Supplemental Appendix to "Disruption Risk and Optimal Sourcing in Multi-tier Supply Networks"

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1 Heterogeneous Unreliable Tier 2 Suppliers

In this section we numerically extend the analytical insights regarding heterogeneous unreliable Tier 2 suppliers derived in the main text to a more general case in which Tier 2 suppliers may differ in both procurement cost and disruption risk, which we denote c_j and λ_j, $j = 1, 2$. As in the main text, we assume that $\lambda_1 \leq \lambda_2$, but the cost parameters may be ordered in any way. Our chief goal is to confirm the following four key results. (1) Holding all else equal, a diamond shaped supply chain results in less reliance on manufacturer mitigation and more reliance on supplier mitigation than a V shaped supply chain. (2) Tier 1 suppliers are less likely to select a V shaped supply chain as the manufacturer's unit revenues increase, while the manufacturer always prefers a V shaped supply chain. (3) A preference conflict between the manufacturer and Tier 1 suppliers over the supply chain configuration is more likely for more severe disruptions and more heterogeneous Tier 2 suppliers. (4) Penalty contracts can eliminate the perverse incentives for Tier 1 suppliers to select a diamond shaped supply chain.

It is clearly the case that (1) and (4) hold even under cost heterogeneity. To show (1), we must consider the limit as cost and risk become equal between the Tier 2 suppliers, as in the main text, so that comparisons between the supply chain configurations are truly made "all else being equal." Because we have already made this comparison in the paper for the case of $c_1 = c_2$ and $\lambda_1 = \lambda_2$, there is no additional analysis necessary. In addition, (4) is also clearly true even with heterogeneous costs, as the manufacturer can still extract all surplus from Tier 1 suppliers using appropriate penalties, thereby making Tier 1 suppliers indifferent between the supply chain configurations. Thus, in our numerical analysis we must only show that (2) and (3) continue to qualitatively hold under cost and risk heterogeneity.

To accomplish this, we numerically calculate the manufacturer's optimal strategy in the diamond and V shaped supply chains, as well as the equilibrium to the supply chain configuration game, for

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Parameter	**Values**
D	10
π	$\{15, 30, 45, 60, 75, 90, 105\}$
c_r	$\{5, 6, 7, 8, 9, 10\}$
c_1	$\{3\}$
c_2	$\{1, 2, 3, 4, 5\}$
λ_1	0.2
λ_2	$\{0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$
K	$\{0, 1, 2, 3, 4\}$

Table 1. Parameter values in numerical analysis.

	{1,1}	{1,2}	{2,1}	{2,2}
$\pi = 15$	100%	91%	91%	68%
$\pi = 30$	99%	66%	66%	61%
$\pi = 45$	98%	48%	48%	60%
$\pi = 60$	96%	35%	35%	60%
$\pi = 75$	93%	26%	26%	60%
$\pi = 90$	92%	20%	20%	60%
$\pi = 105$	91%	15%	15%	60%

Table 2. The impact of manufacturer revenues on the supply chain game equilibrium: the percentage of cases in which each configuration is an equilibrium.

each combination of the parameter values in Table 1. This results in 8,400 problem instances covering a wide range of the feasible parameter space. First, we consider how the manufacturer and Tier 1 prefer the various configurations as a function of the manufacturer's unit revenues (Table 2). Note that while the behavior of the equilibrium is more complex than in our base model (in particular, {1,1} is much more likely to be an equilibrium due to the fact that costs and risk are both heterogeneous), our insights that diamond shaped equilibria are more prevalent than V shaped equilibria continues to hold at any given π. In addition, as in our base model, the Tier 1 suppliers are less likely to select a V shaped supply chain as the manufacturer's unit revenues increase. In all 8,400 instances, however, the V shaped supply chain—either {1,2} or {2,1}—was optimal for the manufacturer. Thus, result (2) is confirmed. Next, we consider the impact of K on the supply chain configuration game equilibrium (Table 3). Because $\{1, 2\}$ and $\{2, 1\}$ are less likely to be equilibria as K decreases, a preference conflict between Tier 1 and the manufacturer is relatively more likely for more severe disruptions. Finally, we consider the impact of cost heterogeneity on the supply chain game equilibrium. To facilitate a fair comparison, we restrict attention to the case where $\lambda_1 = \lambda_2 = 0.2$ and determine the frequency of equilibria as c_2 deviates away from $c_1 = 3$ (holding c_1 constant; Table 4). As the table shows, more heterogeneous costs (weakly speaking) lead to less instances of a V shaped equilibrium, meaning a

	{1,1}	{1,2}	{2,1}	{2,2}
$K = 4$	100%	56%	56%	67%
$K = 3$	96%	47%	47%	64%
$K = 2$	94%	41%	41%	61%
$K = 1$	93%	36%	36%	58%
$K = 0$	95%	35%	35%	57%

Table 3. The impact of disrupted capacity on the supply chain game equilibrium: the percentage of cases in which each configuration is an equilibrium.

	{1,1}	{1,2}	{2,1}	{2,2}
$c_2 = 1$	100%	87%	87%	100%
$c_2 = 2$	100%	87%	87%	100%
$c_2 = 3$	100%	87%	87%	100%
$c_2 = 4$	100%	76%	76%	100%
$c_2 = 5$	100%	69%	69%	71%

Table 4. The impact of cost heterogeneity on the supply chain game equilibrium: the percentage of cases in which each configuration is an equilibrium.

preference conflict is more likely for more heterogeneous Tier 2 suppliers. Thus, result (3) is confirmed.

2 Emergency Sourcing in Tier 2

In practice, a disruption in Tier 2 may not result in a shortage of components and, ultimately, lost sales; in many cases, firms have access to outside sources of emergency supply that can be leveraged after a disruption to satisfy demand. In these cases, disruptions result in an increase in costs rather than lost sales. In this section, we analyze the impact of such a source of emergency supply on the manufacturer's optimal sourcing strategy. Specifically, we assume that in addition to the reliable and unreliable Tier 2 suppliers, all Tier 1 suppliers have access to an "emergency" supplier selling at unit cost c_e that is capable of fulfilling an order of any size even after a disruption has occurred.[1] Following disruptions caused by unpredictable natural disasters such as earthquakes or floods, downstream manufacturers often help subsidize emergency sourcing; for example, Toyota and other members of JAMA, the Japanese Automobile Manufacturer Association, used joint funds following the Tōhoku earthquake to help stabilize supply and set up emergency capacity for key components (Tabuchi 2011). Thus, disrupted Tier 1 suppliers are frequently left with only a portion of the cost burden. To model this, we assume that the Tier 1 suppliers pay a fraction γ of the emergency procurement cost, while the downstream manufacturer pays the remaining fraction $1 - \gamma$. This parameter γ is a decision variable of the manufacturer, whose contractual offer to Tier 1 supplier i is denoted by the triple (p_i, Q_i, γ_i).

Rather than rederive all of our previous results with the addition of emergency supply, we focus on how an emergency supplier impacts the manufacturer's overall choice of sourcing strategy, as illustrated

[1] This emergency supplier may be an outside party or may be one of the existing suppliers, e.g. the reliable supplier, producing at a higher than normal cost to deliver units in an expedited fashion, or even the disrupted unreliable supplier leasing facilities or equipment from third parties.

in the following theorem. We consider specifically the case of identical unreliable Tier 2 suppliers, i.e., $\lambda_1 = \lambda_2 = \lambda$, to streamline our discussion.

Lemma 1. *With emergency supply available at unit cost c_e,*

(i) In either a V shaped or diamond shaped supply chain, profit in structure 3 is greater than in the case of no emergency supply, while profit in structure 4 is unchanged.

(ii) There exists a critical value $\bar{c}_e > c_u$ such that the manufacturer's optimal sourcing strategy in both supply chain configurations is to induce structure 3 whenever the cost of emergency supply does not exceed $\bar{c}_e$.

Proof. All proofs for the supplemental appendix appear in §5.

□

The lemma shows two key results. Part (i) demonstrates that, as one would expect, the presence of emergency supply makes it favorable for the manufacturer to induce single sourcing by Tier 1 suppliers– in other words, the manufacturer is less likely to invest in supplier mitigation when emergency supply is available, in either supply chain configuration. Part (ii) shows that, when emergency supply is cheap, both configurations have the same optimal sourcing strategy for the manufacturer: always induce single sourcing from each Tier 1 supplier, i.e., induce structure 3. Conversely, it is clear that when emergency supply is very expensive ($c_e > \pi$), it is never utilized, leading to the same results that we derived in the main text, and in particular leading to different optimal sourcing strategies for the manufacturer, depending on the supply chain configuration. In other words, the differences in the manufacturer's sourcing strategy between the two configurations are minor if emergency supply is inexpensive, but can be significant if emergency supply is expensive.

Returning to our motivating example of Toyota's experiences following the Tōhoku earthquake, recall that Toyota experienced disruptions from both commodity suppliers (like Fujikura, a rubber manufacturer), and specialized suppliers (like Renesas, a semiconductor manufacturer). Theorem 1 implies that for the commodity supplier Fujikura, for which emergency supply is likely to be relatively inexpensive, the optimal sourcing strategy does not depend heavily on the configuration of the supply chain, and investing effort to learn or influence the configuration of upper tiers of the supply chain may be a fruitless activity for the downstream manufacturer. Moreover, inducing passive acceptance in Tier 1 is likely to be favored over a more costly strategy of induced supplier mitigation. In contrast, engaging Tier 1 suppliers in disruption mitigation efforts may be critical for Tier 2 suppliers like Renesas, for which emergency supply is practically non-existent, given the very long leadtimes of semiconductor manufacturing, and the small number of specialized automotive semiconductor suppliers. In addition, the optimal sourcing strategy depends heavily on the configuration of the supply chain, and hence knowing—and perhaps influencing—this configuration is potentially quite valuable to the manufacturer.

3 Centralized System

We now consider a centralized system, in which the manufacturer sources directly from the Tier 2 suppliers, and in which the unreliable Tier 2 suppliers are identical (i.e., $\lambda_1 = \lambda_2 = \lambda$). All other assumptions remain identical to our original setting. In particular, the manufacturer faces a deterministic demand of D units, with unit revenues of π. Tier 2 consists of a single perfectly reliable supplier, with unit cost c_r, and either (a) two unreliable suppliers that disrupt independently, with probability λ, and have disrupted capacity of K each, or (b) a single unreliable supplier that disrupts with probability λ, and has disrupted capacity K.

We first consider the case of minor disruptions, i.e., $D \leq 2K$ when two independent unreliable suppliers exist, and $D \leq K$ when a single unreliable supplier exists, respectively. Under these circumstances, the manufacturer can always source the entire quantity D from the unreliable supplier(s). This strategy and the resulting profit exactly correspond to those under a multi-tier supply chain, when Tier 1 suppliers are offered contracts with price c_u. As such, the manufacturer's optimal sourcing strategy and profit are identical in the centralized and decentralized settings. This argument is formalized in the following lemma.

Lemma 2. *When disruptions are minor, i.e., $D \leq 2K$ ($D \leq K$) when two (one) independent unreliable suppliers exist(s), the manufacturer's optimal sourcing strategy and profit under a centralized setting are identical to those under a multi-tier (i.e., decentralized) setting.*

We now consider the case of severe disruptions, when two unreliable Tier 2 suppliers (denoted by A and B) exist. The manufacturer's sourcing strategy can be summarized with the triple (q_u^A, q_u^B, q_r), corresponding to the quantities sourced from the two unreliable suppliers and the reliable supplier, respectively. The following lemma characterizes the optimal strategy.

Lemma 3. *If two unreliable Tier 2 suppliers exist and disruptions are severe ($K < D/2$), then:*
(i) If the reliable Tier 2 supplier is "cheap" ($c_r \leq (2-\lambda)c_u$), the manufacturer's optimal strategy is:

$$(q_u^A, q_u^B, q_r)^* = \begin{cases} \big(\theta D, (1-\theta)D, 0\big) \ \textit{for any } \theta \in [\frac{K}{D}, \frac{D-K}{D}], & \text{if } c_u \leq \pi < c_u + \frac{c_r - c_u}{\lambda} \\ (K, K, D-2K), & \textit{otherwise.} \end{cases}$$

(ii) If the reliable Tier 2 supplier is costly ($c_r > (2-\lambda)c_u$), the manufacturer's optimal strategy is:

$$(q_u^A, q_u^B, q_r)^* = \begin{cases} \big(\theta D, (1-\theta)D, 0\big) \ \textit{for any } \theta \in [\frac{K}{D}, \frac{D-K}{D}], & \text{if } c_u \leq \pi < \frac{c_u}{\lambda} \\ (D-K, D-K, 0), & \text{if } \frac{c_u}{\lambda} \leq \pi < \frac{c_r - c_u(2-2\lambda)}{\lambda^2} \\ (K, K, D-2K), & \textit{otherwise.} \end{cases}$$

The lemma confirms that the manufacturer's optimal strategy is qualitatively similar to the one under a multi-tier (decentralized) supply chain. In particular, at low unit revenues (π), the manufacturer always finds it optimal to employ *dual sourcing* (DS) from the two unreliable suppliers, splitting the total quantity D between the two suppliers so as to ensure that the "risk free" capacity K is utilized at each. As unit revenues increase, the manufacturer's strategy critically depends on the marginal cost of reliable supply. When reliable supply is cheap ($c_r \leq (2-\lambda)c_u$), the manufacturer switches to a *triple-sourcing* (TS) strategy, which makes minimal use of the unreliable suppliers (by sourcing only the "risk free" quantity K), and switches the focus to reliable supply. Note that the manufacturer's use on reliable supply is directly related to the severity of disruptions—when these are extreme (e.g., $K = 0$), the strategy collapses to sourcing exclusively from the reliable supplier.

When reliable supply is costly ($c_r > (2-\lambda)c_u$), the manufacturer avoids using reliable supply even at intermediate profit margins, i.e., $\frac{c_u}{\lambda} < \pi < \frac{c_r - c_u(2-2\lambda)}{\lambda^2}$. Instead, the DS strategy is complemented with *inventory mitigation*, by increasing the amount sourced from each unreliable supplier to $D-K$. At sufficiently high unit revenues, however, the manufacturer eventually switches to the same TS strategy, shifting the focus to reliable supply. As intuition would dictate, having costly reliable supply causes the manufacturer to rely more heavily on unreliable suppliers (through dual-sourcing and possibly over-sourcing), restricting the use of the TS strategy and reliable supply to only higher unit revenues. It is interesting to note that the per-unit threshold governing this switch in the strategy, i.e., $\frac{c_r - c_u(2-2\lambda)}{\lambda^2}$, is the same as when penalty contracts are used in a decentralized setting, confirming our observation that the latter mechanism can successfully induce the first-best sourcing strategy from Tier 2.

The strategies and insights derived here are qualitatively similar to those in a multi-tier supply chain. It can be checked that $\bar{c}_r < c_u(2-\lambda)$, so that "cheap" reliable supply in a centralized setting may appear as "costly" in a decentralized one. Furthermore, even when reliable suppliers are "costly" for both centralized and decentralized settings, the threshold under which the manufacturer switches from a DS + IM strategy is larger in a decentralized system. Both of these observations imply that having a multi-tier supply chain causes the manufacturer to rely *more* on direct dual sourcing and inventory mitigation, and *less* on an alternative strategy involving reliable supply. This is intuitive, and a direct manifestation of the agency costs inherent in a decentralized setting, where having to contract for (instead of having direct access to) reliable supply leads to less reliance on it.

We now consider the case when there is a unique unreliable Tier 2 supplier. In this case, the manufacturer's strategy is characterized by the pair (q_u, q_r) of quantities sourced from the unreliable and the reliable supplier, respectively. The next result characterizes the optimal strategy.

Lemma 4. *If there is a unique unreliable Tier 2 supplier with disrupted capacity K, and disruptions are severe ($K < D$), the manufacturer's optimal strategy is*

$$(q_u, q_r)^* = \begin{cases} (D, 0) & \text{if } c_u \leq \pi < \frac{c_r - (1-\lambda)c_u}{\lambda}, \\ (K, D-K) & \text{if } \pi \geq \frac{c_r - (1-\lambda)c_u}{\lambda}. \end{cases} \tag{1}$$

The result confirms that single-sourcing from the unreliable supplier is optimal at sufficiently low profit margins, followed by dual-sourcing, with a higher quantity sourced from the reliable supplier. As in the case of a decentralized supply chain, the latter strategy occurs at sufficiently high profit margins (i.e., $\pi > \frac{c_r-(1-\lambda)c_u}{\lambda}$), and completely eliminates the risk. These insights again closely parallel those in a decentralized, multi-tier setting. It can be checked that $\frac{c_r-(1-\lambda)c_u}{\lambda} < \bar{\pi}_1$, which suggests that a manufacturer operating in a multi-tier supply chain relies *more heavily* on (inducing) single sourcing from unreliable suppliers, and *less* on (inducing) strategies that involve reliable supply, when compared with a centralized system. This confirms the earlier intuition that the agency problems inherent in a decentralized supply chain cause the manufacturer to make less use of reliable supply.

4 Technical Results

Lemma 5. *With an emergency source of supply at unit cost c_e, when $D > 2K$ and $D > K$,*

1. In structure 1, the manufacturer's optimal expected profit is $\Pi^M = (\pi - c_u)\,D - \lambda(c_e - c_u)(D-K)$.

2. In structure 2, the manufacturer's optimal expected profit is $\Pi^M = \left(\pi - \frac{c_r - c_u(1-\lambda)}{\lambda}\right) D$.

3a. In structure 3 with independent Tier 2 suppliers, the manufacturer's optimal expected profit is

$$\Pi^M = \begin{cases} (\pi - c_u)\,D - \left(\lambda^2\pi + (1-2\lambda)c_u\right)(D-2K), & \text{if } c_e > \frac{c_u}{\lambda} \text{ and } \pi < \frac{\lambda c_e - (1-\lambda)c_u}{\lambda^2} \\ (\pi - c_u)D - \lambda(c_e - c_u)(D-2K), & \text{otherwise.} \end{cases}$$

In particular, the profit is always at least as large as the profit when no emergency supply is available.

3b. In structure 3 with shared Tier 2 suppliers, the manufacturer's optimal expected profit is $\Pi^M = (\pi - c_u)D - \lambda(c_e - c_u)(D-K)$, and always exceeds the profit achievable when no emergency supply is available.

4a. In structure 4 with independent Tier 2 suppliers, the manufacturer's optimal expected profit is the same as without emergency sourcing, i.e., $\Pi^M = \pi D - \frac{c_r-(1-\lambda)c_u}{\lambda}(D-K) - c_u K$.

4b. In structure 4 with shared Tier 2 suppliers, the manufacturer's optimal expected profit is the same as without any emergency sourcing, i.e., $\Pi^M = \left(\pi - \frac{c_r-(1-\lambda)c_u}{\lambda}\right)(D-K/2) + (\pi - c_u)\,K/2$.

5. In structure 5, the manufacturer's optimal expected profit is $\Pi^M = \left(\pi - \frac{c_r - c_u(1-\lambda)}{\lambda}\right) D$.

If $\frac{c_r-c_u}{c_e-c_u} \le \lambda$ the manufacturer can induce any of the five structures; if $\frac{c_r-c_u}{c_e-c_u} > \lambda$, the manufacturer can only induce single sourcing with its Tier 1 suppliers, and hence structures 2, 4, and 5 are unavailable.

Proof of Lemma 5. 1. If $p \ge c_u, \gamma c_e$, supplier profit in structure 1 is $\Pi^S(Q,0) = (p-c_u)Q + \lambda(c_u - \gamma c_e)(Q-K)$, and the manufacturer's profit is $\Pi^M = (\pi - p)\,D - \lambda(1-\gamma)c_e(D-K)$. The manufacturer clearly wants the lowest price that will induce this outcome, which is $p = \gamma c_e$ (meaning the supplier has incentive to use emergency supply if necessary), resulting in profit $\Pi^M = (\pi - \gamma c_e)\,D-$

$\lambda(1-\gamma)c_e(D-K)$. We note that $\frac{d\Pi^M}{d\gamma} = -c_e D(1-\lambda) - \lambda c_e K < 0$, meaning the manufacturer wants the smallest γ that can achieve this outcome, $\gamma = c_u/c_e$. This implies that the offered price is $p = c_u$ and the Tier 1 supplier is "fully subsidized" when purchasing units from the emergency supplier, i.e., its effective cost of emergency sourcing is c_u. Moreover, because $p < c_r$, the Tier 1 supplier will indeed single source from the unreliable Tier 2 supplier (rather than use the reliable supplier in any way). Manufacturer profit is $\Pi^M = (\pi - c_u)\,D - \lambda(c_e - c_u)(D-K)$.

2. Note that structure 2 with a non-zero quantity sourced from the reliable Tier 2 supplier implies that the Tier 1 supplier never uses the emergency source. This is because either the emergency supplier is more expensive than the reliable supplier on expected cost basis (meaning the problem is effectively the same as in the no emergency source case), or vice versa (implying that the effective structure is structure 1). Thus the Tier 1 supplier's profit in structure 2 is $\Pi^S(K, Q-K) = (p-c_r)Q + (c_r - c_u)K$, and in order to induce this outcome the manufacturer must set p and γ such that (i) $\Pi^S(K, Q-K) \geq 0$ and (ii) $\Pi^S(K, Q-K) \geq \Pi^S(Q, 0)$. Comparing the supplier profit expressions, dual sourcing is preferred if $\gamma \geq \frac{(c_r - (1-\lambda)c_u)}{\lambda c_e}$, satisfying condition (ii) for any $p \geq \gamma c_e$. The manufacturer clearly prefers the smallest price that can induce the supplier to dual source, which implies both the smallest γ that can accomplish this, and a price equal to $p = \gamma c_e$, i.e., $\gamma = \frac{c_r - c_u(1-\lambda)}{\lambda c_e}$ and $p = \frac{c_r - c_u(1-\lambda)}{\lambda}$. This price is the same as the price that can induce dual sourcing in the absence of an emergency supply option, and manufacturer profit is $\Pi^M = \left(\pi - \frac{c_r - c_u(1-\lambda)}{\lambda}\right) D$. The requirement that $\gamma = \frac{c_r - c_u(1-\lambda)}{\lambda c_e} \leq 1$ implies that we require that $\frac{c_r - c_u}{c_e - c_u} \leq \lambda$ for this solution to be interior; otherwise, the supplier can never be induced to dual source.

3a. From part (1), to induce single sourcing with the use of emergency supply by a Tier 1 supplier (say, A), the manufacturer should offer a price c_u, and charge the respective supplier a fraction $\gamma_A = c_u/c_e$ of the emergency supply cost. This would immediately generate an additional manufacturer cost burden of $c_e - c_u$ when using emergency supply through that Tier 1 supplier.

Note that the manufacturer has three alternatives, depending on the number of Tier 1 suppliers induced to use emergency sourcing: both, only one, or none. Inducing both Tier 1 suppliers to use emergency sourcing is never optimal. When no Tier 1 suppliers use emergency sourcing, the manufacturer's profits are identical with those in part (iii) of Lemma 1 in the main text. When only Tier 1 supplier A is induced to use emergency sourcing, the manufacturer's profit becomes

$$\Pi^M(Q_A, Q_B) = (1-\lambda)\big[\pi D - c_u(Q_A + Q_B)\big] + \lambda(\pi - c_u)(Q_A + K) - \lambda(c_e - c_u)(Q_A - K),$$

where $Q_A, Q_B \geq K$, $Q_A + Q_B \geq D$, $D \geq K + Q_B$, $D \geq Q_A + K$. It can be checked that $\frac{\partial \Pi^M}{\partial Q_B} < \frac{\partial \Pi^M}{\partial Q_A}$, so that the optimal contract for the manufacturer is $Q_A^* = D - K, Q_B^* = K$, resulting in a profit of $(\pi - c_u)D - \lambda(c_e - c_u)(D - 2K)$. This is less than the profit obtained without emergency sourcing (in part (iii) of Lemma 1) only when $\pi > \frac{c_u}{\lambda}$ and $\pi < \frac{\lambda c_e - (1-\lambda)c_u}{\lambda^2}$, which is possible only if $c_e > \frac{c_u}{\lambda}$. Thus, the manufacturer's optimal profit in structure 3 with independent Tier 2 suppliers is given by

$$\Pi^M = \begin{cases} (\pi - c_u)\, D - \big(\lambda^2\pi + (1-2\lambda)c_u\big)\,(D-2K), & \text{if } c_e > \frac{c_u}{\lambda} \text{ and } \pi < \frac{\lambda c_e - (1-\lambda)c_u}{\lambda^2} \\ (\pi - c_u)D - \lambda(c_e - c_u)(D-2K), & \text{otherwise.} \end{cases}$$

The corresponding optimal contracts are $\{(c_u, D{-}K, \bar{\gamma}), (c_u, D{-}K, \bar{\gamma})\}$, and $\{(c_u, D{-}K, \frac{c_u}{c_e}), (c_u, K, \bar{\gamma})\}$, respectively, where $\bar{\gamma} > \frac{c_u}{c_e}$ is arbitrary.

3b. If Tier 2 suppliers are shared, structure 3 is degenerate with structure 1, as in the case with no emergency supply. Following the same reasoning as in part 1, the manufacturer will source D from one of its Tier 1 suppliers, and induce that supplier to use emergency supply, resulting in an optimal profit of $\Pi^M = (\pi - c_u)D - \lambda(c_e - c_u)(D-K)$.

4a. By the same argument as for structure 2, emergency sourcing should be induced only for the Tier 1 supplier B, who does *not* have access to a reliable Tier 2 supplier, yielding a profit of

$$\Pi^M(Q_A, Q_B) = \pi D - \frac{c_r - (1-\lambda)c_u}{\lambda} Q_A - c_u Q_B - \lambda(c_e - c_u)(Q_B - K).$$

It can be readily checked that $\frac{\partial \Pi^M}{\partial Q_A} < 0$ and $\frac{\partial \Pi^M}{\partial Q_B} < 0$, and $\frac{\partial \Pi^M}{\partial Q_A} > \frac{\partial \Pi^M}{\partial Q_B}$ if and only if $c_e > c_u + \frac{c_r - c_u}{\lambda^2}$. When the latter condition holds, no emergency sourcing is used, and the profit is the same as in part (iv) of Lemma 1. When the condition is false, it is optimal to not source anything from Tier 1 supplier A, which would mean that structure 4 essentially degenerates into structure 1.

4b. The argument follows similarly to that in part (4a), and is omitted. 5. Structure 5 is degenerate with structure 2, as in the case with no emergency supply.

5 Proofs

Proof of Lemma 1. (i) By Lemma 5, note that the manufacturer's optimal profit in structure 3 is at least as large when emergency sourcing is possible (see parts (3a) and (3b)), while it remains the same in structure 4 (see parts (4a) and (4b)). The result is immediate.

(ii) By parts (3a) and (3b) of Lemma 5, note that the manufacturer's optimal profit in structure 3 increases to $(\pi - c_u)D$ as $c_e \searrow c_u$, irrespective of whether Tier 2 suppliers are independent or shared. The latter expression is the maximal profit available to the manufacturer, achievable under direct access to Tier 2 suppliers, and is always strictly greater than the profit obtained by inducing structure 4. Since the profit expressions are continuous in c_e, the result is immediate.

Proof of Lemma 3. Let $\Pi(q_u^A, q_u^B, q_r)$ denote the resulting expected profit for the manufacturer. First, note that when $\frac{\partial \Pi}{\partial q_r} \leq 0$, it is optimal to source only from the unreliable suppliers, and the manufacturer's profit expression becomes identical to that achieved in Structure 3 under independent

Tier 2 unreliable suppliers. As such, by part (iii) of Lemma 1, the optimal quantities will be

$$(q_u^A, q_u^B, q_r)^* = \begin{cases} \big(\theta D, (1-\theta)D, 0\big), & \text{if } \pi \le \frac{c_u}{\lambda} \\ (D-K, D-K, 0), & \text{otherwise,} \end{cases}$$

where $\theta \in [\frac{K}{D}, \frac{D-K}{D}]$ is arbitrary. When $\frac{\partial \Pi}{\partial q_r} > 0$, we can make similar observations to those in the main text. In particular, it is always optimal to have $q_u^A \ge K$, $q_u^B \ge K$, $q_u^A + q_u^B + q_r \ge D$, and $2K + q_r \le D$. Furthermore, since Π is concave and symmetric in q_u^A and q_u^B, we must have

$$\Pi(q_u^A, q_u^B, q_r) = \Pi(q_u^B, q_u^A, q_r) \le \Pi\Big(\frac{q_u^A + q_u^B}{2}, \frac{q_u^A + q_u^B}{2}, q_r\Big),$$

so that it is always optimal for the manufacturer to source the same quantity from A and B, which we henceforth denote by q_u. In this case, all optimal sourcing strategies (q_u, q_r) must satisfy the constraints $q_u \ge K$ and $D - 2q_u \le q_r \le D - 2K$. In particular, note that $q_r + q_u \le D - 2K + q_u \le D - K$, and if $q_u = K$, then $q_r = D - 2K$. With these observations, the profit function becomes:

$$\begin{aligned} \Pi(q_u, q_r) = \lambda^2\Big[\pi \cdot (q_r + 2K) - c_r q_r - 2c_u K\Big] + 2\lambda(1-\lambda)\Big[\pi \cdot (q_r + q_u + K) - c_r q_r - c_u\big(q_u + K\big)\Big] \\ + (1-\lambda)^2\big(\pi \cdot D - c_r q_r - 2c_u q_u\big). \end{aligned}$$

Using $\Pi(D/2, 0)$ as a proxy for any sourcing strategy $\big(\theta D, (1-\theta)D, 0\big)$, it can be readily checked that

$$\begin{aligned} \Pi(D/2, 0) &= (\pi - c_u)[\lambda 2K + D(1-\lambda)] \\ \Pi(D-K, 0) &= (\pi - c_u)[\lambda^2 2K + 2D\lambda(1-\lambda)] + (1-\lambda)^2[\pi D - 2c_u(D-K)] \\ \Pi(K, D-2K) &= \pi D - c_r(D-2K) - c_u 2K \end{aligned}$$

Comparing these three expressions, we see that if $c_r < c_u(2-\lambda)$, we can express the overall optimal sourcing strategy as follows:

$$(q_u^A, q_u^B, q_r)^* = \begin{cases} \big(\theta D, (1-\theta)D, 0\big), & \text{if } \pi < c_u + \frac{c_r - c_u}{\lambda} \\ (K, K, D-2K), & \text{otherwise.} \end{cases}$$

If $c_r > c_u(2-\lambda)$, then the optimal strategy becomes:

$$(q_u^A, q_u^B, q_r)^* = \begin{cases} \big(\theta D, (1-\theta)D, 0\big), & \text{if } \pi < \frac{c_u}{\lambda} \\ (D-K, D-K, 0), & \text{if } \frac{c_u}{\lambda} \le \pi < \frac{c_r - c_u(2-2\lambda)}{\lambda^2} \\ (K, K, D-2K), & \text{otherwise.} \end{cases}$$

Proof of Lemma 4. In this case, the manufacturer's decision problem is effectively identical to that of a Tier 1 supplier in our original model. As such, the optimal strategy is characterized by results from the main text, with π and D replacing p_i, and Q_i, respectively.