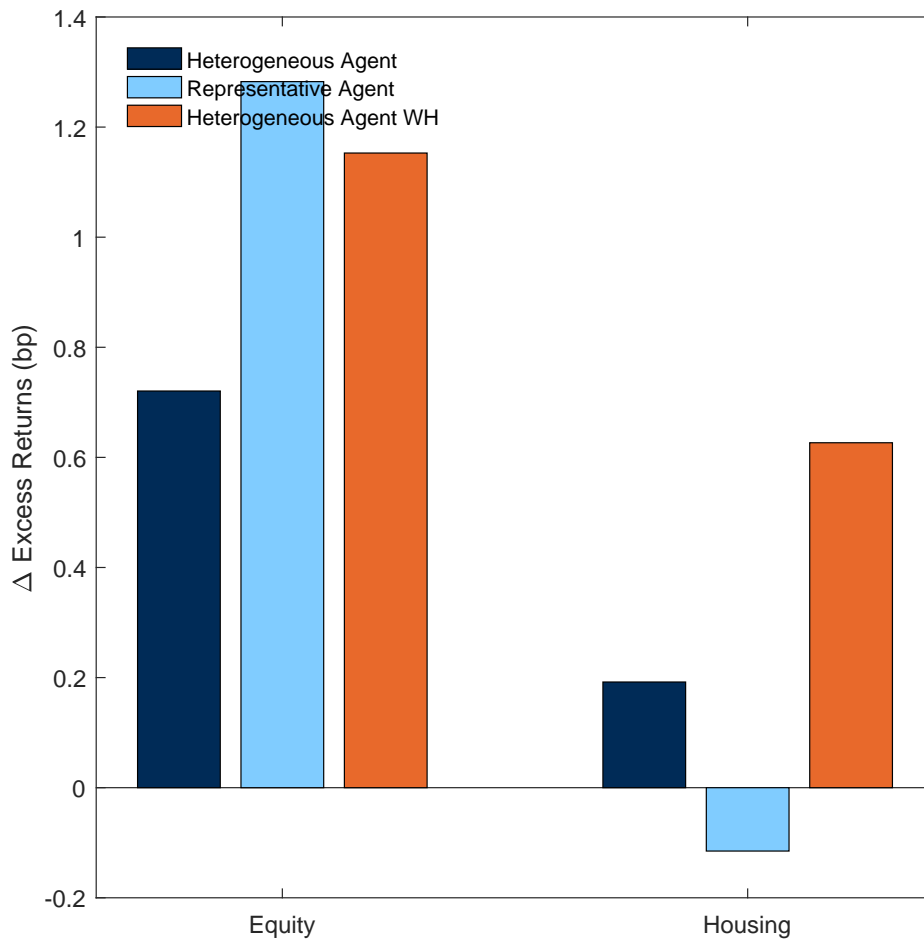
**Figure 8.** Households portfolio rebalancing

### *B. Monetary Policy Shock*

We then simulate a monetary policy shock under the same assumptions detailed in Section VI. In this case, the change in the expected return on bonds is again going to spur a portfolio re-balancing. Figure 8 plots the change in the portfolio of the three assets by the representative agent. In this case, we find also that the households will borrow more in order to buy more units of housing. The change in equity and consumption is small.

As in the heterogeneous agents case, the shock induces an outward shift in the demand curve for both assets, due to the combination of a portfolio rebalancing effect, and an additional wealth effect caused by rising asset prices. This change in asset prices will determine the equilibrium effect on risk premia, as outlined in Figure 9 below.

Figure 9 demonstrates the change in risk premia, comparing the outcomes with heterogeneous agents (in dark blue) vs. the representative agent (in light blue). What appears to happen is that, relative to the heterogeneous agent case, the risk premia of equity rises by more in the representative agent model, and vice versa for housing. Intuitively, this is because, in



**Figure 9.** Change in Risk Premia

order to rationalise high housing shares in the representative setting, the Sharpe ratio of housing is made more attractive than equity because we no longer have this young cohort that is highly attracted to housing. However, the flip side of this is that, when rates now fall, housing demand rises by even more because of this superior relative Sharpe ratio. As a result, the young cohort's high propensity to take risk mitigates housing's superior risk-return profile in the baseline. This then causes housing risk premia to rise, not fall, in response to monetary policy shock as cohorts rebalance by less.

## VIII. Conclusion

In this paper, we attempt to understand the role of the household portfolio rebalancing channel for the aggregate and redistributive effects of QE shocks. We introduce a heterogeneous household life-cycle model with multiple assets and combine it with an incomplete markets asset-pricing framework. After using both micro and macro-level data to derive initial household endowments, we calibrate the model to the data by targeting key empirical aggregates. We then find that, in response to a reduction in safe asset supply, households rebalance portfolios towards riskier assets due to a rise in their risk premia in equilibrium, thereby increasing asset prices and so household wealth.

In our model, the positive wealth effect on consumption is offset by an increase in the saving margin induced by the overall reduction in expected return on household portfolio. However, the strength of these two forces notably varies depending on households age. We find that, absent wealth effects, older cohorts reduce consumption while younger cohorts increase their consumption only slightly. The positive wealth-effect increases the consumption response for all cohorts: it strengthens the positive consumption response of the young and more than offsets the reduction in consumption of the old. Nevertheless, the heterogeneity in responses remain the same: the young raise consumption by more than the old.

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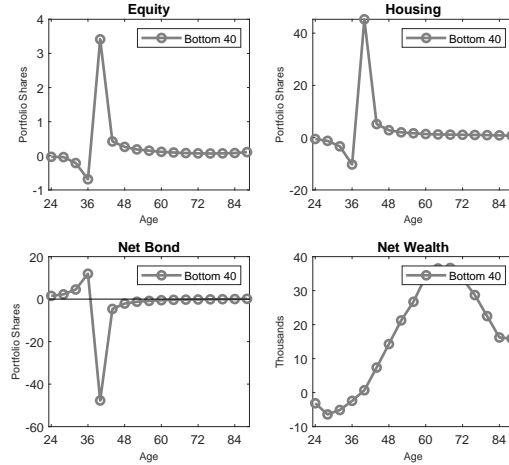
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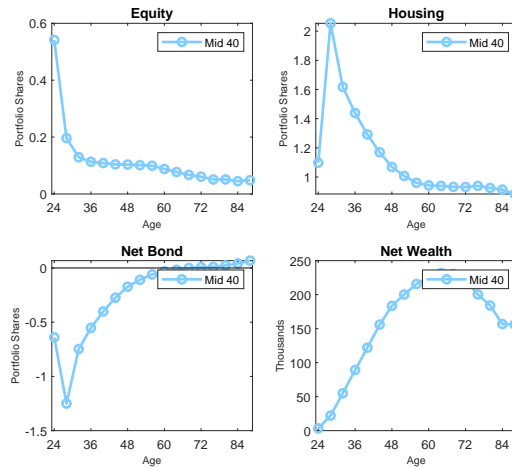


## A. Portfolio Shares Sub-cohorts

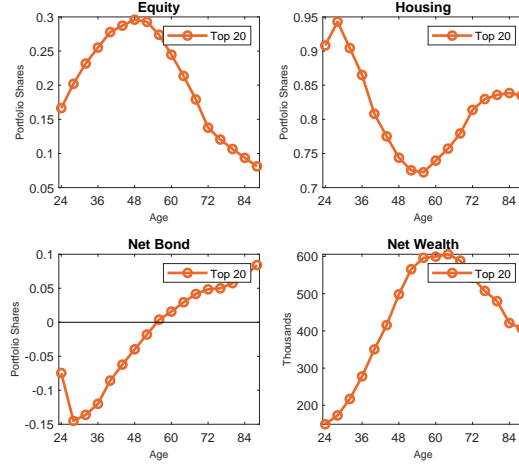
Figure 11 - 12 show the portfolio shares for the bottom 40 percentile, the mid 40 percentile and the top 20.



**Figure 10.** Bottom 40: Average portfolio shares and net wealth of households by cohort in 2014.



**Figure 11.** Mid 40: Average portfolio shares and net wealth of households by cohort in 2014.



**Figure 12.** Top 20: average portfolio shares and net wealth of households by cohort in 2014.

## B. Labour Income Process

Following [Gourinchas and Parker \(2002\)](#), the labour income process for household  $i$  is given by

$$Y_{i,t} = G_t P_{i,t} U_{i,t} \quad (42)$$

$$P_{i,t} = \exp(f(a)) P_{i,t-1} N_{i,t} \quad (43)$$

$$G_t = \exp(\mu^G) \exp(\epsilon_t^G) G_{t-1} \quad (44)$$

To estimate  $f(a)$ , we take advantage of the HFCS household-level income data. Households within cohort  $a$  vary in income through their idiosyncratic permanent and temporary components. We calculate the average income within each cohort:

$$\frac{1}{n_a} \sum_{i \in a} Y_{i,t} = G_t \frac{1}{n_a} \sum_{i \in a} P_{i,t} U_{i,t} \quad (45)$$

Assuming we have a large number of households within each cohort and knowing that  $\ln U_{i,t}$  and  $\ln N_{i,t}$  are independent and iid normal with mean  $\{-0.5 * \sigma_u^2, -0.5 * \sigma_n^2\}$  and variances  $\sigma_u^2$  and  $\sigma_n^2$ , respectively, then by law of large numbers:

$$\text{plim} \frac{1}{n_a} \sum_{i \in a} Y_{i,t} = G_t e^{\sum_{j=1}^a f(j)} P_0 \quad (46)$$

If we also estimate the average income for cohort  $a - 1$  we obtain:

$$\text{plim} \frac{1}{n_{a-1}} \sum_{i \in a-1} Y_{i,t} = G_t e^{\sum_{j=1}^{a-1} f(j)} P_0 \quad (47)$$

Applying these results, we then know that:

$$\text{plim} \left( \log \left( \frac{1}{n_a} \sum_{i \in a} Y_{i,t} \right) - \log \left( \frac{1}{n_{a-1}} \sum_{i \in a-1} Y_{i,t} \right) \right) = \log \left( \text{plim} \frac{1}{n_a} \sum_{i \in a} Y_{i,t} \right) - \log \left( \text{plim} \frac{1}{n_{a-1}} \sum_{i \in a-1} Y_{i,t} \right) \quad (48)$$

$$= f(a) \quad (49)$$

Consequently,  $f(a)$  can be consistently estimated by the log difference in mean income between cohorts  $a$  and  $a - 1$ .

### C. Value Function Normalization

$$V_{i,a,t}(W_{i,t}, P_{i,t}) = \left[ C_{i,t}^{1-\frac{1}{\sigma}} + \beta E_t \left[ p_a V_{i,a+1,t+1}^{1-\gamma}(W_{i,t+1}, P_{i,t+1}) + (1-p_a) \phi_B^{1-\gamma} W_{i,t+1}^{1-\gamma} \right]^{\frac{1-\frac{1}{\sigma}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\sigma}}} \quad (50)$$

To show that this can be normalized, we guess and verify that  $V_t(W_{i,t}, P_{i,t}) = \hat{V}_t(\omega_{i,t}) P_{i,t}$ . The verification argument involves conveying that the only state variable necessary is  $\omega_{i,t}$ . Plugging in the guess, we have that:

$$\hat{V}_{i,a,t}(\omega_{i,t}) = \left[ c_{i,t}^{1-\frac{1}{\sigma}} + \beta E_t \left[ p_a \hat{V}_{i,a+1,t+1}^{1-\gamma}(\omega_{i,t+1}) \left( \frac{P_{i,t+1}}{P_{i,t}} \right)^{1-\gamma} + (1-p_a) \phi_B^{1-\gamma} \omega_{i,t+1}^{1-\gamma} \left( \frac{P_{i,t+1}}{P_{i,t}} \right)^{1-\gamma} \right]^{\frac{1-\frac{1}{\sigma}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\sigma}}}$$

where

- $\omega_{i,t+1} = (e_{i,t} R_{t+1}^E + h_{i,t} R_{t+1}^H + b_{i,t}^+ R_{t+1}^B - b_{i,t}^- (R_{t+1}^B + sp)) \left( \frac{P_{i,t+1}}{P_{i,t}} \right) + \frac{Y_{i,t+1}}{P_{i,t+1}}$
- $\frac{P_{i,t+1}}{P_{i,t}} = \exp(f(a+1)) N_{i,t+1}$
- $\frac{Y_{i,t+1}}{P_{i,t+1}} = G_t \exp(\mu^G) \exp(\epsilon_{t+1}^G) U_{i,t+1}$

where lower-cased policy functions are choices relative to the permanent component of income,  $P_{i,t}$ . As  $\ln N_{i,t+1}$ ,  $\ln U_{i,t+1}$  and  $\epsilon_{t+1}^G$  are independent and i.i.d. normal, then conditional on knowing  $\omega_{i,t}$ , no additional variable is informative.

## D. Representative Agent

Details of the calibration of the representative agent here.

## E. Appendix: Historical Returns

We use historical data to estimate the mean and variance of different assets. Below we detail for each asset class the series we use and the sample.

**Bond:** We use quarterly data on the Eonia 3m OIS yield downloaded from Bloomberg (ticker: EUSWEC Curncy).

**Equity:** We use quarterly data on the The MSCI EMU Index (ticker: GDDLEMU Index). The index captures large and mid cap representation across the 10 Developed Markets countries in the EMU. With 246 constituents, the index covers approximately 85% of the free float-adjusted market capitalization of the EMU.

**Housing:** We use quarterly data on the Residential Property Prices for Euro area. We download the data from Fred (ticker: QXMN628BIS). We use data on price-to-rent index from the OECD to estimate the rental yield evolution. However, in order to transform the index in level terms, we calibrate the time series to match the average rental yield estimated from the HFCS in 2014: 4%. We then use historical price index together with the rental yield series to construct a total return index for housing.

**Inflation:** We use the quarterly Euro area consumer price index. We download data from Bloomberg (ticker: EUHICPI Index)

We transform the bond, housing and equity series into a real index, using the Euro area price index. Our sample runs from Q1-1996 through Q4-2018 for all the series. The only exception is the bond series, which only starts in Q1-2000.

We use the quarterly log-change for all the series. We then calculate the mean and variance of real returns for bonds, housing and equity.

## F. Appendix: Data Collection

Each variable is equal to the sum of the corresponding listed items.

*Income:*

- DI1200 Self-employment income
  - Sum of PG0210 Gross self employment income (profit/losses of unincorporated enterprises) for household members

- DI1200 Self-employment income
- DI1500 Income from pensions
- DI1600 Regular social transfers (except pensions)

*Consumption:*

- HI0100 amount spent on food at home
- HI0200 amount spent on food outside home
- HI0210 amount spent on utilities
- HI0220 amount spent on consumer goods and services

*Equity:*

- DA2104 Value of non self-employment private business
  - HD1010 value of non-selfemployment not publicly traded businesses
- DA2105 Shares, publicly traded
  - HD1510 value of publicly traded share
- DA1140 Value of self-employment businesses

*Unlisted Equity:*

- DA2104 Value of non self-employment private business
- DA1140 Value of self-employment businesses

Listed Equity:

- DA2105 Shares, publicly traded

*Housing:*

- DA1110 Value of household's main residence.  $DA1110 = HB0500 * HB0900$ 
  - HB0500 % of ownership of household main residence
  - HB0900 current price of household main residence
- DA1120 Value of other real estate property.  $DA1120 = \text{Sum of } (HB270x * HB280x) + HB2900$ 
  - HB270\$x other property \$x: % of the property belonging to household

- HB280 other property \$x: current value
- HB2900 additional properties current value

*Bonds:*

- DA2103 Bonds
  - HD1420 market value of bond

*Deposits:*

- DA2101 Deposits.  $DA2101 = HD1110 + HD1210$ 
  - HD1110 value of sight accounts
  - HD1210 value of saving accounts

*Debt:*

- DL1000 Total outstanding balance of household's liabilities.  $DL1000 = DL1100 + DL1200$ .
  - DL1100 Outstanding balance of mortgage debt.  $DL1100 = DL1110 + DL1120$ 
    - \* DL1110 Outstanding balance of HMR mortgages.  $DL1110 = \text{Sum of } (HB170x) + HB2100$ 
      - HB170 other property mortgage \$x: amount still owed
      - HB2100 money still owed on additional HMR loans
    - \* DL1120 Outstanding balance of mortgages on other properties.  $DL1120 = \text{Sum of } (HB370x) + HB4100$ 
      - HB370 other property mortgage \$x: amount still owed
      - HB4100 money still owed on additional other property loans
  - DL1200 Outstanding balance of other, non-mortgage debt.  $DL1200 = HC0220 + HC0320 + \text{Sum of } (HC080x) + HC1100$ 
    - \* HC0220 amount of outstanding credit line/overdraft balance
    - \* HC0320 amount of outstanding credit cards balance
    - \* HC080 other property mortgage \$x: outstanding balance of loan
    - \* HC1100 total amount owed for additional non-collateralised loans

*Pensions:*

- Occupational pension plan non-defined benefit
  - Sum of PF0710 current value of all occupational pension plans that have an account.

- Select only non-defined benefit pension (PF0800 = 2). PF0800 occupational plan has regular benefit in retirement (1 means yes, 2 means no).
- Voluntary pension.
  - Sum of PF0920 voluntary pension schemes - value of accounts
- Social security plans
  - Sum of PF0510 current value of all social security plans that have an account

*Mutual Funds:*

- DA2102 Mutual funds, total. DA2102=HD1320g OR sum of(HD1320a-f)
  - HD1320x market value of mutual funds.
    - \* a - Funds predominantly investing in equity
    - \* b - Funds predominantly investing in bonds
    - \* c - Funds predominantly investing in money market instruments
    - \* d - Funds predominantly investing in real estate
    - \* e - Hedge funds
    - \* f - Other fund types (specify)
    - \* g - Aggregate amount of all funds together