Abstract—We conduct a literature survey of three papers applying game theory to study incentives in cooperative networks. The papers study, respectively, the Stackelberg equilibrium of a scheduling game between an access point and user nodes; price-based and reputation mechanisms; and virtual MIMO coalitions between service providers, all in pursuit of incentivizing cooperation. We discuss the two main, yet distinct, motivations for applying game theory: understanding incentives and designing distributed algorithms. For each of the three papers, we examine the classes of networks for which each approach is applicable, the assumptions and metrics of performance and fairness used in each, and their potential impact on protocol design. We identify some avenues for further exploration of each concept.

I. INTRODUCTION

It is known that wireless networks can achieve better performance when users cooperate with each other to transmit and receive data, than when they compete for access to the same resources. Cooperation can take a number of forms, including relaying and virtual MIMO with techniques such as beamforming. By doing this, they can save power or increase their throughput.

There has, therefore, been a growing body of research to understand the mechanics of cooperation, how it affects capacity and how it can be incorporated into future wireless networks. This brings both opportunities and problems. On one hand, as well as performance gains, cooperation between peers can be implemented using distributed algorithms, leading to new paradigms that can overcome the scaling limitations of infrastructure-based networks.

On the other hand, cooperation requires the willingness of participating nodes. In some contexts, this can be assumed: if the same organization is responsible for an entire network, all nodes can be programmed to cooperate. However, in networks with independent users, each with their own self-interests, there is potential for some nodes to be designed to maximize their own throughput at the expense of their trusting colleagues. It is therefore imperative to study the interaction between nodes as they cooperate—or refuse to cooperate, leading to suboptimal performance.

Game theory has gained attention as a tool to meet both challenges. Game theory provides an analytical framework to study how rational, self-interested actors make strategic decisions when interacting with each other. Traditionally a subject of economics, it has also been applied to political science, psychology and biology. Cooperative wireless networks have both benefited from this body of knowledge, and will need to extend the theory to cover its own needs.

In this project, we undertook a literature survey of applications of game theory in cooperative networks. Our focus was primarily on three papers, chosen for their representation of different network structures.

The first paper, by Canzian, Badia and Zorzi [1], proposes a scheduling framework to incentivize cooperation in a multiuser network with a single access point. They model the situation as a Stackelberg game, with the access point choosing its policy first and the others acting second, and they derive the equilibrium for the game.

The second, by Li and Shen [2], considers price-based and reputation mechanisms for cooperative networks. After showing flaws in those schemes, they propose a new system that integrates both price and reputation to incentivize cooperation.

The third, by Yerramalli, Jain and Mitra [3], considers virtual MIMO coalitions between base stations and femtocells transmitting to a mobile user. They study the conditions under which the grand coalition, that of all transmitters, forms. They show that it does in most scenarios that they consider, which include two receiver types, both sum-power and per-power constraints and both high and low SNR.

The questions we are interested in are:

- In what classes of networks is each approach applicable?
- What knowledge (e.g., channel states, other nodes) is required by players for each approach to work?
- What metrics are used in each approach, and to what extent do they allow comparisons between networks analyzed with different approaches?
- How could the authors’ findings potentially affect system and protocol design?
- What further work would need to be done to make this idea practicable?

The three primary papers focus on the traditional application of game theory, to analyze incentives and strategies between players. Since many researchers have also deployed game theory towards finding distributed algorithms, we also provide a brief discussion of this area.

The rest of this report is organized as follows. In Section II, we discuss the motivations for using game theory in wireless networks in general. In Section III, we summarize the main papers considered in this survey and take note of their network structures and underlying assumptions. In Section IV, we compare and contrast the papers with reference to questions we proposed. Section V discusses the potential impact on system design of the papers, identify the questions left open by the authors and place them in a broader context. Section VI provides a brief treatment of the use of game theory for distributed algorithms. Finally, we conclude in Section VII.
II. Game Theory in Wireless Networks

We begin with an overview of how game theory is applied in wireless networks, to provide some context for our analysis that follows.

A. Motivations for application

Traditionally at home in economics and political science, game theory has gained attention in wireless network research mostly in the last decade. The motivations for using game theory in wireless networks differ between works and do not always follow the conventional aims of game theory. Broadly speaking, we can divide them into two groups.

The first group concerns incentives, in what MacKenzie and DaSilva [4] call a “direct application of game theory”. In this group, researchers seek to understand how transmitters and receivers in the network interact as they make strategic decisions in their interests, knowing that their peers are also acting in theirs.

Since they are interested in understanding behavior, the system models proposed in these works are attempt to model reality. They may consider, for example, the conditions under which nodes do and don’t co-operate, how to incentivize certain behaviors, or the difference between the equilibrium and the optimal operating point. They typically assume nodes seek to optimize for some performance metric, e.g. their own data rate or power consumption, and define their utility functions accordingly.

The second group is motivated by finding distributed algorithms. Here, researchers take advantage of the fact that game theory studies situations in which many actors make independent decisions. With the growing demand in wireless networks, there is a need to develop network management schemes that can scale well to networks with many devices. Distributed algorithms are a promising approach for the large-scale networks [5, p. 6], and the results of game theory can be usefully applied towards this goal.

In [4], MacKenzie and DaSilva characterize this more generally as an “engineering” application of game theory”, which they note assumes that “the engineer is capable of programming the devices in the system to behave as if they are maximizing a chosen utility function” (emphasis in original). This contrasts with the effort to model reality in incentives-motivated works: here, the system designer dictates what a node is trying to optimize for. She could even program the devices to be altruistic, as the authors did in [6]—a sensible decision for design, but a foolish assumption if we were modeling self-interested nodes.

MacKenzie and DaSilva presented these groups as two “philosophies” that are “mutually exclusive”. In practice, they are not so distinct. Oftentimes, work of a descriptive nature is a stepping stone to defining algorithms that incentivize cooperation. Moreover, in multi-stage games, the equilibria of sub-games are often incorporated into the strategies of other players, and this is only practical if there exists an efficient algorithm to calculate those equilibria. This was the case, for example, in the Stackelberg scheduling policy proposed in [1].

On the other hand, distributed algorithms rely on nodes’ adherence to the protocol. In networks fully controlled by one organization, this can be assumed; indeed, in those cases, methods of optimization without game theory will suffice. In networks involving devices manufactured by different organizations, however—for example, large-scale consumer networks—there is a need to ensure that whatever distributed algorithms are proposed are robust to tactics that might be employed by self-interested nodes.

Nonetheless, at their core, works tend to be motivated by one or the other. The first approach is more aligned with traditional uses of game theory: understanding strategies and incentives. The second approach has been driven more by engineering researchers more recently, particularly wireless researchers. As Lasaulce and Tembine remarked in 2011 [7]:

For a long time, game theory was used quite marginally and more like an analysis tool in communication problems. With recent technological progress and the arrival of new wireless paradigms, the era of using game theory for design has come.

There has been a lot of research that uses game theory for designing distributed algorithms, and we provide a brief treatment of some of these works in Section VI. However, the papers that form the primary basis of this survey all fall into the incentives-based category.

B. Branches of game theory

Techniques in game theory can largely be divided into two groups: noncooperative and cooperative. The latter is also known as coalitional game theory. Both have been widely used in the context of wireless networks.

Noncooperative game theory is the study of how individual players (in our case, nodes) in a game (using the network) make strategic decisions. Each player chooses from a set of strategies, and does so to maximize its utility, taking into account what it knows about the other players. Despite its name, noncooperative game theory is a useful tool for studying cooperation: the available strategies to nodes are to help out or to refuse (or some combination of the two), and they choose the one that best suits them. We will see that this is applicable in modeling relay decisions.

Coalitional game theory studies how players form alliances. They are still, ultimately, trying to maximize utility. However, this branch considers coalitions of players to behave like a single unit with respect to other coalitions, and in some constructions of games they are allowed to transfer utility within a coalition. The goal is to understand which coalitions will form and which will be stable. We will see that this is applicable with joint operations such as virtual MIMO, and elsewhere if agreements are enforceable.

The two branches are distinct, but in many works, authors have used a combination of both to understand the network. Of the primary papers in this survey, [1] and [2] are based on noncooperative game theory, and [3] is based on coalitional game theory.
TABLE I
Characteristics of main papers in this survey

<table>
<thead>
<tr>
<th>Network structure</th>
<th>Incentivization strategy</th>
<th>Utility function</th>
<th>Strategy decision</th>
<th>Game theory applied</th>
<th>Main result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Access point and multiple users</td>
<td>Scheduling priority</td>
<td>Increasing in data rate, decreasing in power consumption</td>
<td>Users: Whether to co-operate with each other user: Access point: scheduling reward weights</td>
<td>Non-cooperative multi-follower Stackelberg game</td>
<td>Existence of Stackelberg equilibrium</td>
</tr>
<tr>
<td>Ad-hoc, single routing path considered</td>
<td>Virtual currency and reputation values</td>
<td>Based on fixed rewards and prices of sending and forwarding</td>
<td>Co-operate with everyone or no-one</td>
<td>Non-cooperative two-player game</td>
<td>Integration of reputation and price better than each individually</td>
</tr>
<tr>
<td>Relies on superadditive property of MIMO</td>
<td>Maximum achievable rate</td>
<td>Which coalition to join (if any)</td>
<td>Canonical coalitional game</td>
<td>Grand coalition stable in low and high SNR with SUD receiver, and in low SNR with SIC receiver</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 1. Scenario in Stackelberg scheduling network by Canzian et al. [1]

III. SUMMARY OF PAPERS

We now turn to a summary of the three works primarily concerned in this survey. A comparison of the core characteristics of the networks studied in each paper is in Table I.

A. Stackelberg scheduling

Canzian et al. [1] consider a network with many user nodes and an access point (or, equivalently in this context, base station), focusing on the uplink. This is shown in Figure 1. Only one user may transmit to the access point at a time. The access point schedules which user may transmit each in time slot. The users are at varying distances from the centrally-placed access point, so some users have better channels to the access point than others, and those users are sometimes asked to relay message from user nodes further away. The authors study how scheduling can be adjusted to create incentives for user nodes with good channels to relay for those with poor channels.

For simplicity, the authors consider just one relay hop from the origin to the destination. The access point acts as a central arbiter, rewarding nodes that forward others’ packets with a higher access probability for some number of time slots after the cooperation occurs. The access point’s strategy is to choose coefficients for computing this probability change for each pair of nodes, in order to maximize overall network performance. Because it announces its scheduling policy before nodes transmit, the interactions can be modeled as a Stackelberg game, in which a leader moves first and the other players follow. This means the leader can take its followers’ strategies into account when optimizing its strategy, as the access point does in this work.

Rather than adopt a utility function of any particular form, the authors present results for general user utility functions with assumptions as necessary to complete the analysis. Specifically, they assume that the utility of each ith user $\psi_i(\Delta B_i, \Delta E_i)$ is continuous, increasing in data rate $B_i$ and decreasing in power consumption $E_i$, and where $\Delta B_i$ and $\Delta E_i$ represent the variations in those quantities relative to the no-cooperation case, each $\psi_i$ can be expressed as a sum of contributions from each other user $\psi_{ij}(\Delta B_{ij}, \Delta E_{ij})$, that is,

$$\psi_i(\Delta B_i, \Delta E_i) = \sum_{j \in \mathcal{U} \setminus \{i\}} \psi_{ij}(\Delta B_{ij}, \Delta E_{ij}),$$  \hspace{1cm} (1)$$

where $\mathcal{U}$ is the set of all users. They also do not specify exactly how the access point measures “network performance”: they merely assert that it should be a function of the utilities of all of the users, and that that function should be upper semi-continuous. This generalization makes their results more powerful than adopting particular utility measures like the achievable data rate itself.

Canzian et al. show that the sub-game between users, given a particular access point strategy, has a unique Nash equilibrium, that the game overall has at least one Stackelberg equilibrium, and that all Stackelberg equilibria have equivalent network performance.

They compare simulation results of this system, which they call “voluntary cooperation”, to a “no cooperation” scheme and a “forced cooperation” scheme where users are required to cooperate if the access point deems it optimal. They consider both the throughputs of users and their utility. They find that the users with the weakest channels gain significantly in throughput from voluntary cooperation, but not by as much.
as forced cooperation. The users with strong channels that are also positioned well as relays also gain in throughput with voluntary cooperation, because of the rewards they earn, unlike forced cooperation. However, since those with stronger channels also use more power, in terms of utility only those with weaker channels stand to gain.

They use the Jain index (discussed in Section IV-C) to compare how “fair” the different schemes are. They find that fairness in utility increases very significantly in voluntary cooperation over both forced and no cooperation. They do note, however, that fairness in throughput is worse with voluntary cooperation. They defend this by pointing out that, although inequality between users might have increased, no-one is worse off in throughput than they were in the no cooperation case. They also argue that, because forced cooperation does not operate in a Nash equilibrium, it is not stable game-theoretically and cannot be relied on if users are rational.

B. Reputation and price incentives

Li and Shen [2] consider a single routing path in a mobile ad-hoc network (MANET), a class of distributed wireless networks. This is shown in Figure 2. Because nodes in the network have a short transmission range, successful delivery of a packet requires every node in the path to cooperate by relaying the packet on to the next node. The authors study how reputation and price schemes can be used to incentivize nodes to cooperate, and propose an “integrated” system that uses both reputation and price to achieve this.

Reputation and price systems have been widely studied. In a reputation system, each node maintains “reputation” ratings for each other node, and bases its decision on whether to cooperate with another node (i.e., forward its packet) on its rating for that node. Naturally, one’s reputation increases when it cooperates, and decreases when it refuses. In a price-based system, nodes maintain accounts of a virtual currency that is exchanged whenever one relay forwards another’s packet; an insufficient account value means a node cannot ask for its packet to be forwarded.

First, Li and Shen break the relay path into a series of two-player sub-games between adjacent nodes, such that each node is part of two sub-games. They show that in order for every node to cooperate, it is necessary to show that in every subgame, it is a Nash equilibrium and Pareto optimal for both nodes to cooperate. They then proceed to analyze the routing path as a set of independent two-player games with payoffs that are sums of fixed costs and rewards. They consider three systems: a reputation system, a price-based system and an “integrated” system that uses both reputation and prices.

In their reputation system, nodes increase reputation values by a fixed amount if the relay node forwards a packet, and decrease them by a (possibly different) fixed amount if it doesn’t. Nodes forward packets if and only if the source node’s reputation is above a threshold. Because this is a binary choice, the authors observe that nodes will strategically maintain their reputation values just over the threshold. Because nodes drop packets when far in excess of the threshold, this system does not always encourage cooperation.

In the “price-based system”, nodes maintain virtual currency accounts, buy and sell forwarding services from other nodes. The reward earned and price paid are assumed to be constant throughout the network (i.e., not a market price) and not necessarily equal to each other (i.e., not strictly an exchange of funds). They expose a few flaws in this system: it cannot detect selfish and wealthy nodes and it advantages nodes in high-traffic regions. Like the reputation system, because there is a binary choice, nodes can strategically maintain their account value just above zero.

Finally, they propose a new “integrated system”, which is a combination of the above two systems. However, they introduce a variable price: here, the price a node pays for its packet to be forward is inversely proportional to its reputation, \( m_p / R_i \), where \( R_i \) is the reputation of the \( i \)th node and \( m_p \) is a constant. (The reward stays the same.)

They argue that, because each of price and reputation is capable of catching the shortfalls of the other, their integrated system can avoid the abovementioned problems of both. For example, a selfish wealthy node would eventually fall to its dropping reputation. However, their argument for why the integrated system would prevent nodes from gaming their reputation to be near the threshold relies on the new definition of forwarding price. Specifically, there is an incentive to maintain a large \( R_i \) to make \( m_p / R_i \) small. While it is true that \( R_i \) could not be incorporated into the forwarding price without reputation integration, the authors did not consider other ways of making the payoffs in the reputation and price-based systems graduated rather than binary. It is not clear whether the improvement is a consequence of the fact of integration, or because there is now graduated pricing, or some combination of the two.
C. Coalitions for virtual MIMO

Yerramalli et al. [3] consider a situation where there are several multiple-antenna base stations and femtocells, operated by competing service providers, in range of a multiple-antenna mobile user. This is shown in Figure 3. They consider the downlink, studying the circumstances in which the competing service providers would cooperate by forming coalitions to make a virtual MIMO link. Every player is assumed to wish to maximize its achievable rate.

In doing so, the authors use the framework of canonical coalitional games, as classified by Saad et al. [8]. The main characteristic of canonical games is that it is never detrimental to any player to cooperate.

In a virtual MIMO situation, assuming that cooperation itself entails no costs, a larger coalition always achieves a higher total rate than the sum of smaller ones, and is therefore preferred. Therefore, assuming that payoffs can be shared to within a coalition make no-one worse off than they would be outside the coalition, the grand coalition (of all nodes) is optimal. The authors study the conditions required for the grand coalition to be able to stabilized.

Although they ignore cooperation costs, they account for interference between coalitions in their utility function, which means they frame their system as a partition form game, in which the utility of each coalition depends also on the structure of other coalitions, rather than the analytically-simpler characteristic form game, in which they do not. They then use non-cooperative theory to derive utility functions in terms of the coalition structure, and use those to examine the stability of the grand coalition using the framework in [8].

They consider two receivers: the single user decoding (SUD) receiver and the successive interference cancellation (SIC) receiver. In the latter, unlike the former, the receiver decodes the received signals in some order, iteratively sub-

tracting the re-encoded signals from the received signal [9]. The decoding order for the SIC receiver is predetermined and announced by the receiver. Because showing stability is a difficult problem, the authors consider the low SNR case, i.e. with noise $N_0 \rightarrow \infty$, and the high SNR case, $N_0 \rightarrow 0$.

Yerramalli et al. show that the grand coalition can be stabilized in both low and high SNR regimes when an SUD receiver is used, but only under low SNR when an SIC receiver is used. In high SNR with an SIC receiver, the grand coalition is unstable in general, because of the asymmetry introduced by the fixed decoding order. However, they show that by time-sharing between decoding orders, the system can make the grand coalition stable in this regime. They also note that, even where the grand coalition is not naturally stable, a receiver can stabilize it by penalizing nodes for leaving the coalition by a sufficient amount, say, by refusing to decode certain signals.

As a remark, in canonical coalitional games such as this one, unlike the noncooperative approaches in [1], [2], there is no real question of suboptimality. It is a premise of this class of games that cooperation is always beneficial. Therefore, by showing that the grand coalition is stable under certain conditions, they showed that the optimal arrangement is achieved.

IV. Discussion

A. Classes of networks

The approaches taken by the three works each have characteristics that make them suitable for the game-theoretic approaches taken. In other words, their methods are not applicable in general. It is then natural to ask for what classes of networks each approach is suitable.

The Stackelberg game used by Canzian et al. [1] assumes that one of the players in the game is a leader, i.e. they get to move first. In general, this will be true whenever there is a node that can set some resource allocation policy. Canzian et al. examined scheduling, but similar analyses could be applied in principle to, say, dynamically-allocated frequency permissions. This is generally possible in networks with infrastructure. It need not be limited to the uplink; similar ideas could be applied to the downlink.

In contrast, the asymmetry of the game means that Stackelberg games could not be used in a distributed network where all nodes are peers, such as a wireless sensor network.

The authors assume time-invariant channels, which implies that users are stationary. If, for example, users are mobile, this may change the impact of a scheduling reward or penalty on their utility: a reward that is valuable now may be less so when they move further away. Nonetheless, the basic idea of the authors would still be extensible to situations with time-varying channels.

Li and Shen’s contribution [2], like price and reputation systems generally, in principle suit any situation where nodes can be thought of as providing “services” to each other (perhaps in subtle tension with the notion of “cooperation”).

1 The authors do not explicitly argue that this is also sufficient, but it is implicit in their argument.

2 They also consider a “defenseless” system that uses no incentive scheme, but that is not material to this report.

3 To show that the grand coalition is stable, one can use the Bondareva-Shapley theorem; according to Yerramalli et al., showing satisfiability of this theorem is co-NP-complete.
whether there is infrastructure or not. This is most commonly done for cooperative relay networks (for example [10], [11]).

Reputation systems (unlike price systems) can also apply anywhere where trust between nodes is important. This can extend to virtual MIMO systems, but in a different manner. There, nodes might be interested in the likelihood that other nodes will break their contracts, and may refuse to enter virtual MIMO agreements with nodes that do so. There has been little (if any) literature on this topic. Cooperation in virtual MIMO is a more recent topic than relay networks and, to date, analyses such as [3] have been concerned mainly with coalition formation, assuming that coalitions would be honored.

What [1] and [2] have in common is that they both rely on non-cooperative game theory, in which the strategy sets of players are all independent of the others’. When a node decides whether to relay the packet, there is no natural immediate benefit from the exercise. It must do so trusting that the favor will be returned at some point. That is, with relaying, cooperation is reciprocal, not concurrent. There is no inherent mutual trust—even with reputation systems, that trust is earned—and the incentive schemes are designed with that in mind. This contrasts with our third case study.

In the system studied by Yerramalli et al. [3], the authors use coalitional game theory. The premise of coalitional games is that players will commit to coalitions if the outcome of the coalition is better for them. There is, in the first instance, no assessment of the risk that a player might commit and (for example) waste resources as other players withdraw or deceive their new partners.

This is suitable for virtual MIMO systems because, by their nature, they jointly encode and decode signals. If any one node does not fulfill their role, then in general, the entire coalition collapses and no-one can recover data, so there is no incentive to do so.

Furthermore, in MIMO systems, the achievable rate of a coalition is often greater, and never less, than the sum of the individual links. This means that the grand coalition offers the best total payoff. Because the authors assume transferable utility (that payoffs within a coalition can be distributed arbitrarily among its members), they can apply the canonical coalitional games proposed in [8]. Still, other classes of coalitional games (also in [8]) provide tools to analyze coalitions where these conditions do not hold. The basic idea of applying coalitional games to virtual MIMO systems still holds.

Another requirement, common to all virtual MIMO systems, is that there must be some other link between the cooperating nodes. In the present case, this is in the form of backhaul links between the service providers. To be practical, the links between cooperating nodes must be much faster than the main wireless links. So, for example, it may also apply to (fully) wireless nodes that are close to each other, cooperating to form a MIMO link to another set of nodes that are further away.

Coalitional game theory can be applied to relay networks, as Canzian et al. did in a different paper [12]. That work, however, required an assumption that nodes in a coalition would uphold their agreements. To model incentives fully, therefore, it needs to be supplemented with non-cooperative theory to study the incentives for nodes to renge on their agreements. Coalitional game theory applies more naturally when cooperation is concurrent.

### B. Knowledge assumed

In general, games are more likely to be analytically tractable if there is full knowledge of the network. The analyses in these three works reflect that. The knowledge their algorithms assume is summarized in Table II.

The coalitional game of Yerramalli et al. requires perfect channel knowledge for the entire network because it accounts for interference, and knowledge of the interfering channels is required to calculate achievable rates.

The scheduling mechanism of Canzian et al. requires probabilities of successful reception for its own links to other nodes, as well as some links to which it is not party, like those between nodes that will forward its packets and the access point. Not all links in the network are material, but local channel knowledge is not sufficient. The algorithm proposed also depends on modulation schemes and channel SNRs (through reception probability). It requires each node to know which others will agree to cooperate with it, though these can be inferred from their other knowledge of the network (since that is how nodes calculate their decisions themselves).

The system proposed by Li and Shen does not require any such exchange of knowledge, but that is only because all nodes are assumed to be identical in cost and reward of sending a packet. This simplifies the analysis greatly, but also limits the applicability of the result, since in any real network the nodes will not be face the same channel states, costs and rewards.

### C. Metrics

The metrics used by authors to assess their proposals fall into three groups: (1) network performance, (2) fairness and (3) degree of cooperation. Which ones authors chose offers some insight into their priorities.

<table>
<thead>
<tr>
<th>Knowledge of</th>
<th>Stackelberg scheduling</th>
<th>Price and reputation incentives</th>
<th>Coalitions for virtual MIMO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel state</td>
<td>Most of network</td>
<td>Complete, since all nodes seek to maximize achievable rate, which depends on channel states</td>
<td></td>
</tr>
<tr>
<td>Others nodes’ preferences</td>
<td>Access point has full knowledge; nodes have knowledge of others’ cooperation strategies and modulation schemes</td>
<td>Not relevant, since all nodes are identical</td>
<td></td>
</tr>
<tr>
<td>Past behavior</td>
<td>Access point keeps finite memory</td>
<td>Price and reputation serve as states</td>
<td>None</td>
</tr>
</tbody>
</table>

As examples of these three kinds of metrics, Canzian, Badia and Zorzi [1] assume knowledge of others’ operation strategies and modulation schemes. Li and Shen [2] assume a MIMO link to another set of nodes that are further away and reputation systems. Yerramalli, Jain and Mitra [3] assume full knowledge of all network conditions. Most of network in the table reflects that. The knowledge their algorithms assume is summarized in Table II.
1) Network performance: Canzian et al. assess the asymptotic throughput both for each user and for the network under each of their three regimes (voluntary, forced and no cooperation). They also examine the utility of each user.

While not directly a performance metric, from a game-theoretic perspective, utility is what is important to the user nodes. As a single performance measure may not capture the whole story, utility can be a means of reconciling different aspects of network performance, e.g., data rate, power consumption and latency, in a manner in accordance with the actual priorities of possibly heterogeneous nodes. Still, because different system models make different assumptions (for good reason) about utility, utility fulfillment cannot in general be used to compare different proposals.

Li and Shen use a different measure: they show how the packet drop rate varies with different cost–reward ratios. They do not present results for throughput, but since with their simulation, packets are all the same size and are sent at a constant rate, packet drop rate relates directly to throughput. Although their simulation randomly chooses nodes, the analysis is fundamentally concerned with a single routing path, so they are not as interested in comparing the performance seen by each user as Canzian et al. were.

It may seem intuitive that a throughput-like metric should be generally useful in some sense, but Yerramalli et al. do not present results for throughput, but since with their simulation, packets are all the same size and are sent at a constant rate, packet drop rate relates directly to throughput. Although their simulation randomly chooses nodes, the analysis is fundamentally concerned with a single routing path, so they are not as interested in comparing the performance seen by each user as Canzian et al. were.

2) Fairness: In most situations, performance metrics such as throughput, outage probability and packet drop rate are of prime importance to the network. However, in multiuser networks, the equity of resource allocation between users is also of concern. To assess this, Canzian et al. use the Jain index, a measure of computer network fairness proposed in 1984 [13]. This is defined for \( n \) users, where the \( i \)th user is allocated a quantity \( x_i \) of a shared resource, as

\[
 f(x) = \frac{\left( \sum_{i=1}^{n} x_i \right)^2}{n \sum_{i=1}^{n} x_i^2} \tag{2}
\]

They compute this index for both throughput and utility. However, throughput fairness is worse for voluntary cooperation than forced or no cooperation, and they note that this alone is an incomplete measure: it does not take into account the associated power consumption. Their results for utility fairness are much more promising.

In conventional game theory, utility is a better measure. It makes sense for energy-limited devices like in sensor networks, where this trade-off is strong. In consumer networks, throughput may be of greater importance, but this can still be accounted for in the utility function. However, as discussed in Section IV-C, because utility is specifically defined for each analysis, it does not allow for comparisons between proposals.

It is interesting to note that, of the three works in this survey, only one of them had any explicit concern for fairness.

The setup proposed by Li and Shen had no real concept of fairness—either there exists a complete routing path, or there doesn’t, and in the latter case no data is transferred so fairness is somewhat moot.

The work of Yerramalli et al. is not primarily about fairness, but they note briefly that there are uncountably many possible payoff divisions within the coalition, all of which are stable. For characteristic-form games, the Shapley value [8] gives a unique solution to some suitable fairness axioms, but the virtual MIMO game is a partition-form game. The Shapley value was therefore not directly applicable to their work. Nonetheless, since most coalitional games in the literature are in characteristic form, we provide a discussion here to compare this fairness to the Jain index. It will help to first explain what it is in formal terms.

**Theorem 1 (Shapley value):** Let \( \mathcal{N} = \{1, \ldots, N\} \) be the set of all players in a coalitional game in which payoffs can be transferred within a coalition, let \( v(S) \in \mathbb{R} \) be the total payoff available to a coalition \( S \subseteq \mathcal{N} \), known as the characteristic function. There exists a unique allocation of payoffs to each player in the coalition \( \phi(v) = [\phi_1(v), \ldots, \phi_n(v)] \), known as the Shapley value, that satisfies the following four axioms:

1. **Efficiency:** \( \sum_{i \in \mathcal{N}} \phi_i(v) = v(\mathcal{N}) \).
2. **Symmetry:** If \( i, j \) are two players such that \( v(S \cup i) = v(S \cup j) \) for every \( S \) not containing \( i \) or \( j \), then \( \phi_i(v) = \phi_j(v) \).
3. **Dummy:** If, for any player \( i \), \( v(S) = v(S \cup i) \forall S \subseteq \mathcal{N} \), then \( \phi_i = 0 \).
4. **Additivity:** If \( u, v \) are characteristic functions, then \( \phi(u + v) = \phi(u) + \phi(v) \).

This allocation is given, for each player \( i \), by

\[
\phi_i(v) = \sum_{S \in \mathcal{N} \setminus i} \frac{|S|!(N - |S| - 1)!}{N!} [v(S \cup i) - v(S)] \tag{3}
\]

While the Jain index could in principle be applied to any (hypothetical) virtual MIMO payoff allocation, the notions of fairness proposed by Shapley and Jain are very different. The Jain index in (2) is maximized when all nodes are allocated an equal amount of the resource, \( x_1 = \cdots = x_n \). The Shapley axioms, on the other hand, do not account for equality at all.

Because nodes would leave a coalition if they would be better off alone, and insisting on equality could bring about this situation, equality is not necessarily a desirable property of an intra-coalition allocation. Therefore, the Shapley value takes into account the additional value that each node brings to the coalition (in the last term of the summation), rather than merely dividing the resource equally. In an approach common in economics, the Shapley axioms specify only what is needed for the Shapley value to be sensible at a basic level, and allow the result to follow. While the Shapley axioms are not necessarily the only or best measure of fairness, they capture the observation that fairness in coalitional games is different from fairness in networks generally.

The Jain index, on the other hand, was proposed for users competing for a shared resource. It is an assumption that, in

\[4\] The authors note that a “suitable fairness metric optimized over the core” could work.
general, a gain for one will be a loss for another. This will often be the case—but not always, as we see with this example.

3) Degree of cooperation: The proportion of nodes that cooperate was the primary concern of Li and Shen. They showed that their integrated system is able to incentivize nodes to cooperate much more quickly than the price-based system, while in the reputation system, not all nodes will cooperate at any given time. Canzian et al. did not pay much attention to the degree of cooperation, and the result of the work of Yerramalli et al. was to ensure all nodes cooperate (as is the case in canonical coalitional games generally).

Although it made sense in this case, it is questionable whether the degree of cooperation is an appropriate metric for cooperative networks in general. Although cooperation is known to improve network performance, it is not necessarily the case that total cooperation is optimal. For example, in the network studied by Canzian et al., it is optimal for the node with the best SNR to resend a failed transmission itself rather than have it relayed by a worse-placed node.

4) Further remarks: The measures of success used by these three works reflect the differing focuses of their proposals. Cooperative networks have many objectives, of which each contribution focuses on a subset. Metrics chosen are relevant to the subproblem of cooperative networks at hand.

Nonetheless, cooperation in networks is not merely good for its own sake. It is important to assess how effective proposed mechanisms are in increasing cooperation, but this is ultimately a means to more generic objectives in wireless networks. It remains a challenge to build cooperation theory to a point where different proposals can be assessed using more general metrics.

Even then, there is often a trade-off between capacity (or resource use) and fairness that will need to be assessed. As we saw in canonical coalitional games, the latter is sometimes not a problem, but as assumptions are progressively relaxed, the trade-off will reappear.

V. POTENTIAL IMPACT AND FURTHER WORK

We now discuss the potential impact of these authors’ findings on system design, as well as future work necessary to bring the concept closer to fruition, with a view to the assumptions that the authors made in their analysis.

A. Using network resources as incentives

Canzian et al. show that, in cases where some central node controls a shared resource, it is possible and useful to use the allocation of network resources to incentivize cooperation. Most current wireless networks that fall into this category. The technique may also be applicable to “hybrid” networks that use both infrastructure and peer-to-peer modes, like the approach described in [14].

This may therefore be a promising approach for future-generation systems. However, as the authors observe, when a central arbiter is responsible for dividing a resource between competing users, it raises questions of fairness. Further study is required on what it means to be “fair” in cooperative network design. This question has been studied in computer network engineering and ideas could be adapted from there, as Canzian et al. did with the Jain index.

If successful, the ideas proposed would allow protocols to be designed without any charitable assumptions about user nodes. It is of significant advantage to have a system that is robust to users who try to manipulate the cooperation protocol. Since it is generally an objective of service providers to provide an acceptable service level to all clients, the approach taken by Canzian et al. is a promising concept.

Their analysis assumes time-invariant channels and that traffic is always backlogged. It does not take into account traffic patterns, which may affect the system significantly: a user is less likely to care about losing (or not gaining) access temporarily if it does not plan to transmit for that time anyway. Relaxing these assumptions is one area for future work; another is studying how to integrate notification of the scheduling changes into network control messages.

B. Using price and reputation systems

The contribution of Li and Zhen furthers the development of price and reputation incentive schemes. However, being a mechanism extrinsic to the network, it has some properties which may not be desirable.

Any price-based system based on virtual currency accounts requires the protocol to specify how to maintain it, and must also include safeguards against fraud. This may well be feasible in combination with research in network security, but it introduces an additional layer, and hence cost, to cooperation.

Reputation schemes also entail overhead to maintain, but have the advantage that it is not necessary for them to rely on accurate information from requesting nodes. (Some authors have proposed community-based reputation or measures that include third-party information, which increases overhead again, but offers more information that is considered trustworthy, for example, [15].)

It may be possible for nodes to have their own reputation metric rather than have measures specified by a protocol. Some developers may choose this approach if they believe they have a better prediction mechanism. In this case, a protocol need not necessarily be developed at all—relays can just choose to drop packets. There has, so far, been little (if any) work analyzing how this would work. Most literature to date has focused on schemes where reputation is measured similarly by all nodes in the network.

Nonetheless, despite its extrinsic nature, in ad-hoc networks with competing interests, such an artificial incentive mechanism may be the most reliable way of ensuring cooperation.

In this work, the authors make a number of assumptions to simplify their analysis. Most significantly, they assume that all nodes face the same costs and rewards, and that these are constant with time, which is highly unlikely for any real network. They also ignore any concept of message ownership: no distinction is made between the utility of sending a packet and forwarding another’s (after the decision to forward is made), whereas in practice the incentives for the former will often be higher than the latter. Furthermore, nodes make no distinction between different nodes: either they co-operate with everyone, or they co-operate with no-one.
Their work therefore provides some insight, but further work is needed to move the model towards resembling something more realistic.

C. Using virtual MIMO coalitions

Understanding which coalitions are naturally stable is an important step towards network models that use cooperation to improve performance. In particular, capturing interference between coalitions using partition-form games is a significant step forward in understanding wireless coalitional games.

However, there is much more to understand before these results become directly applicable. Not all situations will have the property that cooperation is always beneficial, particularly if the costs of cooperation are taken into account. The authors provided suggestions for what to do when the grand coalition is not stable, but relatively briefly and there is much more to study on this question. One of their proposed solutions would have the receiver refuse to decode some signal. While this would stabilize the coalition again, it may be suboptimal as it wastes resources.

To further understand MIMO coalitions in less ideal situations, the other frameworks proposed in [8] could be applied. These allow for the study of coalitions other than the grand coalition, and those where not all nodes in a coalition are in direct communication with each other. The latter might be useful, say, in a network where mobile nodes form a relay mini-network among themselves in order to form a MIMO coalition.

In situations where optimal coalitions do form naturally, all that is needed from compatibility standards is a format for exchanging information and negotiating agreements. Similarly to using network resources as incentives, the lack of extrinsic mechanism to ensure cooperation makes it promising. From the current literature, however, it is unlikely that natural cooperation is enough by itself.

D. Information and costs of co-operation

We note that none of the three studies accounted for costs of cooperation. These costs may include the direct overhead of cooperation and the difficulty in managing large coalitions, but they may also include impacts on competition, especially if (say) the cooperating base stations are operated by competing service providers. Incorporating these costs into cooperative network analysis remains a challenge.

On the other hand, we noted in Section IV-B that the three studies all assumed either complete or near-complete knowledge of the network (except where all nodes were identical). Relaxing this assumption would help make these models more realistic, especially if channel state information could be restricted to channels local to the node in question. We note that there has been some work involving incomplete information; for example, the authors in [16] use a Bayesian game to assess its effects in a co-operative diversity context.

To some extent these two possibilities are substitutes: running systems with incomplete information reduces the need for information exchange, and hence the cost of cooperation. Another aspect of imperfect information is incorporating the effects of imperfect channel estimation into system models, and understanding how uncertainty in the estimate can effect nodes’ strategies.

VI. DISTRIBUTED ALGORITHMS

This report has largely concerned analyses of incentives to cooperate in wireless networks. As we noted, another use of game theory is to devise distributed algorithms for managing networks. In this section, we provide an overview of some approaches using coalitional games. (This project does not address distributed algorithms using noncooperative games.)

In [17], Khayatian et al. study a network of many source nodes transmitting to a base station. The nodes also act as relays, and form coalitions to reduce their power consumption. Within each coalition, after every transmission, all the nodes in the coalition forward the packet to the base station.

The authors provide a distributed algorithm for finding stable coalitions. Nodes self-organize into coalitions using a merge rule based on the Shapley value: coalitions continue to merge until no node can improve its Shapley value without harming another’s. The algorithm must converge, since there are a finite number of nodes in the network. Its benefit is that it can be executed in a distributed manner, and so is easier to implement. Specifically, they note that a centralized coalition formation algorithm would run in exponential time, while the distributed algorithm runs in polynomial time. The cost is that the distributed algorithm is suboptimal. The authors do not address the stability of the coalition or the incentives of nodes.

In [6], Baidas and MacKenzie study “altruistic coalition formation”. Nodes form coalitions if the sum of rates of all the nodes when in the coalition, would be greater than the sum of rates of all the nodes when not in the coalition. They propose doing this using the merge-split rule, where coalitions merge or split if the action would benefit the sum of rates of all nodes in the transaction. Note that a node will join a coalition even if it its own achievable rate is lower, if the coalition as a whole is better off.

Because there are a finite number of coalition combinations, the merge-split algorithm converges. This can be run in a distributed manner: coalitions propose to each other to merge or split, and agree on a decision between them, rather than liaising through a central base station.

Clearly, self-interested nodes would not be acting in this manner. The authors defined the utility function to promote cooperation, not to reflect the true utility of each node. This is, therefore, an example of the “engineering applications” MacKenzie and DaSilva wrote about in [4].

In [14], Akkarajitsakul et al. point out a need for a cooperative structure that is stable with selfish nodes. They use the merge-split rule in their system to find a stable coalitional structure.

However, the merge-split algorithm does not always yield a unique result [18]. The incentives of nodes during the distributed merge-split algorithm—e.g. whether they will follow

5Specifically, it yields a unique result if and only if certain conditions are met, as shown in [18].
it—have not been studied and it is not obvious whether they align with the algorithm itself. What is clear is that, once the algorithm is complete, no node will have an incentive to leave or merge from the solution. A key benefit, however, is the fact that the algorithm can be distributed. We therefore see an application with objectives in both encouraging cooperation (to some extent) and providing an efficient, distributed means of achieving it.

The point of presenting these works is that applications of game theory in cooperative networks are not always, and indeed often are not, motivated by an understanding of the incentives of nodes, which is the traditional application game theory. In these works, researchers take advantage of algorithms arising from game theory for a non-traditional “engineering” purpose: to design distributed algorithms for network management.

VII. CONCLUSIONS

Game theory has been used in cooperative networks with two main motivations, which are distinct in nature, though in practice can lead into each other. The first is the traditional motivation, to understand the incentives and strategies of nodes and how to design systems to incentivize cooperation. The second is an engineering motivation, to devise distributed algorithms that can scale efficiently to networks with many nodes. We compared and contrasted three papers from the first category in detail.

We noticed that the constructions of games used in the three works are each appropriate in different classes of networks. Various approaches are useful where a central node controls a resource, where cooperation can be considered a “service”, and where cooperation is concurrent rather than reciprocal. We found that full knowledge of the network tends to simplify analysis, but also increases costs of cooperation, and either accounting for those costs or relaxing knowledge assumptions is for future work.

While network performance is the end goal, these metrics are not always appropriate for the subproblems these authors were considering. Utility is the true measure of a game’s efficacy, but does not allow for meaningful comparisons between different network proposals. We compared two conceptions of fairness: the Jain index and the Shapley value. In particular, the Jain index considers equality to be optimal, which if strictly followed could stymie coalition formation. More generally, what is “fair” depends on the context.

We explored how the contributions of these authors might impact protocol design and what further work can be done with their ideas. We provided some examples of works where game theory is used towards the second motivation, distributed algorithms.

Although this survey focuses on three papers in particular, it touches on a range of ideas in the literature on game theory in cooperative networks, and we have identified a number of avenues for further exploration of the topic.

REFERENCES


