Appendix to
"Sustaining Smallholders and Rainforests by Eliminating Payment-Delay in a Commodity Supply Chain—It Takes a Village"

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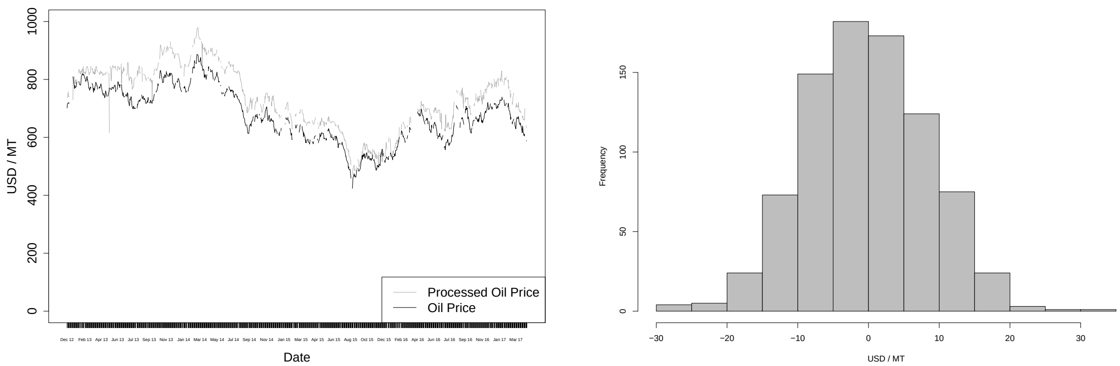
1 Supporting Materials



(a) Historical CPO and RBD prices

(b) Histogram of historical CPO price increments

Figure Notes: Crude Palm Oil (CPO) prices are from the *KO1 Comdty* series in Bloomberg. We converted CPO prices, which are given in Malaysian Ringgits (MYR), to USD by applying the *MYR Curncy* series in Bloomberg. Refineries process CPO into 80% Refined, Bleached, Deodorized (RBD) palm olein and 20% RBD palm stearin. Prices for these outputs were retrieved from the *OLEPRRCM* index series and *TTNSPSMY* index series, respectively, on Bloomberg. We combined 0.8× *OLEPRRCM* and 0.2× *TTNSPSMY* to arrive at the RBD Composite Price Index in panel 1a.

Figure 1: Crude Palm Oil (CPO) and Refined, Bleached, Deodorized (RBD) Prices per metric ton 2012-2017.

Figure 2: Field vehicle stuck on unpaved road after rain.

(a) **(b)**

Figure 3: Farmer receipts from a transport provider, under normal road conditions (a) and poor road conditions (b) ('jalan' is short-hand for 'kondisi jalan' in Indonesian, referring to 'road conditions').

2 Proofs

Proof of Theorem 1. Recall that under a fixed payment delay τ, the problem horizon is divided into $N+1$ periods of length τ. The first N such periods are payment periods, indexed by $n \in \{1, \ldots, N\}$, with the n-th period corresponding to the interval $[(n-1)\tau, n\tau)$. In the terminal period $N+1$, each farmer consumes all his remaining cash net of any interest payments at a constant rate, and the mill shuts down. Since all parties (farmers and the mill) take their decisions at discrete points of time $n\tau$ corresponding to the start of each payment period, we can discretize their decision problems accordingly.

Our proof starts by allowing the mill to offer a different price p_n^f to each farmer, possibly depending on the farmer-specific parameters. The sequential game between the mill and the farmers can be analyzed through a dynamic programming (DP) formulation. In principle, in the DP model, the state for the mill and for every farmer at the beginning of each period $n \in \{1, \ldots, N+1\}$ would consist of all available information, including the cash positions x_n^f, discount rates β^f, land areas ℓ^f, realized cost parameters $\mathcal{W}_n^f$, etc. We seek to show that despite the apparent complexity, several dramatic simplifications take place:

1. the game decouples into F separate games, one for each mill-farmer interaction;

2. in the game between the mill and farmer f, a sufficient state in period $n \in \{1, \ldots, N\}$ for the mill is the outstanding oil price $\mathcal{P}_{(n-1)\tau}$, and a sufficient state for the farmer is his cash position x_n^f and the mill's fruit price p_n^f;

3. in equilibrium, the mill's value function at the beginning of period $n \in \{1, \ldots, N\}$ is given by the sum of the F individual value functions, from each of the F separate games.

Our goal will be to prove this, together with several additional structural results, by backwards induction. To streamline the notation and exposition, we start by first *assuming* the results above, which will allow us to analyze every game between the mill and each farmer separately. We will then return to discuss how these results are preserved by the induction hypothesis.

Consider the game between the mill and a generic farmer f. For $n \in \{1, \ldots, N\}$, let $V_n^f(\mathcal{P}_{(n-1)\tau})$ and $J_n^f(x_n^f, p_n^f)$ denote the mill's and the farmer's equilibrium value functions. We show the following.

Proposition 1. *In the unique subgame-perfect Nash equilibrium for this game, for any $n \in \{1, \ldots, N\}$,*

$$p_n^f(\mathcal{P}_{(n-1)\tau}) = \frac{y\mathcal{P}_{(n-1)\tau} + k}{2} \tag{A.1a}$$

$$V_n^f(\mathcal{P}_{(n-1)\tau}) = Z_n^f \mathcal{P}_{(n-1)\tau}^2 + Y_n^f \mathcal{P}_{(n-1)\tau} + W_n^f \tag{A.1b}$$

$$r_n^f(p_n^f) = C^f(p_n^f - k), \quad \text{where } C^f = \frac{1}{2q + (1 - e^{-\beta^f \tau})\alpha\tau^2 \ell^f \sigma^2}, \tag{A.1c}$$

$$c_n^f(x_n^f, p_n^f) = \left(x_n^f - (e^{-\alpha\tau x_n^f} - 1) - \left(q(r_n^f(p_n^f))^2 + k\, r_n^f(p_n^f)\right)\tau\ell^f - g_n^f(p_n^f)\right)/\tau \tag{A.1d}$$

$$J_n^f(x_n^f, p_n^f) = \frac{1 - e^{-\beta^f \tau}}{\beta^f \tau}\left[x_n^f - (e^{-\alpha\tau x_n^f} - 1) + \bar{A}_n^f (p_n^f)^2 + \bar{B}_n^f p_n^f + \bar{C}_n^f\right], \tag{A.1e}$$

where $g_n^f(p) = \Gamma_{1,n}^f + \Gamma_{2,n}^f p + \Gamma_{3,n}^f p^2$, and the constants $Z_n^f, Y_n^f, W_n^f, C^f, \bar{A}_n^f, \bar{B}_n^f, \bar{C}_n^f$, $\Gamma_{1,n}^f, \Gamma_{2,n}^f, \Gamma_{3,n}^f$ only depend on parameters specific to farmer f and the oil price process $(\mathcal{P}_s)_{s \geq 0}$.

Proof. For ease of exposition, we suppress the superscript f when no confusion can arise. We first check that (A.1e) adequately captures the farmer's value function in the terminal period $n = N + 1$. In this period, the mill stops operating, which can be seen as setting a price $p_{N+1} = 0$ and deriving value $V_{N+1} \equiv 0$, consistent with (A.1b) with $Z_{N+1} = Y_{N+1} = W_{N+1} = 0$. The farmer starts with a cash position x_{N+1}, stops producing fruit, and consumes at a constant rate $c_{N+1}^f = (x_{N+1} - (e^{-\alpha\tau x_{N+1}} - 1))/\tau$, to deplete all cash net of interest payments over a period of length τ. Thus, his value function can be written:

$$J_{N+1}(x_{N+1}, p_{N+1}) = \int_0^\tau \left(\frac{x_{N+1} - (e^{-\alpha\tau x_{N+1}} - 1)}{\tau}\right) e^{-\beta s} ds = \frac{1 - e^{-\beta\tau}}{\beta\tau}\Big(x_{N+1} - (e^{-\alpha\tau x_{N+1}} - 1)\Big),$$

which is consistent with (A.1e) with $\bar{A}_{N+1} = \bar{B}_{N+1} = \bar{C}_{N+1} = 0$.

We can now prove all the results by induction. Assume the induction hypothesis holds in period $n + 1$, and consider the farmer's decision problem in period n. The farmer starts with cash position x_n, is offered a price p_n by the mill, and anticipates the mill's future pricing strategy in period $n + 1$, and his resulting value function. Thus, his value-to-go function in period n becomes:

$$J_n(x_n, p_n) = \max_{r_n \geq 0, c_n} \left\{ c_n \int_0^\tau e^{-\beta s} ds + e^{-\beta\tau}\, \mathbb{E}_n\Big[J_{n+1}\big(x_{n+1}, p_{n+1}(\mathcal{P}_{n+1})\big)\Big] \right\},$$

where we use the shorthand $\mathbb{E}_n[\cdot]$ to denote expectation conditional on all information available at the beginning of period n (at time $(n-1)\tau$), and x_{n+1} is given by (1), replicated below for convenience:

$$x_{n+1} = x_n + r_n \ell\tau p_n - c_n\tau - \big(q\,(r_n)^2 + \mathcal{W}_n\, r_n\big)\ell\tau - (e^{-\alpha\tau x_n} - 1).$$

Recall that $\mathcal{W}_n \sim \mathcal{N}(k, \sigma)$ is independent of $\mathcal{P}_{(n-1)\tau}$. We now introduce a change of variables, where instead of choosing the consumption rate c_n, the farmer chooses his expected cash savings g_n, defined as follows:

$$g_n := x_n - c_n \tau - (q\,(r_n)^2 + k\,r_n)\ell\tau - (e^{-\alpha\tau x_n} - 1).$$

With this definition, we have that given any x_n, p_n, a production rate r_n and a choice g_n, we can rewrite:

$$c_n = \frac{1}{\tau}\Big(x_n - (e^{-\alpha\tau x_n} - 1) - (q r_n^2 + k r_n)\tau\ell - g_n\Big), \tag{A.2a}$$

$$x_{n+1} = g_n + r_n \ell\tau(p_n - \sigma\,\varepsilon_n), \tag{A.2b}$$

where $\varepsilon_n := \frac{\mathcal{W}_n - k}{\sigma} \sim \mathcal{N}(0,1)$ is independent of p_n. Substituting these and replacing J_{n+1} from (A.1e) yields:

$$\begin{aligned} J_n(x_n, p_n) = \max_{r_n \geq 0, g_n} \Big\{ & x_n - (e^{-\alpha\tau x_n} - 1) - (q r_n^2 + k r_n)\tau\ell - g_n \\ & + e^{-\beta\tau}\mathbb{E}_n\Big[x_{n+1} - (e^{-\alpha\tau x_{n+1}} - 1) + \bar{A}_{n+1}p_{n+1}^2 + \bar{B}_{n+1}p_{n+1} + \bar{C}_{n+1}\Big]\Big\}\frac{1 - e^{-\beta\tau}}{\beta\tau} \\ = \frac{1 - e^{-\beta\tau}}{\beta\tau}\Big\{ & x_n - (e^{-\alpha\tau x_n} - 1) + e^{-\beta\tau}\mathbb{E}_n\big[1 + \bar{A}_{n+1}p_{n+1}^2 + \bar{B}_{n+1}p_{n+1} + \bar{C}_{n+1}\big] \\ & + \max_{r_n \geq 0, g_n} \underbrace{\Big[-(q r_n^2 + k r_n)\tau\ell - g_n + e^{-\beta\tau}\mathbb{E}_n\big[x_{n+1} - e^{-\alpha\tau x_{n+1}}\big]\Big]}_{\triangleq h_n(r_n, g_n)}\Big\}, \end{aligned} \tag{A.3}$$

where the second step follows since p_{n+1} is independent of r_n, g_n, by (A.1a). The expression for the maximand $h_n(r_n, g_n)$ can be simplified by noting that conditional on information at time $(n-1)\tau$, the distribution of x_{n+1} is Gaussian, by (A.2b). Using the Gaussian Moment Generating Function yields:

$$\mathbb{E}_n\big[x_{n+1} - e^{-\alpha\tau x_{n+1}}\big] = g_n + r_n\ell\tau p_n - \exp\big(-\alpha\tau(g_n + r_n\ell\tau p_n) + (\alpha r_n \ell\tau^2\sigma)^2/2\big),$$

so that the maximand becomes:

$$h_n(r_n, g_n) = -(q r_n^2 + k r_n)\tau\ell - g_n + e^{-\beta\tau}\Big[g_n + r_n\ell\tau p_n - \exp\big(-\alpha\tau(g_n + r_n\ell\tau p_n) + (\alpha r_n\ell\tau^2\sigma)^2/2\big)\Big]. \tag{A.4}$$

It can be checked through standard composition rules that h_n is jointly concave (Boyd and Vandenberghe, 2009). The first-order-conditions (FOCs) become:

$$\begin{aligned} \frac{\partial h_n}{\partial r_n} = 0 \;\Leftrightarrow\; & e^{\beta\tau}(k + 2q r_n) - p_n + \\ & \exp\big(-\alpha\tau(g_n + r_n\ell\tau p_n) + (\alpha r_n\ell\tau^2\sigma)^2/2\big)\tau\alpha(-p_n + \ell r\alpha\sigma^2\tau^2) = 0 \end{aligned} \tag{A.5a}$$

$$\frac{\partial h_n}{\partial g_n} = 0 \;\Leftrightarrow\; \exp\big(-\alpha\tau(g_n + r_n\ell\tau p_n) + (\alpha r_n\ell\tau^2\sigma)^2/2\big) = \frac{e^{\beta\tau} - 1}{\tau\alpha}. \tag{A.5b}$$

Using (A.5b) in (A.5a), we can solve for r_n^*, and then use the solution in (A.5b) to find g_n^*. We obtain:

$$r_n^* = C(p_n - k), \qquad \text{where } C := \frac{1}{2q + (1-e^{-\beta\tau})\alpha\tau^2\ell\sigma^2} \tag{A.6}$$

$$g_n^* = \Gamma_{1,n} + \Gamma_{2,n}p_n + \Gamma_{3,n}p_n^2, \qquad \text{where } \begin{cases} \Gamma_{1,n} := -\frac{1}{\alpha\tau}\log\left(\frac{e^{\beta\tau}-1}{\alpha\tau}\right) + \frac{(k\sigma\ell C\tau)^2\alpha\tau}{2} \\ \Gamma_{2,n} := \left(1-\alpha\tau^2\ell\sigma^2 C\right)k\ell\tau C \\ \Gamma_{3,n} := \left(\frac{\alpha\tau^2\ell\sigma^2 C}{2} - 1\right)\ell\tau C. \end{cases} \tag{A.7}$$

If $r_n^* \geq 0$, this would be the optimal solution in (A.3), and the proof for (A.1c) and (A.1d) would be complete. To confirm this, let us now consider the mill's problem in the separate game between the mill and a single farmer, which can be written using (A.1b) as:

$$\begin{aligned} V_n(\mathcal{P}_{(n-1)\tau}) &= \max_{p_n}\left\{e^{-\kappa\tau}\mathbb{E}_n\left[\left(y\mathcal{P}_{n\tau} - p_n\right)r_n\ell\tau + Z_{n+1}\mathcal{P}_{n\tau}^2 + Y_{n+1}\mathcal{P}_{n\tau} + W_{n+1}\right]\right\} \\ &= e^{-\kappa\tau}\max_{p_n}\left\{\left(y\mathcal{P}_{(n-1)\tau} - p_n\right)r_n\ell\tau\right\} + e^{-\kappa\tau}\mathbb{E}_n\left[Z_{n+1}\mathcal{P}_{n\tau}^2 + Y_{n+1}\mathcal{P}_{n\tau} + W_{n+1}\right], \end{aligned} \tag{A.8}$$

where r_n is the fruit delivery from farmer f. Thus, when r_n is given by (A.6), the maximand above becomes a concave quadratic in p_n, which is maximized by a choice:

$$p_n^* = \frac{y\mathcal{P}_{(n-1)\tau} + k}{2}. \tag{A.9}$$

In turn, this implies that $r_n^* = \frac{C(y\mathcal{P}_{(n-1)\tau} - k)}{2} \geq 0$, due to our standing assumption, so that the proof for (A.1a), (A.1c), (A.1d) is complete.

To complete our inductive proof, it only remains to confirm the expressions for V_n and for J_n. For the former, using p_n^*, r_n^* and that $\mathcal{P}_s$ is a Brownian motion with zero mean and volatility σ, we can simplify the expression for V_n in (A.8) as:

$$\underbrace{\left(\tau C\ell y^2/2 + e^{-\kappa\tau}Z_{n+1}\right)}_{\triangleq Z_n}\mathcal{P}_{(n-1)\tau}^2 + \underbrace{\left(e^{-\kappa\tau}Y_{n+1} - \tau C\ell yk/2\right)}_{\triangleq Y_n}\mathcal{P}_{(n-1)\tau} + \underbrace{\frac{k^2}{4} + e^{-\kappa\tau}Z_{n+1}\nu^2\tau + e^{-\kappa\tau}W_{n+1}}_{\triangleq W_n},$$

which proves (A.1b).

Lastly, consider the expression in (A.3) for J_n. Using p_{n+1} from (A.1a), we rewrite the third term as:

$$\begin{aligned} &\mathbb{E}_n\left[\bar{A}_{n+1}p_{n+1}^2 + \bar{B}_{n+1}p_{n+1} + \bar{C}_{n+1}\right] \\ \text{(by (A.1a) at } n+1) \quad &= \mathbb{E}_n\left[\bar{A}_{n+1}\left(\frac{y\mathcal{P}_{n\tau} + k}{2}\right)^2 + \bar{B}_{n+1}\left(\frac{y\mathcal{P}_{n\tau} + k}{2}\right) + \bar{C}_{n+1}\right] \\ (*) \quad &= \mathbb{E}_n\left[\bar{A}_{n+1}\left(\frac{y(\mathcal{P}_{(n-1)\tau} + \sqrt{\tau}\nu\tilde{\epsilon}_n) + k}{2}\right)^2 + \bar{B}_{n+1}\left(\frac{y\mathcal{P}_{n\tau}(\mathcal{P}_{(n-1)\tau} + \sqrt{\tau}\nu\tilde{\epsilon}_n + k}{2}\right) + \bar{C}_{n+1}\right] \\ \text{(by (A.1a) at } n) \quad &= \mathbb{E}_n\left[\bar{A}_{n+1}\left(p_n + \frac{y\sqrt{\tau}\nu\tilde{\epsilon}_n}{2}\right)^2 + \bar{B}_{n+1}\left(p_n + \frac{y\sqrt{\tau}\nu\tilde{\epsilon}_n}{2}\right) + \bar{C}_{n+1}\right] \\ (**) \quad &= \bar{A}_{n+1}p_n^2 + \bar{B}_{n+1}p_n + \left(\bar{A}_{n+1}\frac{y^2\tau\nu^2}{4} + \bar{C}_{n+1}\right), \end{aligned} \tag{A.10}$$

where (*) and (**) follow since $\tilde{\epsilon}_n := \frac{\mathcal{P}_{n\tau}-\mathcal{P}_{(n-1)\tau}}{\sqrt{\tau}\nu} \sim \mathcal{N}(0,1)$ is independent of $\mathcal{P}_{(n-1)\tau}$.

Finally, using (A.10) and replacing p_n^*, r_n^* and g_n^* in (A.3), we can confirm (after some tedious algebra) that the expression for J_n in (A.1e) holds, where $\bar{A}_n, \bar{B}_n, \bar{C}_n$ are given by the recursions:

$$\bar{A}_n := -(1-e^{-\beta\tau})\Gamma_{3,n} - q\ell\tau C_n^2 + e^{-\beta\tau}\ell\tau C_n + e^{-\beta\tau}\bar{A}_{n+1}, \tag{A.11a}$$

$$\bar{B}_n := -(1-e^{-\beta\tau})\Gamma_{2,n} - \big(1+e^{-\beta\tau} - 2qC_n\big)k\ell\tau C + e^{-\beta\tau}\bar{B}_{n+1} \tag{A.11b}$$

$$\bar{C}_n := -(1-e^{-\beta\tau})\Gamma_{1,n} + (1-qC)\ell\tau k^2 C - \frac{1-e^{-\beta\tau}}{\alpha\tau} + e^{-\beta\tau}\Big(1+\bar{A}_{n+1}\frac{y^2\tau\nu^2}{4} + \bar{C}_{n+1}\Big). \tag{A.11c}$$

This completes the proof of Proposition 1. □

To complete the proof of Theorem 1, note that the mill's equilibrium pricing strategy was the same in all the separate games, according to (A.1a). By considering a DP formulation for the overall problem and following similar arguments to Proposition 1, it can be readily checked that in equilibrium, the same pricing strategy would remain optimal for the mill, the mill's value function would be additive (given by $\sum_{f=1}^{F} V_n^f$ in period n, with V_n^f from (A.1b)), and farmers would respond with the equilibrium production and consumption decisions in (A.1c)-(A.1d), respectively, achieving the value functions in (A.1e). We omit the details for brevity. ■

Remark: It can be shown that these results also hold for general length of the terminal period $T \geq \tau$.

Proof of Theorem 2.

The result follows from Proposition 2 and Proposition 3. ■

Proposition 2. *Farmer f's equilibrium productivity $\mathbb{E}\Big[\frac{\int_0^D r^f_{\lfloor s/\tau \rfloor} ds}{D}\Big]$ decreases with τ, σ, α, q, k, β^f, and ℓ^f.*

Proof of Proposition 2. From Theorem 1 (results (A.1a) and (A.1c)), the equilibrium production rate in the n-th period is given by $r_n^* = C\frac{y\mathcal{P}_{(n-1)\tau}-k}{2}$, where:

$$C = \frac{1}{2q + (1-e^{-\beta\tau})\alpha\tau^2\ell\sigma^2}.$$

The expression for productivity is:

$$\mathbb{E}\left[\frac{1}{D}\int_0^D r^*_{\lceil s/\tau\rceil} ds\right] = \frac{\tau}{D}\,\mathbb{E}\left[\sum_{n=1}^{N} C\frac{y\mathcal{P}_{(n-1)\tau}-k}{2}\right] = \frac{\tau(y\mathcal{P}_0 - k)}{2D}NC = \frac{y\mathcal{P}_0-k}{2}C \tag{A.12}$$

where the penultimate step follows since the process $\mathcal{P}_s$ is a Brownian motion with zero drift, and the last step follows by recognizing that the production horizon $D = N\tau$ is fixed.

To show that **productivity decreases with** $\alpha, \sigma, q, k, \ell, \beta$ **and** τ, note that we have (by inspection):

$$\frac{\partial C}{\partial \tau} \leq 0,\ \frac{\partial C}{\partial \alpha} \leq 0,\ \frac{\partial C}{\partial \sigma} \leq 0,\ \frac{\partial C}{\partial q} \leq 0,\ \frac{\partial C}{\partial k} = 0,\ \frac{\partial C}{\partial \ell} \leq 0,\ \frac{\partial C}{\partial \beta} \leq 0. \tag{A.13}$$

Since $y\mathcal{P}_0 \geq k$ by our standing assumption, the desired results readily follow from (A.12). □

Remark: It can be shown that these results also hold for general length of the terminal period $T \geq \tau$.

Proposition 3. *Farmer f's equilibrium welfare decreases with τ, σ, α, q, k, and increases with ℓ^f.*

Proof of Proposition 3. The farmer's welfare is given by (A.1e) for $n = 1$ and $x_1 = 0$:

$$\frac{1-e^{-\beta\tau}}{\beta\tau}\left\{\bar{A}_1 p_1^2 + \bar{B}_1 p_1 + \bar{C}_1\right\}, \tag{A.14}$$

where $\bar{A}_1, \bar{B}_1, \bar{C}_1$ are given by recursions (A.11a)-(A.11c), and $p_1 = \frac{y\mathcal{P}_0+k}{2}$ from (A.1a). In particular, with $\bar{A}_{N+1} = \bar{B}_{N+1} = \bar{C}_{N+1} = 0$, the recursions can be solved to obtain:

$$\bar{A}_1 = \frac{(1-e^{-\beta D})e^{-\beta\tau}}{2(e^{\beta\tau}-1)}C\ell\tau \tag{A.15a}$$

$$\bar{B}_1 = -\frac{(1-e^{-\beta D})e^{-\beta\tau}}{2(e^{\beta\tau}-1)}kC\ell\tau \tag{A.15b}$$

$$\begin{aligned}\bar{C}_1 = {} & \frac{e^{-\beta(D-\tau)}\big[4(e^{\beta D}-1)(e^{\beta\tau}-1)k^2 + \big(\tau e^{\beta D} - \tau - De^{\beta\tau} + D\big)y^2\nu^2\big]C\ell\tau}{8(e^{\beta\tau}-1)^2} \\ & + \frac{(1-e^{-\beta D})\Big(1 - e^{\beta\tau} + \alpha\tau + (e^{\beta\tau}-1)\log\big[\frac{e^{\beta\tau}-1}{\alpha\tau}\big]\Big)}{(e^{\beta\tau}-1)\alpha\tau},\end{aligned} \tag{A.15c}$$

where $C = \frac{1}{2q+\alpha(1-e^{-\beta\tau})\tau^2\ell\sigma^2}$ is given by (A.6). Substituting yields an expression for welfare:

$$\underbrace{e^{-\beta D}\frac{(e^{\beta D}-1)(e^{\beta\tau}-1)(\mathcal{P}_0 y - k)^2 + \big(\tau e^{\beta D} - \tau - De^{\beta\tau} + D\big)y^2\nu^2}{8\beta(e^{\beta\tau}-1)}C\ell}_{f_1} + \underbrace{\frac{e^{-\beta\tau}(1-e^{-\beta D})\Big(1 - e^{\beta\tau} + \alpha\tau + (e^{\beta\tau}-1)\log\big[\frac{e^{\beta\tau}-1}{\alpha\tau}\big]\Big)}{\alpha\beta\tau^2}}_{f_2}. \tag{A.16}$$

To show that **welfare decreases with** τ, we treat each term separately. We have:

$$\frac{\partial f_1}{\partial\tau} = -\frac{Ce^{-\beta D}(e^{\beta D}-1)\big(1+e^{\beta\tau}(-1+\beta\tau)\big)\ell y^2\nu^2}{8(e^{\beta\tau}-1)^2\beta} \leq 0 \tag{A.17a}$$

$$\frac{\partial f_1}{\partial C} = e^{-\beta D}\frac{(e^{\beta D}-1)(e^{\beta\tau}-1)(\mathcal{P}_0 y - k)^2 + \big(\tau e^{\beta D} - \tau - De^{\beta\tau} + D\big)y^2\nu^2}{8\beta(e^{\beta\tau}-1)}\ell \geq 0. \tag{A.17b}$$

Above, (A.17a) follows since

$$1 + e^x(-1+x) \geq 0,\ \forall\, x \in [0,\infty), \tag{A.18}$$

which holds by Lemma 6(ii). Similarly, (A.17b) holds since:

$$\tau e^{\beta D} - \tau - De^{\beta\tau} + D \geq 0, \tag{A.19}$$

which follows since its derivative with respect to D is $1-e^{\beta\tau}+\beta\tau e^{\beta D}$, which is greater than $1-e^{\beta\tau}+\beta\tau e^{\beta\tau}$ (since $D \geq \tau$), and the latter function is non-negative by (A.18). Therefore, recalling from (A.13) that $\frac{\partial C}{\partial \tau} \leq 0$, we can use (A.17a) and (A.17b) to conclude that:

$$\frac{df_1}{dC} = \frac{\partial f_1}{\partial \tau} + \frac{\partial f_1}{\partial C}\frac{\partial C}{\partial \tau} \leq 0. \tag{A.20}$$

For the second term in (A.16), we have:

$$\frac{df_2}{d\tau} = \frac{e^{-\beta\tau}(1-e^{-\beta D})h(\alpha,\beta,\tau)}{\alpha\beta\tau^3}$$

$$\text{where} \;\; h(\alpha,\beta,\tau) := (-1+e^{\beta\tau}-\alpha\tau)(1+\beta\tau) + (2-2e^{\beta\tau}+\beta\tau)\log\Big(\frac{e^{\beta\tau}-1}{\alpha\tau}\Big).$$

The sign of this expression is the same as the sign of h. Note that:

$$\begin{aligned}
\frac{\partial^2 h}{\partial \alpha^2} &= \frac{2-2e^{\beta\tau}+\beta\tau}{\alpha^2} \leq 0 \\
\frac{\partial h}{\partial \alpha}\bigg|_{\alpha=\beta} &= \frac{-2+2e^{\beta\tau}-\beta\tau(2+\beta\tau)}{\beta} \geq 0,
\end{aligned}$$

where the latter inequality follows from Lemma 6(ii). Therefore, since h is concave in α, we have that:

$$\begin{aligned}
h(\alpha,\beta,\tau) &\leq \max_{\alpha\in[0,\frac{4\beta}{5}]} h(\alpha,\beta,\tau) \\
&= h\Big(\frac{4\beta}{5},\beta,\tau\Big) \\
&= \Big(-1+e^{\beta\tau}-\frac{4\beta\tau}{5}\Big)(1+\beta\tau) + (2-2e^{\beta\tau}+\beta\tau)\log\Big(\frac{5e^{\beta\tau}-5}{4\beta\tau}\Big) \\
&\leq \Big(-1+e^{\beta\tau}-\frac{4\beta\tau}{5}\Big)(1+\beta\tau) + (2-2e^{\beta\tau}+\beta\tau)\log\frac{5}{4} \\
&\leq 0,
\end{aligned}$$

where the last two inequalities follow by applying results (ii) and (iv) of Lemma 6, respectively. In turn, this implies that $\frac{df_2}{d\tau} \leq 0$, which together with (A.20) and (A.16) implies that welfare is decreasing in τ.

To show that **welfare decreases with σ and q**, note from (A.16) that only f_1 depends on these parameters, through C. By (A.17b) and (A.13), we immediately obtain the desired results.

To show that **welfare decreases with k**, note that welfare only depends on k through f_1 in (A.16), both directly and through C. Since $\frac{\partial C}{\partial k} = 0$ by (A.13), and

$$\frac{\partial f_1}{\partial k} = -\frac{e^{-\beta D}(e^{\beta D}-1)(y\mathcal{P}_0-k)C\ell}{4\beta} \leq 0$$

since $y\mathcal{P}_0 \geq k$, we obtain the desired result.

To show that **welfare decreases with α**, note that f_1 only depends on α through K, and f_2 depends on α explicitly. Since $\frac{\partial f_1}{\partial C} \geq 0$ from (A.17b), $\frac{\partial C}{\partial \alpha} \leq 0$ by (A.13), and

$$\frac{\partial f_2}{\partial \alpha} = -\frac{e^{-\beta\tau}(e^{\beta D}-1)(e^{\beta\tau}-1)\log\big(\frac{e^{\beta\tau}-1}{\alpha\tau}\big)}{\alpha^2\beta\tau^2} \leq 0,$$

we obtain the desired result.

Finally, to show that **welfare increases with** ℓ, note that welfare only depends on ℓ through f_1 in (A.16), both directly and through C. Recall that $\frac{\partial f_1}{\partial C} \geq 0$ from (A.17b), and $\frac{\partial C}{\partial \ell} \geq 0$ by (A.13); since

$$\frac{\partial f_1}{\partial \ell} = e^{-\beta D} \frac{(e^{\beta D}-1)(e^{\beta\tau}-1)(\mathcal{P}_0 y - k)^2 + \big(\tau e^{\beta D} - \tau - D e^{\beta\tau} + D\big) y^2 \nu^2}{8\beta(e^{\beta\tau}-1)} C \geq 0,$$

which follows from (A.19), we obtain the desired result. □

Proof of Theorem 3. We first show the part of the theorem related to the refinery's expected discounted profit. We use n to index the delivery cycles; that is, the refinery receives batches of oil from the mill at each time $n\tau_{sq}$, for $n \in \{1, 2, \ldots, D/\tau_{sq}\}$. We use m to denote the payment periods; that is, the refinery makes payments at each time $(m+1)\tau$. For simplicity, we examine a regime where $\tau \to 1$ so that τ_{sq} remains an integer multiple of τ.

The refinery's expected discounted profit is then given by:

$$\mathbb{E}\Bigg[\sum_{n=1}^{D/\tau_{sq}} \sum_{m=\frac{(n-1)\tau_{sq}}{\tau}}^{(n\tau_{sq}/\tau)-1} \left(e^{-\delta n\tau_{sq}} \mathcal{P}^r_{n\tau_{sq}} r_{m\tau} y\tau - e^{-\delta(m+1)\tau} \mathcal{P}_{(m+1)\tau} r_{m\tau} y\tau\right)\Bigg].$$

We can then show that the refinery's expected discounted profit is decreasing in τ when $\delta < \overline{\delta}$. Let $\mathcal{H}_t$ denote all historical information available to the refinery about prices at time t. Furthermore, note that at time $m\tau$ the refinery knows the delivery rate $r_{m\tau}$; since this remains constant during the next τ units of time, it can thus perfectly predict the batch of oil delivered during this time. However, the refinery does not know its costs and revenues during this time, because the RBD prices $\mathcal{P}^r_s$ and CPO prices $\mathcal{P}_s$ evolve stochastically, as Brownian motions. The conditional expected discounted profit for the m-th payment (with $(n-1)\tau_{sq} \leq m\tau \leq n\tau_{sq}$) is then:

$$\begin{aligned}
\mathbb{E}[\pi_{m\tau} | \mathcal{H}_{m\tau}] &= \mathbb{E}\Big[e^{-\delta(n\tau_{sq}-m\tau)} \mathcal{P}^r_{n\tau_{sq}} r_{m\tau} y\tau - e^{-\delta\tau} \mathcal{P}_{m\tau} r_{m\tau} y\tau \;\Big|\; \mathcal{H}_{m\tau}\Big] \\
&= e^{-\delta(n\tau_{sq}-m\tau)} \mathbb{E}\big[\mathcal{P}^r_{n\tau_{sq}} \;\big|\; \mathcal{H}_{m\tau}\big] r_{m\tau} y\tau - e^{-\delta\tau} \mathcal{P}_{m\tau} r_{m\tau} y\tau \\
&= e^{-\delta(n\tau_{sq}-m\tau)} \mathcal{P}^r_{m\tau} r_{m\tau} \tau - e^{-\delta\tau} \mathcal{P}_{m\tau} r_{m\tau} y\tau \\
&= \left(e^{-\delta(n\tau_{sq}-m\tau)} \mathcal{P}^r_{m\tau} - e^{-\delta\tau} \mathcal{P}_{m\tau}\right) r_{m\tau} y\tau.
\end{aligned}$$

At time $(n-1)\tau_{sq}$, by the law of iterated expectation, the expected discounted expected profit for the

n-th batch is then given by:

$$
\begin{aligned}
\mathbb{E}[\pi_{n\tau_{sq}}|\mathcal{H}_{(n-1)\tau_{sq}}] &= \mathbb{E}\Bigg[\sum_{m=\frac{(n-1)\tau_{sq}}{\tau}}^{(n\tau_{sq}/\tau)-1} e^{-\delta m\tau}\mathbb{E}\big[\pi_{m\tau}|\mathcal{H}_{m\tau}\big] \;\Big|\; \mathcal{H}_{(n-1)\tau_{sq}}\Bigg] \\
&= \mathbb{E}\Bigg[\sum_{m=\frac{(n-1)\tau_{sq}}{\tau}}^{(n\tau_{sq}/\tau)-1} e^{-\delta m\tau}\Big(e^{-\delta(n\tau_{sq}-m\tau)}\mathcal{P}^r_{m\tau} - e^{-\delta\tau}\mathcal{P}_{m\tau}\Big) r_{m\tau} y\tau \;\Big|\; \mathcal{H}_{(n-1)\tau_{sq}}\Bigg] \\
&= \mathbb{E}\Bigg[\sum_{m=\frac{(n-1)\tau_{sq}}{\tau}}^{(n\tau_{sq}/\tau)-1} \Big(e^{-\delta n\tau_{sq}}\mathcal{P}^r_{m\tau} - e^{-\delta(m+1)\tau}\mathcal{P}_{m\tau}\Big) r_{m\tau} y\tau \;\Big|\; \mathcal{H}_{(n-1)\tau_{sq}}\Bigg]. \qquad \text{(A.21)}
\end{aligned}
$$

Recall that the mill receives fruit at a rate $r_{m\tau} = \sum_{f=1}^{F} C^f(\mathcal{P}_{m\tau} - k)$, wherein C^f is given by (A.6). Furthermore, recall that $\mathcal{P}^r_t := K\mathcal{P}_t$ with $K \geq 1$. We can then re-write (A.21) as

$$
\begin{aligned}
&\mathbb{E}\Bigg[\sum_{m=\frac{(n-1)\tau_{sq}}{\tau}}^{(n\tau_{sq}/\tau)-1} \Big(e^{-\delta n\tau_{sq}}K - e^{-\delta(m+1)\tau}\Big) f(\tau)\ (\mathcal{P}^2_{m\tau} - k\mathcal{P}_{m\tau})\ \tau \;\Big|\; \mathcal{H}_{(n-1)\tau_{sq}}\Bigg] \\
&= \sum_{m=\frac{(n-1)\tau_{sq}}{\tau}}^{(n\tau_{sq}/\tau)-1} \Big(e^{-\delta n\tau_{sq}}K - e^{-\delta(m+1)\tau}\Big) f(\tau)\tau\ \mathbb{E}\big[(\mathcal{P}^2_{m\tau} - k\mathcal{P}_{m\tau}) \;\big|\; \mathcal{H}_{(n-1)\tau_{sq}}\big] \\
&= \sum_{m=\frac{(n-1)\tau_{sq}}{\tau}}^{(n\tau_{sq}/\tau)-1} \Big(e^{-\delta n\tau_{sq}}K - e^{-\delta(m+1)\tau}\Big) f(\tau)\ \Big(\tau\mathcal{P}_{(n-1)\tau_{sq}}(\mathcal{P}_{(n-1)\tau_{sq}} - k) + \tau\big(m\tau - (n-1)\tau_{sq}\big)\nu^2\Big) \qquad \text{(A.22)}
\end{aligned}
$$

To show that

$$
\frac{\partial}{\partial\tau}\Big(\mathbb{E}[\pi_{n\tau_{sq}}|\mathcal{H}_{(n-1)\tau_{sq}}]\Big) < 0, \qquad \text{(A.23)}
$$

we look at the Taylor expansion of (A.22) with respect to δ. Specifically, the Taylor expansion of (A.22) with respect to δ is given by:

$$
-\Bigg(\frac{(e^{\beta\tau}(K-1)\ell\tau_{sq}(2e^{\beta\tau}q\nu^2 + (e^{\beta\tau}-1)\ell\alpha\nu^2\sigma^2(2\tau_{sq}-\tau)\tau + \ell\alpha\nu^2\sigma^2\tau^2\beta(\tau_{sq}-\tau) + b(4(e^{\beta\tau}-1)\ell\alpha\sigma^2\tau + 2\ell\alpha\sigma^2\tau^2\beta))}{2(\ell\alpha\sigma^2\tau^2 - e^{\beta\tau}(2q + \ell\alpha\sigma^2\tau^2))^2}\Bigg)
$$
$$
+ \mathcal{O}(\delta),
$$

wherein $b := \mathcal{P}_{(n-1)\tau_{sq}}(\mathcal{P}_{(n-1)\tau_{sq}} - k)$.

Since the dominating term is always negative, it follows from basic continuity arguments that there exists a $\overline{\delta} > 0$ such that (A.23) holds for $\delta \leq \overline{\delta}$. Specifically, it can be shown that $\overline{\delta} := \min(\overline{\delta}_1, \overline{\delta}_2, \overline{\delta}_3)$, wherein:

$$
\overline{\delta}_1 := \max\left\{\delta : \frac{e^{-\delta\tau}(e^{\delta\tau_{sq}} - 1)}{(1 - e^{-\delta\tau})^2}\frac{\delta\tau^2}{\tau_{sq}} < K\right\}, \qquad \text{(A.24)}
$$

$$
\overline{\delta}_2 := \max\left\{\delta : K > \frac{e^{-\delta\tau}(e^{\delta\tau_{sq}} - 1)}{1 - e^{-\delta\tau}}\frac{\tau}{\tau_{sq}}\right\}, \qquad \text{(A.25)}
$$

$$
\overline{\delta}_3 := \max\left\{\delta : K > e^{\delta(\tau_{sq} - \tau)}\right\} \qquad \text{(A.26)}
$$

Thus, conditional on being at the beginning of a batch period, the refinery's expected discounted profit is decreasing in τ for that period/batch when $\delta \leq \overline{\delta}$. What is left to prove is that the refinery's expected discounted profit *at time zero* is decreasing in τ when $\delta \leq \overline{\delta}$. To prove this, note that by iteratively applying the law of iterated expectation we can re-write the refinery's expected discounted profit at time zero as:

$$\sum_{n=1}^{D/\tau_{sq}} \mathbb{E}\Big[e^{-\delta n \tau_{sq}} \pi_{n\tau_{sq}} \Big| \, \mathcal{H}_0\Big] = \sum_{n=1}^{D/\tau_{sq}} \mathbb{E}\Big[e^{-\delta n \tau_{sq}} \mathbb{E}\big[\pi_{n\tau_{sq}} \mid \mathcal{H}_{(n-1)\tau_{sq}}\big] \mid \mathcal{H}_0\Big]. \tag{A.27}$$

We already know that the second term in (A.27) is decreasing in τ and the first term is independent of τ. It follows that

$$\frac{\partial}{\partial \tau}\Bigg(\sum_{n=1}^{D/\tau_{sq}} \mathbb{E}\Big[e^{-\delta n \tau_{sq}} \pi_{n\tau_{sq}} \Big| \, \mathcal{H}_0\Big]\Bigg) < 0 \quad \text{if } \delta \leq \overline{\delta}.$$

Finally, we show the second part of the theorem, related to the mill's expected discounted profit. For this, we remind the reader that the mill pays the farmers when it gets paid by the refinery. The mill's value-to-go function therefore remains as derived in Theorem 1:

$$\begin{aligned} V_n(\mathcal{P}_{(n-1)\tau}) = \Big(\tau \sum_{f=1}^{F} C\ell y^2/2 + e^{-\kappa\tau} Z_{n+1}\Big)\mathcal{P}^2_{(n-1)\tau} + \Big(e^{-\kappa\tau} Y_{n+1} - \tau \sum_{f=1}^{F} C\ell y k/2\Big)\mathcal{P}_{(n-1)\tau} \\ + \frac{k^2}{4} + e^{-\kappa\tau} Z_{n+1}\nu^2\tau + e^{-\kappa\tau} W_{n+1}. \end{aligned}$$

Building on the iterated expectation argument above, it then follows from $\frac{\partial C^f}{\partial \tau} < 0$ (see (A.13)) and $\frac{\partial e^{-\kappa\tau}}{\partial \tau} \leq 0$ that the mill's expected discounted profit strictly decreases in the payment delay τ.

Proof of Theorem 4. Recall from the proof of Proposition 3 that the farmer's welfare when producing on a total land area ℓ is given by (A.16); therein, ℓ impacts welfare through f_1, both directly and through C (given by (A.6)). Thus, a farmer already endowed with productive land ℓ_e solves the following problem in choosing how much additional land ℓ_d to deforest at $t = 0$:

$$\max_{\ell_d \geq 0} \left\{ \frac{\big[(1 - e^{-\beta D})(e^{\beta\tau} - 1)(\mathcal{P}_0 y - k)^2 + \big(\tau e^{\beta D} - \tau - D e^{\beta\tau} + D\big) y^2 \nu^2\big](\ell_d + \ell_e)}{8\beta(e^{\beta\tau} - 1)\big[2q + \alpha(1 - e^{-\beta\tau})\tau^2\sigma^2(\ell_d + \ell_e)\big]} - c_d(\ell_d) \right\}. \tag{A.28}$$

The set of maximizers in (A.28) is non-empty and compact, due to our assumptions that c_d is lower-semicontinuous and coercive (see, e.g., Proposition A.8 in Bertsekas, 1999). Let $g(\tau, \beta, \ell_e, \ell_d)$ denote the maximand in (A.28).

To show that **the set of optimal solutions decreases with** τ, it suffices to show that g is supermodular in $(\tau, -\ell_d)$ (see, e.g., Topkis, 1998), which is equivalent to showing that g is submodular in (τ, ℓ_d). We have:

$$
\begin{aligned}
\operatorname{sign}\Big(\frac{\partial^2 g}{\partial \tau \partial \ell_d}\Big) &= \operatorname{sign}\Big(h_0(\tau, D) + h_1(\tau, D)(\ell_e + \ell_d)\Big)\\
h_0(\tau, D) &:= -2e^{-\beta(D-3\tau)}(e^{\beta D} - 1)\big(1 + e^{\beta\tau}(\beta\tau - 1)\big)q^2y^2\nu^2\\
h_1(\tau, D) &:= +e^{-\beta(D-2\tau)}(e^{\beta\tau} - 1)q\alpha\sigma^2\tau\Big[-2(e^{\beta D} - 1)(e^{\beta\tau} - 1)(-2 + 2e^{\beta\tau} + \beta\tau)(\mathcal{P}_0 y - k)^2 +\\
&\qquad \Big(2D(e^{\beta\tau} - 1)(-2 + 2e^{\beta\tau} + \beta\tau) - (e^{\beta D} - 1)\big(-3 + 2\beta\tau + e^{\beta\tau}(3 + \beta\tau)\big)\tau\Big)y^2\nu^2\Big].
\end{aligned}
$$

We analyze terms separately. To see that h_0 is non-positive, recall that the last parenthesis is non-negative from (A.18). The sign of h_1 equals that of the inner brace. The first term therein is non-negative by Lemma 6(ii) (or simply a Taylor expansion of the exponentials). The second term has the same sign as

$$
h_3(\tau, D) := 2D(e^{\beta\tau} - 1)(-2 + 2e^{\beta\tau} + \beta\tau) - (e^{\beta D} - 1)\big(-3 + 2\beta\tau + e^{\beta\tau}(3 + \beta\tau)\big)\tau.
$$

We have:

$$
\begin{aligned}
\frac{\partial^2 h_3}{\partial D^2} &= -e^{D\beta}\beta^2\tau\big[-3 + 2\beta\tau + e^{\beta\tau}(3 + \beta\tau)\big] \leq 0\\
h_3(\tau, \tau) &= -(e^{\beta\tau} - 1)\tau\big(1 + e^{\beta\tau}(\beta\tau - 1)\big) \leq 0\\
\frac{\partial h_3}{\partial D}\bigg|_{D=\tau} &= -\big(1 + e^{\beta\tau}(\beta\tau - 1)\big)\big(-4 + 2\beta\tau + e^{\beta\tau}(4 + \beta\tau)\big) \leq 0,
\end{aligned}
$$

where all the requisite inequalities follow by (A.18) or Lemma 6(ii) applied to the functions in variable $x = \beta\tau$. By applying the gradient inequality to the concave function h_3 in variable D, we can conclude that:

$$
h_3(\tau, D) \leq h_3(\tau, \tau) + \frac{\partial h_3}{\partial D}\bigg|_{D=\tau}(D - \tau) \leq 0,
$$

which implies that h_1 is also non-positive, and thus g is submodular in (τ, ℓ_d).

To show that **the set of optimal solutions decreases with** ℓ_e, it suffices to show that g is submodular in (ℓ_e, ℓ_d), which can be seen by inspection since:

$$
\operatorname{sign}\Big(\frac{\partial^2 g}{\partial \ell_e \partial \ell_d}\Big) = \operatorname{sign}\Big(-\frac{4e^{2\beta\tau}(e^{\beta\tau} - 1)q\alpha\sigma^2\tau^2}{\big[e^{\beta\tau}\big(2q + \alpha(1 - e^{-\beta\tau})(\ell_e + \ell_d)\tau^2\sigma^2\big)\big]^3}\Big) \leq 0.
$$

To show that **the set of optimal solutions decreases with** β, it suffices to show that g is submodular in (β, ℓ_d). We have:

$$
\begin{aligned}
\operatorname{sign}\Big(\frac{\partial^2 g}{\partial \beta \partial \ell_d}\Big) &= \operatorname{sign}\Big((y\mathcal{P}_0 - k)^2\Big[h_4(\tau, D) + (\ell_e + \ell_d)h_5(\tau, D)\Big] + y^2\nu^2\Big[h_6(\tau, D) + (\ell_e + \ell_d)h_7(\tau, D)\Big]\Big)\\
h_4(\tau, D) &:= -2e^{-\beta(D-3\tau)}(e^{\beta\tau} - 1)^2q^2(-1 + e^{D\beta} - D\beta)\\
h_5(\tau, D) &:= -e^{-\beta(D-2\tau)}(-1 + e^{\beta\tau})^2q\alpha\sigma^2\tau^2\big(1 + e^{\beta(D+\tau)} + D\beta - e^{\beta\tau}(1 + D\beta) - 2\beta\tau + e^{D\beta}(-1 + 2\beta\tau)\big)\\
h_6(\tau, D) &:= 2e^{-\beta(D-3\tau)}q^2\big[D^2(-1 + e^{\beta\tau})^2\beta + D(-1 + e^{\beta\tau})(-1 + e^{\beta\tau} + \beta\tau) - (e^{D\beta} - 1)\tau\big(-1 + e^{\beta\tau}(1 + \beta\tau)\big)\big]\\
h_7(\tau, D) &:= e^{-\beta(D-2\tau)}(-1 + e^{\beta\tau})q\alpha\sigma^2\tau^2\Big[D^2(-1 + e^{\beta\tau})^2\beta + D(-1 + e^{\beta\tau})(-1 + e^{\beta\tau} + 3\beta\tau)\\
&\qquad - (e^{D\beta} - 1)\tau\big(-1 + 2\beta\tau + e^{\beta\tau}(1 + \beta\tau)\big)\Big].
\end{aligned}
$$

The function h_4 is readily non-positive due to (A.18). The sign of h_5 is opposite to the sign of the function:

$$\tilde{h}_5(\tau, D) := 1 + e^{\beta(D+\tau)} + D\beta - e^{\beta\tau}(1 + D\beta) - 2\beta\tau + e^{D\beta}(-1 + 2\beta\tau).$$

For $\tilde{h}_5$, it can be checked that:

$$\begin{aligned}
\frac{\partial^2 \tilde{h}_5}{\partial D^2} &= e^{D\beta}\beta^2(-1 + e^{\beta\tau} + 2\beta\tau) \geq 0 \\
\tilde{h}_5(\tau, \tau) &= (-1 + e^{\beta\tau})(-1 + e^{\beta\tau} + \beta\tau) \geq 0 \\
\left.\frac{\partial \tilde{h}_5}{\partial D}\right|_{D=\tau} &= 2e^{\beta\tau}\beta\Big(-1 + \beta\tau + \frac{e^{\beta\tau} + e^{-\beta\tau}}{2}\Big) \geq 0,
\end{aligned}$$

where all the inequalities follow by applications of Lemma 6(ii). Thus, $\tilde{h}_5$ is non-negative on $D \in [\tau, \infty)$, and therefore h_5 is non-positive.

The sign of h_6 equals the sign of:

$$\tilde{h}_6(\tau, D) := D^2(-1 + e^{\beta\tau})^2\beta + D(-1 + e^{\beta\tau})(-1 + e^{\beta\tau} + \beta\tau) - (e^{D\beta} - 1)\tau\big(-1 + e^{\beta\tau}(1 + \beta\tau)\big).$$

For this function, it can be checked that:

$$\begin{aligned}
\frac{\partial^2 \tilde{h}_6}{\partial D^2} &= \beta\Big[2(-1 + e^{\beta\tau})^2 - e^{D\beta}\beta\tau\Big(-1 + e^{\beta\tau}(1 + \beta\tau)\Big)\Big] \leq 0 \\
\tilde{h}_6(\tau, \tau) &= 0 \\
\left.\frac{\partial \tilde{h}_6}{\partial D}\right|_{D=\tau} &= 1 + \beta\tau - 2e^{\beta\tau}(1 + \beta\tau) + e^{2\beta\tau}[1 + \beta\tau(1 - \beta\tau)] \leq 0.
\end{aligned}$$

The first inequality follows since the left-hand-size decreases in D, and is non-positive at $D = \tau$ by Lemma 6(iv). The last inequality follows by applying the gradient inequality to the function on the left-hand-side in variable $x = \beta\tau$ (which is concave and has a value of 0 and a non-positive derivative at 0). In view of these, we conclude by an application of the gradient inequality that $\tilde{h}_6$ is non-positive, and therefore h_6 is non-positive as well.

Finally, the sign of h_7 equals the sign of:

$$\tilde{h}_7(\tau, D) := D^2(-1 + e^{\beta\tau})^2\beta + D(-1 + e^{\beta\tau})(-1 + e^{\beta\tau} + 3\beta\tau) - (e^{D\beta} - 1)\tau\big(-1 + 2\beta\tau + e^{\beta\tau}(1 + \beta\tau)\big).$$

To prove that $\tilde{h}_7$ is non-positive, it suffices to show that it is smaller than $\tilde{h}_6$. To that end,

$$\tilde{h}_7(\tau, D) - \tilde{h}_6(\tau, D) = 2\beta\tau[D(-1 + e^{\beta\tau}) + \tau - e^{D\beta}\tau].$$

The latter function has a second derivative with respect to D of $-2e^{D\beta}\beta^3\tau^2 \leq 0$, and at $D = \tau$ takes a value of 0 and has a derivative of $2\beta\tau[-1 + e^{\beta\tau}(1 - \beta\tau)]$, which is negative by Lemma 6. Thus the difference is non-positive, which completes our proof.

The following special case will be of interest in a different result, so we discuss it here for convenience. Note that if the cost of deforesting and developing the land is linear $c_d(\ell_d) = d\,\ell_d$, the objective in (A.28) becomes strictly concave in ℓ_d, and the optimal amount of land can be found by solving the first-order-condition. We obtain the following solution, which corresponds to a global maximum:

$$\ell_d^* = \max\Bigg\{0,\ \frac{e^{-D\beta}}{2d(e^{\beta\tau}-1)^3\alpha^2\beta\sigma^4\tau^4}\Bigg(-4de^{\beta(D+\tau)}(e^{\beta\tau}-1)^2q\alpha\beta\sigma^2\tau^2+ \\ \sqrt{de^{\beta(D+2\tau)}(e^{\beta\tau}-1)^3q\alpha^2\beta\sigma^4\tau^4\Big[(e^{D\beta}-1)(e^{\beta\tau}-1)(y\mathcal{P}_0-k)^2-y^2\nu^2\big(D(e^{\beta\tau}-1)+\tau-e^{D\beta}\tau\big)\Big]}\Bigg)-\ell_e\Bigg\}. \tag{A.29}$$

■

Proof of Theorem 5. Under the farmer-level requirement $\mathbb{F}$, farmer $f \in \mathcal{V}$ engages in deforestation if and only if this generates a higher welfare than receiving same-day payment but producing only with his endowed land, *i.e.*, if and only if:[1]

$$J^f(\ell_e^f + \ell_d^{f*}, \tau_{sq}) - c_d(\ell_d^{f*}) \geq J^f(\ell_e^f, 1).$$

Therefore, the set of problem parameters $P_{\mathbb{F}}$ under which no deforestation occurs with the $\mathbb{F}$ requirement is given by the parameters so that:

$$J^f(\ell_e^f, 1) > J^f(\ell_e^f + \ell_d^{f*}, \tau_{sq}) - c_d(\ell_d^{f*}),\ \forall f \in \mathcal{V}. \tag{A.30}$$

By Proposition 4, the set of problem parameters $P_{\mathbb{V}}$ under which no deforestation occurs under $\mathbb{V}$ is given by all parameters so that:

$$\sum_{f\in\mathcal{V}} J^f(\ell_e^f, 1) > \sum_{f\in\mathcal{F}} \Big[J^f(\ell_e^f + \ell_d^{f*}, \tau_{sq}) - c_d(\ell_d^{f*})\Big]. \tag{A.31}$$

By Proposition 5, the set of problem parameters $P_{\mathbb{R}}$ under which no deforestation occurs under $\mathbb{R}$ is given by all parameters so that:

$$\text{(A.31) holds or } \sum_{f\in G}\Big(J^f(\ell_e^f, 1) - J^f(\ell_e^f + \ell_d^{f*}, \tau_{sq})\Big) > \eta\big|\{f \notin G\}\big|, \tag{A.32}$$

where $G = \big\{f : J^f(\ell_e^f, 1) > J^f(\ell_e^f + \ell_d^{f*}, \tau_{sq}) - c_d(\ell_d^{f*})\big\}$.

To prove that set of model parameters satisfying (A.30) is a subset of the model parameters satisfying (A.31), which is a subset of the model parameters satisfying (A.32), note that if (A.30) holds, then (A.31) must hold; and if (A.31) holds, then (A.32) must hold. Furthermore, it is easy to construct a problem instance in which (A.31) holds but (A.30) does not, and (A.32) holds but (A.31) does not, so that the inclusion is strict.

[1] We conservatively assume that a farmer that is indifferent prefers deforestation, but our proof for the nestedness result works under any tie-breaking rule that is consistently applied.

Proposition 4. *The village-level no-deforestation requirement is met in equilibrium if and only if*

$$\sum_{f \in \mathcal{V}} J^f\big(l_e^f,\, 1\big) > \sum_{f \in \mathcal{V}} \Big[J^f\big(l_e^f + l_d^{f*},\, \tau_{sq}\big) - c_d(l_d^{f*})\Big]. \tag{A.33}$$

Proof of Proposition 4. The proof is structured as follows. We first show that when (A.33) holds, (i) an equilibrium always exists, and (ii) the village-level no-deforestation requirement is met in every equilibrium that exists. Finally, we prove that if (A.33) does not hold, then no equilibrium exists in which the village-level no-deforestation requirement is met.

We use $\pi_{\mathcal{V}} := \{\mathcal{V}, \emptyset\}$ to denote the partition containing only the grand coalition. π_0 is the finest partition of singletons. Let $\mathcal{E} = \big\{(d_1^*, \ldots, d_{|\pi|}^*), \pi \in \Pi\big\}$ be the set of all possible Nash equilibria, for any partition $\pi \in \Pi$. Consider a partition $\pi \neq \pi_{\mathcal{V}}$. If

$$\sum_{f \in S_i} J^f\big(l_e^f,\, 1\big) > \sum_{f \in S_i} \Big[J^f\big(l_e^f + l_d^{f*},\, \tau_{sq}\big) - c_d(l_d^{f*})\Big] \text{ for all } S_i \in \pi, \tag{A.34}$$

then coalitions in π play a coordination game and two pure-strategy Nash equilibria exist. In the pay-off dominant Nash equilibrium, no coalition deforests. In the non-pay-off dominant Nash equilibrium, all coalitions deforest. Note that (A.34) implies (A.33), but (A.33) does not imply (A.34). If (A.34) does not hold, then the unique Nash equilibrium is that all coalitions $S_i \in \pi$ deforest.

The set $\mathcal{E}$ therefore consists of a Nash equilibrium for partition $\pi_{\mathcal{V}}$, Nash equilibria for a (potentially empty) set of partitions Z_1 where (A.34) holds and Nash equilibria for a (potentially empty) set of partitions Z_2 where (A.34) does not hold.

We first show that when (A.33) holds, an equilibrium exists wherein the village-level no-deforestation requirement is met. Consider the case where $Z_1 = \emptyset$, *i.e.* all coalitions in all partitions $\pi \neq \pi_{\mathcal{V}}$ deforest. Then there exists a unique partition function and

$$w(S_i, \pi) = \begin{cases} \sum_{f \in S_i} \Big[J^f\big(l_e^f + l_d^{f*},\, \tau_{sq}\big) - c_d(l_d^{f*})\Big] & \text{for all } S_i, \text{ for all } \pi \neq \pi_{\mathcal{V}}, \\ \sum_{f \in \mathcal{V}} J^f\big(l_e^f,\, 1\big) & \text{for } S_i = \mathcal{V}, \pi = \pi_{\mathcal{V}}. \end{cases}$$

If condition (A.33) holds, we then have:

$$\sum_{f \in \mathcal{V}} J^f\big(l_e^f,\, 1\big) > \sum_{f \in \mathcal{V}} \Big[J^f\big(l_e^f + l_d^{f*},\, \tau_{sq}\big) - c_d(l_d^{f*})\Big] = \sum_{S_i \in \pi} \sum_{f \in S_i} \Big[J^f\big(l_e^f + l_d^{f*},\, \tau_{sq}\big) - c_d(l_d^{f*})\Big] \text{ for any } \pi \in \Pi, \tag{A.35}$$

such that $\pi_{\mathcal{V}}$ with allocation

$$a_f(\pi_{\mathcal{V}}, w) := \Big[J^f\big(l_e^f + l_d^{f*},\, \tau_{sq}\big) - c_d(l_d^{f*})\Big] + \frac{1}{|\mathcal{V}|}\Bigg(\sum_{f \in \mathcal{V}} J^f\big(l_e^f,\, 1\big) - \sum_{f \in \mathcal{V}} \Big[J^f\big(l_e^f + l_d^{f*},\, \tau_{sq}\big) - c_d(l_d^{f*})\Big]\Bigg)$$

with the decision $d_1^* = 0$ is an equilibrium. Hence, when (A.33) holds, there exists an equilibrium wherein the village-level no-deforestation requirement is met.

We next show that the village-level no-deforestation requirement is met in every equilibrium that exists when (A.33) holds. The proof proceeds by contradiction. Assume that (A.33) holds. In addition, suppose that there exists an equilibrium wherein all coalitions deforest; more precisely, suppose there exists a partition π, a partition function w, an allocation $a_f(\pi, w)$ and decisions $(d_1^*, \ldots, d_{|\pi|}^*) := (1, \ldots, 1)$ that are an equilibrium. Then the village-level no-deforestation requirement is not met in every equilibrium. Then $a_f(\pi, w) = J^f(l_e^f + l_d^{f*}, \tau_{sq}) - c_d(l_d^{f*})$ for all $f \in S_i$, for all $S_i \in \pi$. But then $\mathcal{V}$ has an objection to π and $a_f(\pi, w)$ because $w(\mathcal{V}, \pi_{\mathcal{V}}) = \sum_{f \in \mathcal{V}} J^f(l_e^f, 1) > \sum_{f \in \mathcal{V}} \big[J^f\big(l_e^f + l_d^{f*}, \tau_{sq}\big) - c_d(l_d^{f*})\big]$, by condition (A.33). Therefore, the partition π and corresponding allocation $a_f(\pi, w)$ cannot be an equilibrium. Hence, if (A.33) holds, there exists no equilibrium wherein the village-level no-deforestation requirement is not met.

Finally, we prove that if (A.33) does not hold, then there exists no equilibrium wherein the village-level no-deforestation requirement is met. The proof is again by contradiction. Assume that (A.33) does not hold. In addition, suppose that there exists an equilibrium wherein the village-level no-deforestation requirement is met in equilibrium; more precisely, suppose there exists a partition π, a partition function w, an allocation $a_f(\pi, w)$ and decisions $(d_1^*, \ldots, d_{|\pi|}^*) := (0, \ldots, 0)$ that are an equilibrium. Such a Nash equilibrium exists only if (A.34) holds. But if (A.34) holds then (A.33) also holds by implication. Hence, we have a contradiction. Therefore, if (A.33) does not hold, then there exists no equilibrium wherein the village-level no-deforestation requirement is met. $\square$

Proposition 5. *The village-level regeneration requirement* $\mathbb{R}$ *is met in equilibrium if and only if* (A.33) *holds or*

$$\sum_{f \in G} \Big(J^f(\ell_e^f, 1) - J^f(\ell_e^f + \ell_d^{f*}, \tau_{sq}) \Big) > \eta \big|\mathcal{V} \setminus G\big|, \tag{A.36}$$

where $G = \big\{ f : J^f(\ell_e^f, 1) > J^f(\ell_e^f + \ell_d^{f*}, \tau_{sq}) - c_d(\ell_d^{f*}) \big\}$.

Proof of Proposition 5. The proof proceeds as follows. We first show that when condition (A.36) holds, an equilibrium exists wherein the village-level regeneration requirement is met. We next show that the village-level regeneration requirement is met in every equilibrium that exists when condition (A.36) holds. Finally, we show that if (A.36) does not hold, but (A.33) holds then the village-level regeneration requirement is met in every equilibrium that exists.

We use $\pi_{\mathcal{V}}$ to denote the partition containing the grand coalition alone. π_0 is the finest partition of singletons. Consider a partition $\pi \neq \pi_{\mathcal{V}}$. Under a village-level regeneration requirement, coalitions in π play the following sequential game. All coalitions first simultaneously decide whether to protect forests. Given the information/observation about decisions by coalitions in the first stage, all coalitions then simultaneously decide whether to prevent fruit production on land that was deforested. Note that coalitions S with $\sum_{f \in S} J^f(\ell_e^f, 1) > \big(\sum_{f \in S} J^f(\ell_e^f + \ell_d^{f*}, \tau_{sq}) - c_d(\ell_d^{f*}) \big)$ either deforest or prevent fruit production, but never deforest *and* prevent fruit production. Coalitions S with $\sum_{f \in S} J^f(\ell_e^f, 1) \leq \big(\sum_{f \in S} J^f(\ell_e^f + \ell_d^{f*}, \tau_{sq}) - c_d(\ell_d^{f*}) \big)$

never prevent fruit production and they deforest only if they anticipate that fruit production will not be prevented on land they deforest. Hence, in the sequential Nash equilibrium, either all coalitions deforest or all coalitions do not deforest. Hence, for each $\pi \neq \pi_{\mathcal{V}}$, either $w(S,\pi) = \sum_{f\in S} J^f(\ell_e^f, 1)$ for all S in π or $w(S,\pi) = \sum_{f\in S} J^f(\ell_e^f + \ell_d^{f*}, \tau_{sq}) - c_d(\ell_d^{f*})$ for all S in π.

We first show that when (A.36) holds, an equilibrium exists wherein the village-level regeneration requirement is met. Assume (A.36) holds. Then the partition $\pi_g = \{G, \mathcal{V}\backslash G\}$ has a unique sequential Nash equilibrium wherein no coalition deforests. Specifically, the unique Nash equilibrium in the second stage of the game is for all farmers $f \in G$ to prevent fruit production on land deforested by farmers $f \in \mathcal{V} \setminus G$. In anticipation of such prevention of fruit production, the unique pure strategy Nash equilibrium for all farmers in the first stage of the game is to not deforest. Then the partition π_g, partition function $w(S_i, \pi_g) = \sum_{f\in S_i} J^f(\ell_e^f, 1)$ with corresponding allocation

$$\begin{cases} a_f(G, w) & = \sum_{f\in G} J^f(\ell_e^f, 1) \\ a_f(\{\mathcal{V}\backslash G\}, w) & = \sum_{f\in \mathcal{V}\backslash G} J^f(\ell_e^f, 1), \end{cases}$$

forest-protection decisions $(d_1, d_2) = (0, 0)$ and blocking decisions $(\mathbf{b}_1, \mathbf{b}_2) = (\mathbf{0}, \mathbf{0})$ is an equilibrium. Therefore, when (A.36) holds, then there exists an equilibrium wherein the village-level regeneration requirement is met.

We next show that the village-level regeneration requirement is met in every equilibrium that exists when (A.36) holds. The proof is by contradiction. Assume that (A.36) holds. Suppose that a partition function w, a partition π, associated decisions $(d_1, \ldots, d_{|\pi|}) = (1, \ldots, 1)$ and $(\mathbf{b}_1, \ldots, \mathbf{b}_{|\pi|}) = (\mathbf{0}, \ldots, \mathbf{0})$, and a corresponding allocation a_f are in the core, such that the village-level regeneration requirement is not met in every equilibrium. Then $a_f(\pi, w) = J^f(\ell_e^f + \ell_d^{f*}, \tau_{sq}) - c_d(\ell_d^{f*})$ for all $f \in \pi$. But then G has an objection to π and $a_f(\pi, w)$ because $w(G, \pi_g) > \sum_{f\in G} \left(J^f(\ell_e^f + \ell_d^{f*}, \tau_{sq}) - c_d(\ell_d^{f*}) \right)$ when (A.36) holds. Therefore, by contradiction, no such partition function w, partition π, associated decisions $(d_1, \ldots, d_{|\pi|}) = (1, \ldots, 1)$ and $(\mathbf{b}_1, \ldots, \mathbf{b}_{|\pi|}) = (\mathbf{0}, \ldots, \mathbf{0})$, and a corresponding allocation a_f can be an equilibrium. Thus, when (A.36) holds, no equilibrium exists wherein the village-level regeneration is not met.

If (A.36) does not hold, but (A.33) holds then it follows from Theorem 4 that the village-level no-deforestation requirement is met in equilibrium. Hence, no deforestation occurs in the village in equilibrium. Therefore, the village-level regeneration requirement is also met in equilibrium. $\square$

Lemma 6. *Consider the functions $f : [0, \infty) \to \mathbb{R}$ and $g : [0, \infty) \to \mathbb{R}$ given by:*

$$f(x) = \alpha_0 + \alpha_1 x + \alpha_2 e^x + \alpha_3\, x\, e^x$$
$$g(x) = \alpha_0 + \alpha_1 x + \alpha_2 e^x + \alpha_3\, x\, e^x + \alpha_4\, e^{2x} + \alpha_5\, x\, e^{2x},$$

where $\alpha_i \in \mathbb{R}$, $\forall i \in \{0, \ldots, 5\}$. Then, *(i)* $f(x)$ *is convex if* $\{\alpha_3 \geq 0, \alpha_2 + 2\alpha_3 \geq 0\}$*;*

(ii) $f(x)$ *is non-negative if it is convex and* $\{\alpha_0 + \alpha_2 \geq 0,\ \alpha_1 + \alpha_2 + \alpha_3 \geq 0\}$*;*

(iii) $g(x)$ *is convex if* $\{\alpha_5 \geq 0,\ \alpha_4 + 3\alpha_5 \geq 0,\ \alpha_3 + 4\alpha_4 + 8\alpha_5 \geq 0,\ \alpha_2 + 2\alpha_3 + 4\alpha_4 + 4\alpha_5 \geq 0\}$*;*

(iv) $g(x)$ *is non-negative if it is convex and* $\{\alpha_0 + \alpha_2 + \alpha_4 \geq 0,\ \alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4 + \alpha_5 \geq 0\}$*.*

Proof of Lemma 6. For f, we have:

$$f''(x) = e^x[\alpha_2 + (2+x)\alpha_3] \tag{A.37}$$

$$f(0) = \alpha_0 + \alpha_2 \tag{A.38}$$

$$f'(0) = \alpha_1 + \alpha_2 + \alpha_3. \tag{A.39}$$

Result (i) follows by (A.37), since the linear function $\alpha_2 + 2\alpha_3 + x\alpha_3$ is non-negative on $[0,\infty)$ under the given conditions. To prove (ii), note that the gradient inequality applied to the convex function f implies:

$$f(x) \geq f(0) + f'(0)x \overset{(A.38,\ A.39)}{\geq} \alpha_0 + \alpha_2 + (\alpha_1 + \alpha_2 + \alpha_3)x \geq 0,\ \forall x \in [0,\infty).$$

Similarly, for g we have:

$$g''(x) = e^x[\alpha_2 + (2+x)\alpha_3 + 4e^x(\alpha_4 + \alpha_5 + x\alpha_5)] \tag{A.40}$$

$$g(0) = \alpha_0 + \alpha_2 + \alpha_4 \tag{A.41}$$

$$g'(0) = \alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4 + \alpha_5. \tag{A.42}$$

By (A.40), g is convex if $\alpha_2 + (2+x)\alpha_3 + 4e^x(\alpha_4 + \alpha_5 + x\alpha_5)$ is non-negative on $[0,\infty)$; applying result (ii) to this function, we obtain the desired conditions. Finally, result (iv) follows by an analogous argument to our earlier one, by applying the gradient inequality to the convex function g and using (A.41) and (A.42). ☐

Proof of Proposition 1. From Proposition 2, the expression for productivity is given by (A.12):

$$\frac{y\mathcal{P}_0 - k}{2}C,\ \text{ where } C = \frac{1}{2q + \alpha(1-e^{-\beta\tau})\tau^2\ell\sigma^2}.$$

It can be readily checked that:

$$\frac{\partial}{\partial\sigma}\left(\left[\frac{y\mathcal{P}_0 - k}{2}C\right]\Bigg|_{\tau=1} - \left[\frac{y\mathcal{P}_0 - k}{2}C\right]\Bigg|_{\tau}\right)$$

$$= \frac{\partial}{\partial\sigma}\left(\frac{1}{2q + \alpha(1-e^{-\beta})\ell\sigma^2} - \frac{1}{2q + \alpha(1-e^{-\beta\tau})\ell\tau^2\sigma^2}\right)\frac{y\mathcal{P}_0 - k}{2}$$

$$= \frac{\alpha\ell 2\sigma\left(\tau^2 - 1 + e^{-\beta} - \tau^2 e^{-\beta\tau}\right)\left(4q^2 + 2q\alpha\ell\sigma^2(1 - e^{-\beta} + \tau^2 - \tau^2 e^{-\beta\tau}) + (\alpha\ell\sigma^2)^2\tau^2(1-e^{-\beta})(1-e^{-\beta\tau})\right)}{\left(4q^2 + 2q\alpha\ell\sigma^2(1 - e^{-\beta} + \tau^2 - \tau^2 e^{-\beta\tau}) + (\alpha\ell\sigma^2)^2\tau^2(1-e^{-\beta})(1-e^{-\beta\tau})\right)^2}$$

$$- \frac{\alpha\ell\sigma^2\left(\tau^2 - 1 + e^{-\beta} - \tau^2 e^{-\beta\tau}\right)\left(2q\alpha\ell(2\sigma)(1 - e^{-\beta} + \tau^2 - \tau^2 e^{-\beta\tau}) + 4\sigma^3(\alpha\ell)^2\tau^2(1-e^{-\beta})(1-e^{-\beta\tau})\right)}{\left(4q^2 + 2q\alpha\ell\sigma^2(1 - e^{-\beta} + \tau^2 - \tau^2 e^{-\beta\tau}) + (\alpha\ell\sigma^2)^2\tau^2(1-e^{-\beta})(1-e^{-\beta\tau})\right)^2},$$

which is non-negative if and only if

$$
\begin{aligned}
&8q^2 \geq 2(\alpha\ell\sigma^2)^2\tau^2(1-e^{-\beta})(1-e^{-\beta\tau}) \\
&\Leftrightarrow q \geq \underbrace{\sqrt{\frac{1}{4}(\alpha\ell\sigma^2)^2\tau^2(1-e^{-\beta})(1-e^{-\beta\tau})}}_{:=q_1^f}
\end{aligned}
\tag{A.43}
$$

Thus, the productivity benefits from eliminating payment delay increase as σ increases if and only if $q \geq q_1^f$.

From Proposition 3, the farmer's welfare (here, abbreviated by $W(\tau)$) is:

$$
\begin{aligned}
W(\tau) := \frac{1-e^{-\beta\tau}}{\beta\tau}\Bigg\{ &\underbrace{e^{-\beta D}\frac{(e^{\beta D}-1)(e^{\beta\tau}-1)(\mathcal{P}_0 y-k)^2+\big(\tau e^{\beta D}-\tau-De^{\beta\tau}+D\big)y^2\nu^2}{8\beta(e^{\beta\tau}-1)}C\ell}_{f_1} + \\
&\underbrace{\frac{e^{-\beta\tau}(1-e^{-\beta D})\Big(1-e^{\beta\tau}+\alpha\tau+(e^{\beta\tau}-1)\log\big[\frac{e^{\beta\tau}-1}{\alpha\tau}\big]\Big)}{\alpha\beta\tau^2}}_{f_2}\Bigg\}.
\end{aligned}
\tag{A.44}
$$

It can readily be checked that:

$$
\begin{aligned}
&\frac{\partial}{\partial\sigma}\Big(W(1)-W(\tau)\Big) \\
&= \frac{1-e^{-\beta}}{\beta}\Bigg(\frac{e^{-\beta D}(e^{\beta D}-1)(e^{\beta}-1)(\mathcal{P}_0 y-k)^2+(e^{\beta D}-1-De^{\beta}+D)y^2\nu^2}{8\beta(e^{\beta}-1)}\Bigg)\ell\Bigg(-\frac{2\sigma\alpha(1-e^{-\beta})\ell}{(2q+\alpha(1-e^{-\beta})\ell\sigma^2)^2}\Bigg) \\
&\quad - \frac{1-e^{-\beta\tau}}{\beta\tau}\Bigg(\frac{e^{-\beta D}(e^{\beta D}-1)(e^{\beta\tau}-1)(\mathcal{P}_0 y-k)^2+(\tau e^{\beta D}-\tau-De^{\beta\tau}+D)y^2\nu^2}{8\beta(e^{\beta\tau}-1)}\Bigg)\ell\Bigg(-\frac{2\sigma\alpha(1-e^{-\beta\tau})\tau^2\ell}{(2q+\alpha(1-e^{-\beta\tau})\tau^2\ell\sigma^2)^2}\Bigg),
\end{aligned}
$$

which is non-negative if and only if

$$
\begin{aligned}
&\underbrace{\frac{(1-e^{-\beta})^2(e^{\beta\tau}-1)}{(1-e^{-\beta\tau})^2(e^{\beta}-1)\tau}\Bigg(\frac{e^{-\beta D}(e^{\beta D}-1)(e^{\beta}-1)(\mathcal{P}_0 y-k)^2+(e^{\beta D}-1-De^{\beta}+D)y^2\nu^2}{e^{-\beta D}(e^{\beta D}-1)(e^{\beta\tau}-1)(\mathcal{P}_0 y-k)^2+(\tau e^{\beta D}-\tau-De^{\beta\tau}+D)y^2\nu^2}\Bigg)}_{:=x} \\
&\qquad \leq \frac{(2q+\alpha(1-e^{-\beta})\ell\sigma^2)^2}{(2q+\alpha(1-e^{-\beta\tau})\tau^2\ell\sigma^2)^2},
\end{aligned}
\tag{A.45}
$$

which after some algebra yields:

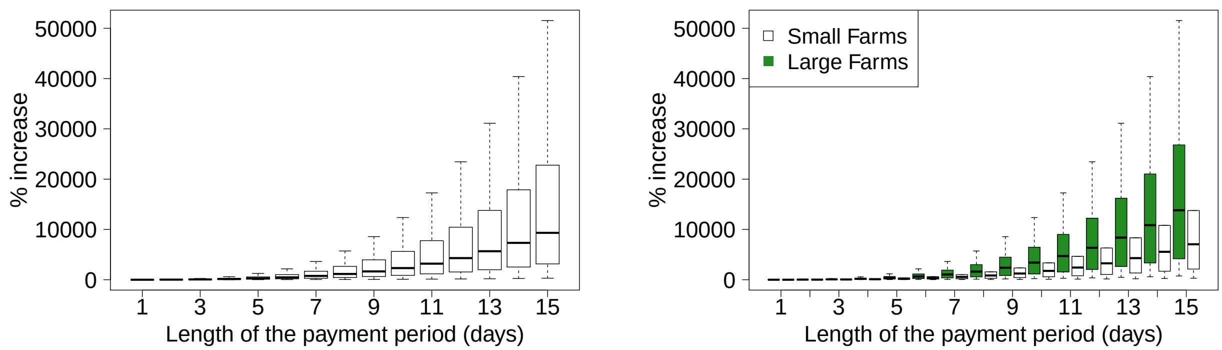
$$
q \geq \underbrace{\frac{\alpha\ell\sigma^2}{2}\frac{(1-e^{-\beta\tau})\tau^2\sqrt{x}-(1-e^{-\beta})}{1-\sqrt{x}}}_{:=q_2^f}
\tag{A.46}
$$

Thus, the welfare benefits from eliminating payment delay increase as σ increases if and only if $q \geq q_2^f$.

Finally, we let $q^f := \max\{q_1^f, q_2^f\}$,

3 Supporting Figures for Empirical Analysis

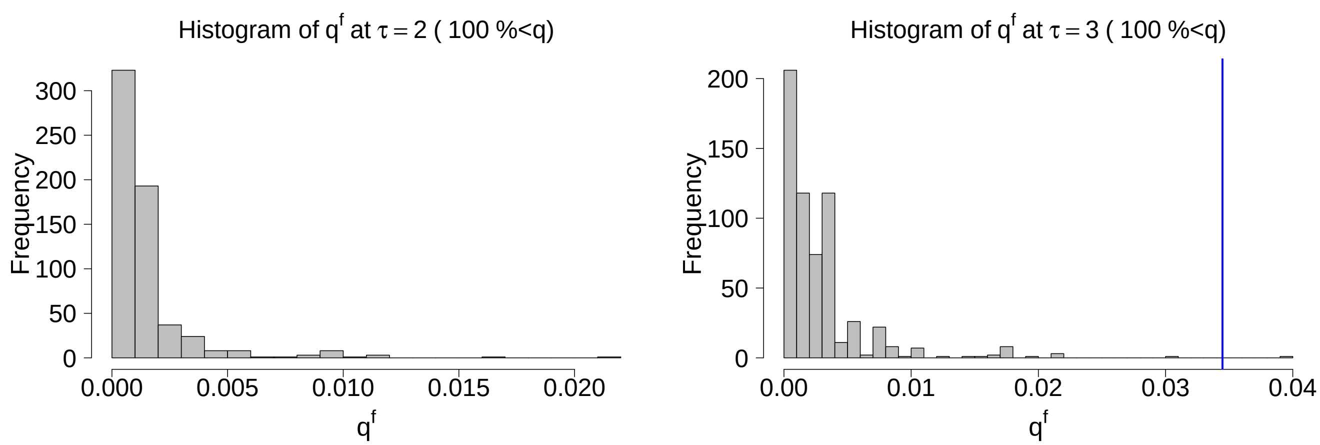
3.1 Robustness checks



(a) **(b)**

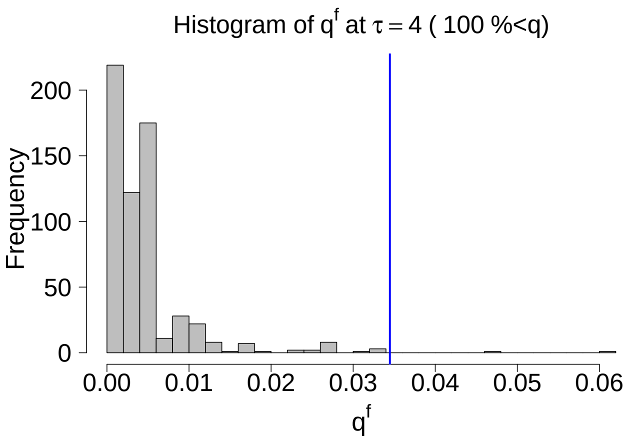
Figure 4: Percentage increase in farmer welfare if the buyer eliminates payment delay, as a function of status quo length of the payment period. The left figure shows box-plots for all 728 farms in our sample. The right figure is obtained by splitting the sample into small and large farms (depending on whether the size is smaller or larger than the median size, respectively), and shows separate box-plots for each category.

3.2 Condition on q

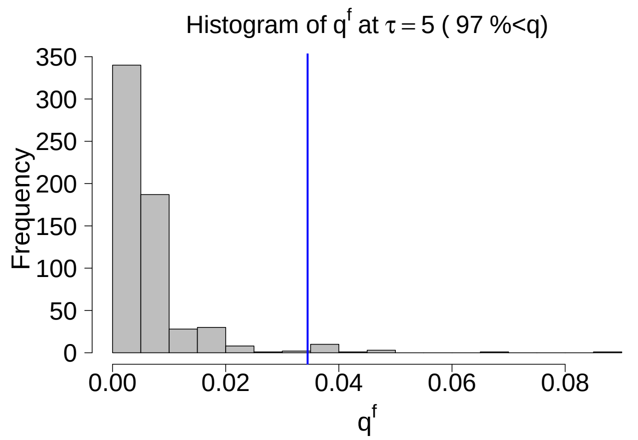


(a) **(b)**

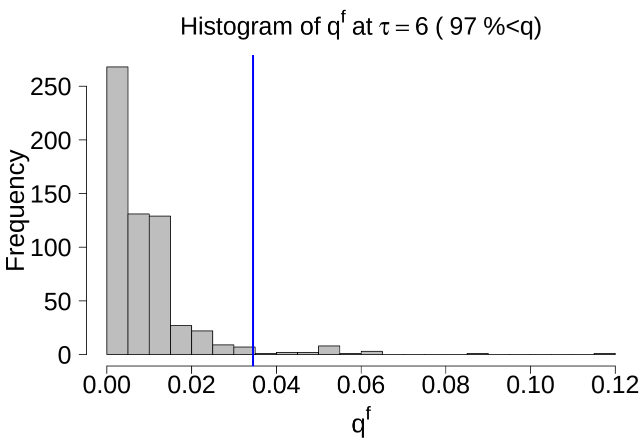
Figure 5: Histograms of q^f relative to q (blue line), for $\tau = 2$ to $\tau = 3$.



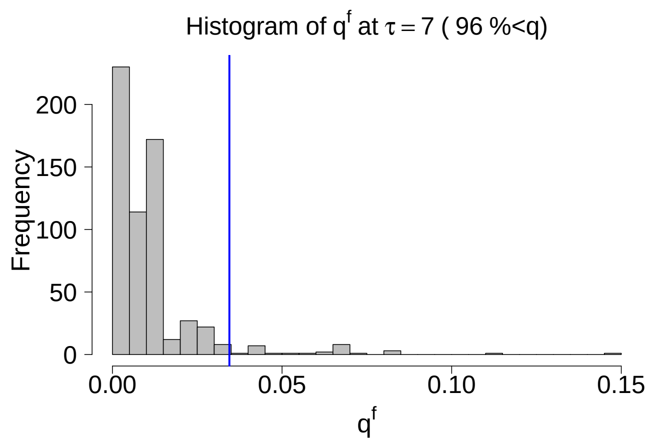
(a)



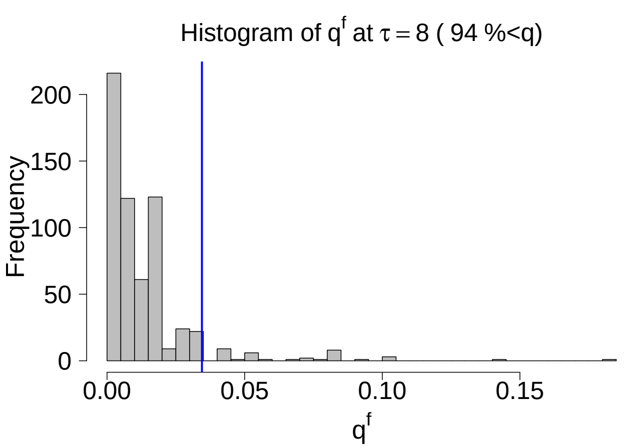
(b)



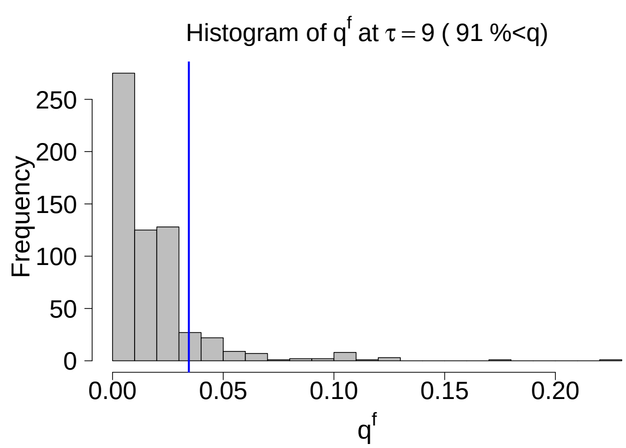
(c)



(d)



(e)



(f)

Figure 6: Histograms of q^f relative to q (blue line), for $\tau = 4$ to $\tau = 9$.

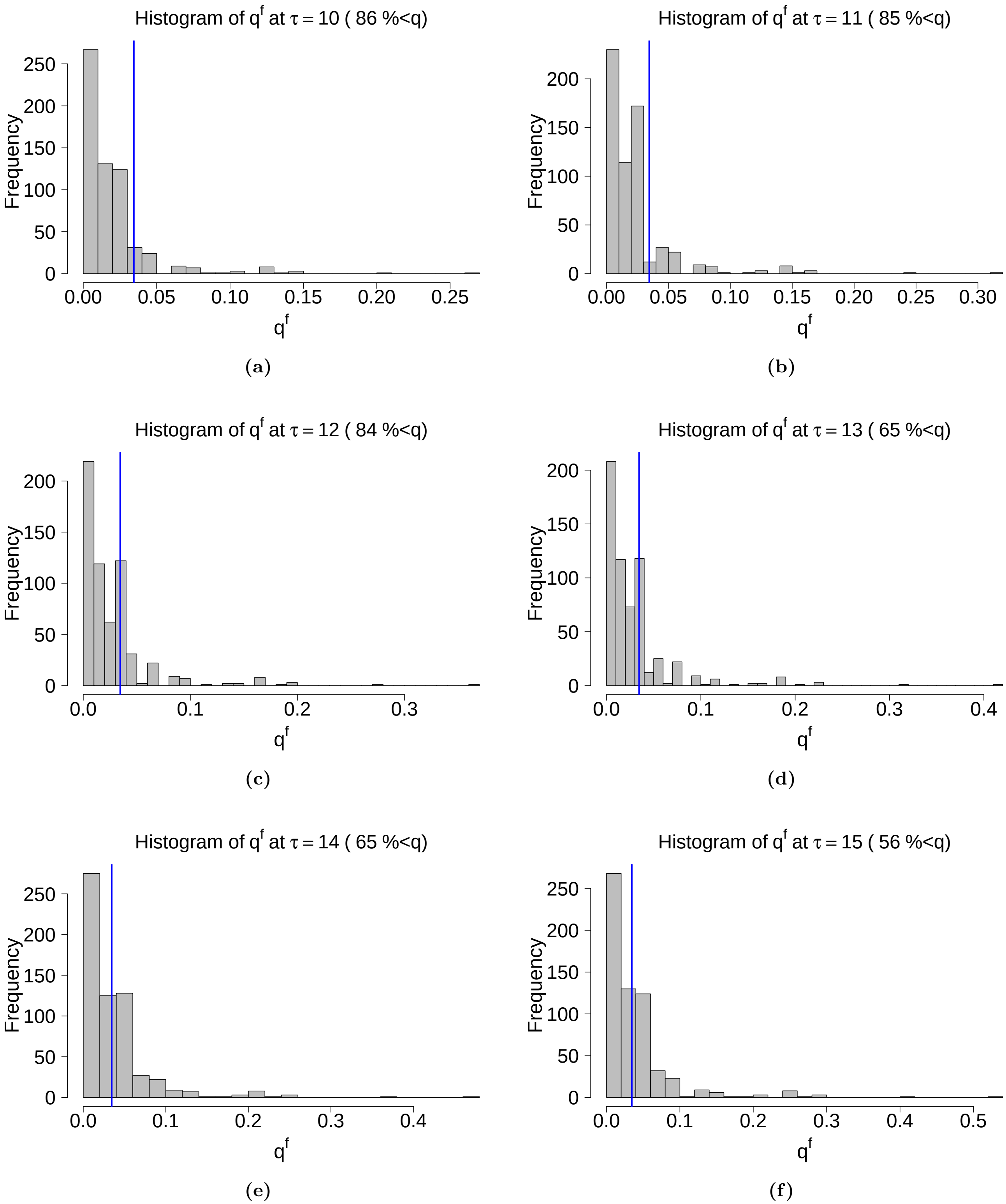


Figure 7: Histograms of q^f relative to q (blue line), for $\tau = 10$ to $\tau = 15$.

References

Bertsekas, D. 1999. *Nonlinear Programming*. Athena Scientific.

Boyd, S., L. Vandenberghe. 2009. *Convex Optimization*. Cambridge University Press, The Edinburgh Building, Cambridge, CV2 2RU, UK.

Topkis, D.M. 1998. *Supermodularity and Complementarity*. Princeton University Press.