Counting Concepts

Cleo Condoravdi Dick Crouch Martin van den Berg
Xerox PARC FXPAL

Abstract: Singular indefinite NPs in prevention statements give rise to an ambiguity between a general and a specific reading. In order to account for this ambiguity, we extend Zimmermann's proposal for referential concept NPs to also allow for quantificational concept NPs. We further motivate the need for quantificational concept NPs on the basis of the interpretation of indefinite NPs with numeral determiners in prevention contexts. We treat numerals as generalized determiners quantifying over concepts and propose a way of counting concepts by counting maximally specific instantiated concepts. □

In [2] we argued that the object argument of *prevent* must be concept denoting rather than individual denoting. The singular indefinite NP *a strike* in (1) existentially quantifies over sub-concepts of the concept Strike, including the concept Strike itself.

(1) Negotiations prevented a strike.

(1) entails that Strike or some sub-concept of it is uninstantiated in the actual world, where negotiations occur. But in a counterfactual world, identical apart from the absence of negotiations, the concept is instantiated.

This paper discusses how this analysis can be extended to deal with concept denoting indefinite NPs with numeral determiners, as in (2).

(2) Negotiations prevented three strikes.

If the NP *three strikes* quantifies over concepts, what is counted by the numeral *three*? We argue that *three* corresponds to a generalized determiner counting appropriately identified sub-concepts of Strike. More direct attempts to count the number of individuals instantiating Strike fail to predict the correct entailments for (2).

Section 1 reviews some of the entailment patterns of sentences like (1). Section 2 points out the difficulties these patterns pose for standard treatments of intensional verbs, and reviews the arguments in [2] that taking NPs to quantify over concepts rather than individuals accounts for these patterns. Section 3 discusses various ways of analyzing plural concept-denoting NPs, and explains why some of the more obvious possibilities fail to do justice to examples like (2).

1 General and Specific Prevention

Perhaps the most striking feature of (1) is its negative existential entailment: the prevented strike does not come into existence. Less immediately obvious is that (1) is ambiguous between a general reading (no strikes occurred, paraphrasable as *Negotiations prevented any strike*) and a specific reading (some particular potential strike did not occur). This ambiguity is brought out more clearly by

(3) Safety procedures at Chernobyl prevented a serious accident.

As a general statement (3) is notoriously false. But as a specific statement (3) might well be true: perhaps there were other occasions when the safety procedures really did prevent an accident.[1]

General and specific interpretations of prevent statement also license distinct monotonicity entailments.

[1]Non-indefinite NPs are restricted to a specific reading. For instance, *Negotiations prevented every strike/most strikes* implies that there is a set of particular potential strikes quantified over.

(4) a. Negotiations prevented a (specific) long strike. $\not\dashv \models$
Negotiations prevented a (specific) strike.

b. Negotiations prevented a (any) long strike. $\not\dashv \not\models$
Negotiations prevented a (any) strike.

Under the specific interpretation (4a), *prevent* is upward monotone in its second argument: if negotiations prevented some particular long strike, then, clearly, they prevented some particular strike. Under the general interpretation (4b), however, *prevent* is neither upward nor downward monotone. Upward monotonicity fails as follows: even if negotiations prevented any long strikes, short ones may still have occurred. Downward monotonicity fails in a subtler way. Preventing any strikes clearly entails that no long strikes occur. But this does not necessarily mean that any long strikes have been prevented: maybe none of the prevented strikes would have been long to begin with.

Specific prevention is thus upward monotone in its factual and its counterfactual entailments. General prevention is downward monotone in its factual entailments but upward monotone in its counterfactual entailments, and hence neither upward nor downward monotone overall.

2 Prevention and Quantification over Concepts

Montague's classical account of intensional verbs like *want* does not properly account for existence entailments and general/specific readings when extended to verbs like *prevent*[2]:

(5) a. General, *de dicto*:
$\mathsf{prevent}(n,^\wedge[\lambda P.\exists x.\ \mathsf{strike}(x) \land P(x)])$ no existential commitment

b. Specific, *de re*:
$\exists x.\ \mathsf{strike}(x) \land \mathsf{prevent}(n,^\wedge[\lambda P.P(x)]))$ existential commitment

The general, *de dicto* reading (5a) is too weak for a predicate like *prevent*: *prevent* requires an entailment of non-existence rather than a mere non-entailment of existence. This can be rectified by a meaning postulate stating that the set of properties serving as the second argument must be empty in the world of evaluation. However, this does not help with the specific, *de re* reading (5b), which carries an (undesired) entailment of existence, regardless of any meaning postulates. Indeed the proposed postulate for *prevent* makes the specific reading self-contradictory: how can an individual sublimation be empty? Put another way, Montague's analysis ensures that all opaque verbs carry existential commitment under specific, *de re* readings, regardless of any lexico-semantic differences between the verbs.

A possible fix that can quickly be dismissed (dismissed at greater length in [2]) would be for quantifiers to range over possible rather than actual individuals. The *de re* reading would then merely entail the presence of a possible individual rather than the actual existence of a real individual. However, technical problems aside, the identity criteria for (un-named) possible individuals are extremely murky.

The core difficulty with Montague's analysis is that while *prevent* takes a concept as an argument, NPs quantify over / refer to individuals. Zimmermann [4] argues for an ambiguity between (i) individual NPs, which quantify over or refer to individuals, and (ii) concept NPs, which are non-quantificational, referential NPs denoting concepts. General readings arise from concept denoting NPs interpreted

[2]The second argument to *prevent* is a concept, i.e., the intension of a set of properties. The first is some constant n referring to a particular set of negotiations.

in situ (6a). Specific readings arise from an quantificational, individual NPs taking scope over the opaque predicate, plus type raising of the individual denoting variable to an individual concept (6b).

(6) a. General: $\mathsf{prevent}(n, \mathsf{Strike})$ no existential commitment

b. Specific: $\exists x.\ \mathsf{strike}(x) \wedge \mathsf{prevent}(n, ^\wedge \lambda y.y = x)$ existential commitment

However, Zimmermann's specific reading suffers exactly the same problem as Montague's: an existential commitment that arises from logical form rather than lexical entailment.

2.1 Singular Reference to and Quantification over Concepts

In [2] we extended Zimmermann's referential concept NPs to also allow quantificational concept NPs. The object of *prevent* is uniformly a concept NP, with a distinction between an NP referring to the nominal concept (general reading) or quantifying over sub-concepts of it (specific reading). This leads to the following (here somewhat simplified) analyses of (1):

(7) a. General (referential):
$\exists Y.Y \sqsubseteq \mathsf{Negotiation}\ \wedge\ \mathsf{prevent}(Y, \mathsf{Strike})$

b. Specific (quantificational):
$\exists X.X \sqsubseteq \mathsf{Strike}\ \wedge\ \exists Y.Y \sqsubseteq \mathsf{Negotiation}\ \wedge\ \mathsf{prevent}(Y, X)$

Here Strike and $\mathsf{Negotiation}$ refer to the concept of strike-events and negotiations respectively, and $\sqsubseteq$ means 'sub-concept'.

To spell out the entailments of (7a,b) we need to say something about the lexical entailments of the *prevent* predicate. Informally, $\mathsf{prevent}(Y, X)$ is true if the following two conditions hold. (i) In the actual world, w_a, the preventor concept Y has an instance but the preventee concept X has no instance; for example, there is an instance of a negotiation and no instance of a strike. (ii) There is a counterfactual world w_c as similar as possible to w_a apart from not having an instance of the preventor Y[3], and in w_c there *is* an instance of X; for example, in the counterfactual world there are no instances of a negotiation but there is an instance of a strike. Stating this more formally

(8) $w_a \models \mathsf{prevent}(Y, X)$ iff
$\mathsf{inst}(w_a, Y)$, $\neg\mathsf{inst}(w_a, X)$, and for all w_c $\neg\mathsf{inst}(w_c, Y)$, $\mathsf{inst}(w_c, X)$

where $\mathsf{inst}(w, C)$ is true if the concept C has a non-null extension in world w, and where we leave the precise relation between w_a and w_c open.

Given that an instance of a specific concept also counts as an instance of a concept that generalizes it:

(9) $\forall X, Y, w.\ (X \sqsubseteq Y\ \wedge\ \mathsf{inst}(w, X)) \rightarrow \mathsf{inst}(w, Y)$

and abbreviating the concepts of *strike* and *long strike* as S and LS, and where $LS \sqsubseteq S$, we obtain the following factual and counterfactual entailments:

[3]For the purposes of this paper we fortunately do not need to commit ourselves on the vexed issue of how to identify the most similar counterfactual world in which there is no instance of Y, or indeed whether there is a unique most similar world.

General	Specific
Prevent long strike $\not\models_a$ Prevent strike $\neg\mathsf{inst}(w_a, LS) \not\models \neg\mathsf{inst}(w_a, S)$ Prevent long strike $\models_c$ Prevent strike $\mathsf{inst}(w_c, LS) \models \mathsf{inst}(w_c, S)$	Prevent long strike $\models_a$ Prevent strike $\exists X.X \sqsubseteq LS \wedge \neg\mathsf{inst}(w_a, X) \models \exists X.X \sqsubseteq S \wedge \neg\mathsf{inst}(w_a, X)$ Prevent long strike $\models_c$ Prevent strike $\exists X.X \sqsubseteq LS \wedge \mathsf{inst}(w_c, X) \models \exists X.X \sqsubseteq S \wedge \mathsf{inst}(w_c, X)$
Prevent strike $\models_a$ Prevent long strike $\neg\mathsf{inst}(w_a, S) \models \neg\mathsf{inst}(w_a, LS)$ Prevent strike $\not\models_c$ Prevent long strike $\mathsf{inst}(w_c, S) \not\models \mathsf{inst}(w_c, LS)$	Prevent strike $\not\models_a$ Prevent long strike $\exists X.X \sqsubseteq S \wedge \neg\mathsf{inst}(w_a, X) \not\models \exists X.X \sqsubseteq LS \wedge \neg\mathsf{inst}(w_a, X)$ Prevent strike $\not\models_c$ Prevent long strike $\exists X.X \sqsubseteq S \wedge \mathsf{inst}(w_c, X) \not\models \exists X.X \sqsubseteq LS \wedge \mathsf{inst}(w_c, X)$

This correctly predicts the monotonicity properties of general and specific prevention discussed in section 1. Specific prevention is compatible with the existence of other strikes, whereas general prevention says there were no strikes at all.[4]

3 Plural Concept NPs

We have argued here, and at greater length in [2], that the object NP of the verb *prevent* must either refer to or quantify over concepts, rather than individuals. We now turn to the interpretation of plural sentences like (2). At issue is integrating our treatment of NPs as quantifiying over concepts and sub-concepts with a treatment of plural quantifiers ([3], [1]). In this section we consider three broad approaches to plural quantification which involve, respectively: (i) counting the number of objects instantiating a concept, (ii) employing plural and singular concepts, and (iii) counting the number of (sub)concepts. Only the last option, where the NP is a generalized quantifier over concepts, gives sensible results when applied to (2).

3.1 Counting Instances

One possible way of capturing the cardinality of the concept NP *three strikes* in (2) would be to say that the concept / subconcept has three instances. This might lead to NP meanings along the lines of

(10) a. $\lambda P.\ \#\mathsf{inst}(\mathsf{Strike}, w, 3)\ \wedge\ P(\mathsf{Strike})$

b. $\lambda P.\ \exists X.\ [X \sqsubseteq \mathsf{Strike}\ \wedge\ \#\mathsf{inst}(X, w, 3)]\ \wedge\ P(X)$

where $\#\mathsf{inst}(X, w, 3)$ says that the number of individuals instantiating the concept X is world w is (at least) three.

It is important to relativize the counting of concept instances to a world. As we have already seen, the same concept can have different instantiations in different worlds. However, this relativization to worlds poses severe problems for sentences like (2). We might represent one reading of (2) as

(11) $\exists X.\ [X \sqsubseteq \mathsf{Strike}\ \wedge\ \#\mathsf{inst}(X, w, 3)]\ \wedge\ \mathsf{prevent}(\mathsf{Negotiation}, X)$

Which world should w be? In the actual world, the concept X should have no objects instantiating it. It is only in the counterfactual world implicit in *prevent* that X has three instances. But (i) how does the numeral quantification obtain access to this implicitly defined counterfactual world? And (ii) how can the numeral

[4] [2] discusses various ways in which the general assertion that there are no strikes at all can be relativized to mean no strikes meeting a contextually salient description of relevant possible strikes.

quantification also be set up to entail that there are zero instances in the actual world, while apparently it quantifies over three strikes?

The root of the problem is that the instantiation (or otherwise) of a concept is the result of the predicates that apply to the concept. In the case of *prevent*, the predication leads to different instantiation claims for different worlds. But the proposed treatment of cardinal NPs also makes instantiation part of the meaning of the quantifier over concepts. This is wrong. The NP itself should not be making any existence / instantiation claims; only the predicates that apply to it should.

3.2 Plural Concepts

The previous approach took counting instances to be part of the NP quantifier, and this led to problems. Suppose instead that counting instances is incorporated into the concepts over which the NP quantifies. For example, suppose that we take the numeral 3 to be an adjective, so that the NP *three strikes* might be represented as

(12) a. $\lambda P.\ P(3 \sqcap \mathsf{Strike})$

b. $\lambda P.\ \exists X.\ X \sqsubseteq 3 \sqcap \mathsf{Strike}\ \wedge\ P(X)$

where $3 \sqcap \mathsf{Strike}$ is the combination of the concept of a strike and the concept of a threesome. Instances of this concept would be groups of (at least) three things, all of which are strikes.

We are now in the improved position where *prevent* takes a concept of three-strikes and evaluates it at different worlds. In the actual world the concept is uninstantiated: there are not three strikes. In the counterfactual world the concept is instantiated: there are three strikes.

Unfortunately, this analysis makes implausible predictions. Sentence (2) is interpreted to claim (generally) that the concept *three strikes* is uninstantiated in the actual world, or that (specifically) some sub-concept of *three strikes* is uninstantiated. What does it mean for such concepts to be uninstantiated? It means that there are two or fewer strikes. This is clearly wrong: under a general interpretation of (2), there should be no strikes in the actual world. Similarly under a specific interpretation there should be no instances of strikes meeting the more restricted sub-concept. In both cases there should be no instances, not just less than three. Put another way, this analysis treats (2) as synonymous with

(13) Negotiations prevented there being three strikes.

but the two sentences differ in meaning.

3.3 Counting Concepts

Suppose that we treat *three* as a generalized determiner quantifying over concepts:

(14) $3(\lambda X.X \sqsubseteq \mathsf{Strike}, \lambda X.\mathsf{Prevent}(X))$

In this case we must be careful about the way that we count concepts. Concepts are ordered by specificity in a way that individuals are not, and this needs to be taken into account when counting concepts. Monotonicity of instantiation (9) means that one individual can lead to the instantiation of multiple concepts. Simply counting all instantiated concepts will not lead to a count of the instantiating individuals. For an accurate count of individuals we need to count maximally specific instantiated concepts. The difficult part is to define what is meant by a maximally specific instantiated concept.

Assuming that the determiner is conservative, (14) is equivalent to (15).

(15) $3(\lambda X.X \sqsubseteq \mathsf{Strike}, \lambda X.X \sqsubseteq \mathsf{Strike}\ \wedge\ \mathsf{Prevent}(X))$

Now we must define a way of counting that will retrieve three maximally specific concepts from the set of prevented strikes, $\lambda X.X \sqsubseteq \mathsf{Strike}\ \wedge\ \mathsf{Prevent}(X)$.

Let's start with a simple case first. Suppose there are only two worlds: a world w_0, in which there are no strikes, and a world w_c in which there are exactly three strikes (a, b, c). The set $\lambda X.X \sqsubseteq \mathsf{Strike}\ \wedge\ \mathsf{Prevent}(X)$ is a partially ordered set of concepts, all of which are instantiated by one or more of the individuals a, b or c in w_c. If a, b and c are distinct individuals, then there have to be incomparable concepts that distinguish them. Two concepts are incomparable iff they do not share any sub-concepts. This means that we can always find a triple of incomparable concepts A B and C (any three that distinguish a, b and c) but not a quadruple of incomparable concepts. Since there are only three strikes, two of the four concepts must be comparable, as they will apply to the same strike. We can thus count the number of individual strikes by finding the maximum n for which there is an n-tuple of incomparable concepts in $\lambda X.X \sqsubseteq \mathsf{Strike}\ \wedge\ \mathsf{Prevent}(X)$.

The general case is slightly more complicated. We find the maximum n for which there is an n-tuple of incomparable concepts in $\lambda X.X \sqsubseteq \mathsf{Strike}\ \wedge\ \mathsf{Prevent}(X)$. But we have to be careful what it means for two concepts X, Y to be incomparable. We know that X, Y are instantiated in every alternative world. And because they are from a maximal set, they will be instatiated by only one individual. But we have to ensure that there is no world where X and Y are instantiated by the same individual. We can do this by demanding that there is no sub-concept of strike, prevented or not, that is more specific than both X and Y. In particular, if X and Y were instantiated by the same individual a in a world w, the concept instantiated by a in w, and uninstantiated anywhere else would be more specific than X and Y. This leads to the following way of counting.

Let $\mathsf{xc}_{\mathcal{X}}(X_1, X_2, \ldots, X_n)$ mean that every X_i, X_j is mutually exclusive in the sense that there is no concept in $\mathcal{X}$ that is more specific than both X_i and X_j.

(16) $3(\mathcal{X}, \mathcal{Y}) \iff \exists X, Y, Z \in \mathcal{Y}\ .\ \mathsf{xc}_{\mathcal{X}}(X, Y, Z) \wedge \neg\exists X, Y, Z, P \in \mathcal{Y}\ .\ \mathsf{xc}_{\mathcal{X}}(X, Y, Z, P)$

This meaning for the generalized quantifier 3 results in the right reading for (14). The maximal set of mutually exclusive concepts that are strike-concepts and satisfy the conditions of $\mathsf{Prevent}$ has a cardinality 3, which means that in all accessible worlds, they are instantiated by 3 strikes.[5]

References

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[3] van der Does, J. *Applied Quantifier Logics: Collectives & Naked Infinitives.* PhD thesis, University of Amsterdam, 1992.

[4] Zimmermann, E. On the proper treatment of opacity in certain verbs. *Natural Language Semantics* 1, 149–179, 1993.

[5]There may still be worlds with more strikes, but at least some worlds should have exactly three.