The Algebraic Structure of Amounts: Evidence from Comparatives

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Abstract. Heim [9] notes certain restrictions on quantifier intervention in comparatives and proposes an LF-constraint to account for this. I show that these restrictions are identical to constraints on intervention in *wh*-questions widely discussed under the heading of *weak islands*. I also show that Heim's proposal is too restrictive: existential quantifiers can intervene. Both of these facts follow from the algebraic semantic theory of weak islands in Szabolcsi & Zwarts [25], which assigns different algebraic structures to amounts and counting expressions. This theory also makes novel predictions about the interaction of degree operators with conjunction and disjunction, which I show to be correct. Issues involving modal interveners [9], interval semantics for degrees [23,1], and density [4] are also considered.

Keywords: Comparatives, weak islands, degrees, algebraic semantics, quantification, disjunction.

1 Introduction

1.1 Two Puzzles

Consider the sentence in (1):

(1) Every girl is less angry than Larry is.

A prominent theory of comparatives, associated with e.g. von Stechow [28] and Heim [9], predicts that (1) should be ambiguous between two readings, the first equivalent to (2a) and the second equivalent to (2b).

(2) a. For every girl, she is less angry than Larry is.
  b. The least angry girl is less angry than Larry is.

But as Kennedy [11] and Heim [9] note, the predicted reading in (2b) is absent: instead, (1) is unambiguously false if there is *any* girl who is angrier than Larry.

The second puzzle involves the relationship between (3) and (4).

(3) John is richer than his father was or his son will be.
  a. OK if John has $1 million, his father has $1,000, and his son has $10,000.

T. Icard, R. Muskens (Eds.): ESSLLI 2008/2009 Student Sessions, LNAI 6211, pp. 38–56, 2010.

b. OK if John has $10,000, his father has $1,000, and his son has $1 million (e.g., with continuation "... but I'm not sure which one").

(4) John is richer than his father was and his son will be.

a. OK if John has $1 million, his father has $1,000, and his son has $10,000.

b. * if John has $10,000, his father has $1,000, and his son has $1 million.

(3) is ambiguous between (3a) and (3b), while (4) is unambiguous. In other words, both (3) and (4) can be read as meaning that John is richer than the *richer* of his father and his son, but only (3) can be read as meaning that he is richer than the *poorer* of the two. This is also a problem for theories of comparatives that follow von Stechow's and Heim's assumptions, because they predict that (3) and (4) should both be ambiguous, and have the same two readings.

Sentences like (3) have been taken as evidence that *or* is lexically ambiguous between the standard logical disjunction and an NPI interpreted as conjunction [23]. I will argue, first, that the ambiguity of (3) is a matter of scope rather than lexical ambiguity; and, second, that the fact that (4) does not display a similar ambiguity is explained by the same principles that rule out the unavailable reading of (1) that is paraphrased in (2b).

1.2 The Plan

Heim [9] states an empirical generalization that quantificational DPs may not intervene scopally between a comparative operator and its trace, essentially to explain the absence of reading (2b) of (1). She suggests that this can be accounted for by an intervention constraint at LF. I demonstrate that this generalization holds with some but not all quantifiers, and that the limitations and their exceptions match quite closely the distribution of interveners in weak islands. Following a brief suggestion in Szabolcsi [24], I show that the algebraic semantic theory of weak islands in Szabolcsi & Zwarts [25] predicts the observed restrictions on comparative scope without further stipulation, as well as the exceptions.

This extension of the algebraic account to comparatives also predicts the existence of maximum readings of *wh*-questions and comparatives with existential quantifiers, and I show that this prediction is correct. In addition, it accounts for the asymmetry between conjunction and disjunction in the comparative complement that we saw in (3) and (4). Certain strong modals appear to provide counter-examples; however, I suggest that the problem lies not in the semantics of comparison but in the analysis of modals.

A great deal of work has been done since 1993 on both comparative scope and weak islands, and we might suspect that the problem discussed here can be avoided by adopting one of these more recent proposals. In the penultimate section I survey two influential accounts of the semantics of degree, one relating to comparatives [23] and the other to negative islands [4]. I suggest that neither of these proposals accounts for the data at issue, but both are compatible with, and in need of, a solution along the lines proposed here.

2 Comparatives and Weak Islands

2.1 Preliminaries

Suppose, following Heim [9], that gradable adjectives like *angry* denote relations between individuals and degrees.

(5) $[\![\; angry \;]\!] = \lambda d \lambda x[angry(d)(x)]$

$[angry(d)(x)]$ can be read as "x's degree of anger is (at least) d". As the presence of "at least" in the previous sentence suggests, we also assume with Heim that expressions of degree in natural language follow the monotonicity condition in (6).

(6) A function $f_{<d,et>}$ is *monotone* iff:
$\forall x \forall d \forall d'[(f(d)(x) \wedge d' < d) \rightarrow f(d')(x)]$

(5) is not uncontroversial, but it is a reasonably standard analysis, and we will examine its relationship to some alternative accounts in the penultimate section. (6) is required to accommodate, e.g., the acceptability of true but underinformative answers to questions: *Is your son three feet tall? Yes – in fact, he is four feet tall.*

I also assume, following von Stechow [28] and Heim [9], that *more/-er* comparatives are evaluated by examining the *maxima* of two sets of degrees:

(7) $[\![\; \mathbf{max} \;]\!] = \lambda D_{<d,t>} \iota d \forall d'[D(d') \rightarrow d \geq d']$

More takes two sets of degrees as arguments and returns 1 iff the maximum of the second (the main clause) is greater than the maximum of the first (the *than*-clause). *Less* does the same, replacing except that the ordering is reversed.

(8) a. $[\![\; more/\text{-}er \;]\!] = \lambda D_{d,t} \lambda D'_{d,t}[\mathbf{max}(D') > \mathbf{max}(D)]$
b. $[\![\; less \;]\!] = \lambda D_{d,t} \lambda D'_{d,t}[\mathbf{max}(D') < \mathbf{max}(D)]$

Finally, we assume that *more/-er* forms a constituent with the *than*-clause to the exclusion of the adjective, and that typical cases of (at least clausal) comparatives involve ellipsis, so that *Larry is angrier than Bill is* = *Larry is angry* [*-er than Bill is ~~angry~~*].

2.2 Quantificational Interveners and the Heim-Kennedy Constraint

Once we consider quantifiers, the treatment of gradable adjectives and comparatives outlined briefly in the previous subsection immediately generates the puzzle in (1)-(2). To see this, note first that, in order for *more/-er* to get its second argument, it must have undergone QR above the main clause. The difference between the two predicted readings depends on whether the quantifier *every girl* raises before or after the comparative clause. So, for example, Heim would assign the first reading of (1) the LF in (9a). The alternative reading is generated when the comparative clause raises to a position higher than the quantifier *every girl*.

(9) Every girl is less angry than Larry is.

a. Direct scope: *every girl* > *less* > *d-angry*
$\forall x[\mathbf{girl}(x) \rightarrow [\mathbf{max}(\lambda d.\mathbf{angry}(d)(x))] < \mathbf{max}(\lambda d.\mathbf{angry}(d)(Larry))]$
"For every girl x, Larry is angrier than she is."

b. Scope-splitting: *less* > *every girl* > *d-tall*
$\mathbf{max}(\lambda d.\forall x[\mathbf{girl}(x) \rightarrow \mathbf{angry}(d)(x)]) < \mathbf{max}(\lambda d.\mathbf{angry}(d)(Larry))$
* "Larry's max degree of anger exceeds the greatest degree to which every girl is angry (i.e., he is angrier than the least angry girl)."

If (9) had the"scope-splitting" reading in (9b), it would be true (on this reading) if the least angry girl is less angry than Larry. However, (9) is clearly false if any girl is angrier than Larry. Heim [9] suggests that the unavailability of (9b) and related data can be treated as a LF-constraint along the lines of (10) (cf. [26]):

(10) **Heim-Kennedy Constraint (HK)**:
A quantificational DP may not intervene between a degree operator and its trace.

The proposed constraint (10) attempts to account for the unavailability of (9b) (and similar facts with different quantifiers) by stipulating that the quantificational DP *every girl* may not intervene between the degree operator *less* and its trace *d-tall*. The puzzle is what syntactic or semantic principles explain this constraint given that structures such as (9b) are semantically unexceptionable on our assumptions.

2.3 Similarities between Weak Islands and Comparative Scope

As Rullmann [21] and Hackl [7] note, there are considerable similarities between the limitations on the scope of the comparative operator and the core facts of weak islands disussed by Kroch [14] and Rizzi [20], among many others. Rullmann notes the following patterns:

(11) a. I wonder how tall Marcus is / # isn't.
b. I wonder how tall this player is / # no player is.
c. I wonder how tall every player is / # few players are.
d. I wonder how tall most players are / # fewer than ten players are.
e. I wonder how tall many players / # at most ten players are.

(12) a. Marcus is taller than Lou is / # isn't.
b. Marcus is taller than this player is / # no player is.
c. Marcus is taller than every player is / # few players are.
d. Marcus is taller than most players are / # fewer than ten players are.
e. Marcus is taller than many players are / # at most ten players are.

These similarities are impressive enough to suggest that a theory of the weak island facts in (11) should also account for the limitations on comparatives in (12). Rullmann suggests that the unavailability of the relevant examples in (11) and (12) is due to semantic, rather than syntactic, facts. Specifically, both *wh*-questions and comparatives make use of a maximality operation, roughly as in (13):

(13) a. I wonder how tall Marcus is.
I wonder: what is the degree d s.t. $d = \mathbf{max}(\lambda d.\text{Marcus is } d\text{-tall})$?
b. Marcus is taller than Lou is.
$(\iota d[d = \mathbf{max}(\lambda d.\text{Marcus is } d\text{-tall})]) > (\iota d[d = \mathbf{max}(\lambda d.\text{Lou is } d\text{-tall})])$

With these interpretations of comparatives and questions, we predict that the sentences in (14) should be ill-formed because each contains an undefined description:

(14) a. # I wonder how tall Marcus isn't.
I wonder: what is the degree d s.t. $d = \mathbf{max}(\lambda d\ .\ \text{Marcus is not } d\text{-tall})$?
b. # Marcus is taller than Lou isn't.
$(\mathbf{max}(\lambda d.\text{Marcus is d-tall}) > (\mathbf{max}(\lambda d.\ \text{Lou is not } d\text{-tall}))$

If degrees of height are arranged on a scale from zero to infinity, there can be no maximal degree d such that Marcus or Lou is not d-tall.

However, the similarities between comparatives and weak island-sensitive expressions such as *how tall* go deeper than Rullmann's discussion would indicate. S&Z point out that several of the acceptable examples in (11) do not have all the readings predicted by the logically possible orderings of *every player* and *how tall.* As it turns out, the same scopal orders are also missing in the corresponding comparatives when we substitute *-er* for *how tall.* For example,

(15) I wonder how tall every player is.
a. *every player* > *how tall* > *d-tall*
"For every player x, I wonder: what is the max degree d s.t. x is d-tall?"
b. *how tall* > *every player* > *d-tall*
"I wonder: what is the degree d s.t. $d = \mathbf{max}(\lambda d.\ \text{every player is } d\text{-tall})$?"

A complete answer to (15) would involve listing all the players and their heights. In contrast, an appropriate response to (15b) would be to intersect the heights of all the players and give the maximum of this set, i.e. to give the height of the shortest player. This second reading is clearly not available. In fact, although S&Z and Rullmann do not notice, similar facts hold for the corresponding comparative:

(16) Marcus is taller than every player is.

a. *every player > -er > d-tall*
"For every player, Marcus is taller than he is."

b. *-er > every player > d-tall*
"Marcus' max height is greater than the max height s.t. every player is that tall, i.e. he is taller than the shortest player."

Rullmann's explanation does not exclude the unacceptable (16b): unlike comparatives with an intervening negation, there **is** a maximal degree d s.t. every player is d-tall on Rullmann's assumptions, namely the height of the shortest player.

Note in addition that (16) is identical in terms of scope possibilities to our original comparative scope-splitting example in (1)/(9), although its syntax is considerably different. Like (9), (16) falls under Heim's proposed LF-constraint (10), which correctly predicts the unavailability of (16b).[1]

3 Comparative Scope and the Algebraic Structure of Amounts

3.1 Szabolcsi & Zwarts' (1993) Theory of Weak Islands

Like Rullmann [21], S&Z argue that no syntactic generalization can account for the full range of weak islands, and propose to account for them in semantic terms. They formulate their basic claim as follows:

(17) Weak island violations come about when an extracted phrase should take scope over some intervener but is unable to.

S&Z explicate this claim in algebraic terms, arguing that weak islands can be understood if we pay attention to the operations that particular quantificational elements are associated with. For instance,

(18) Universal quantification involves taking *intersections* (technically, meets).
Existential quantification involves taking *unions* (technically, joins).
Negation involves taking *complements.*

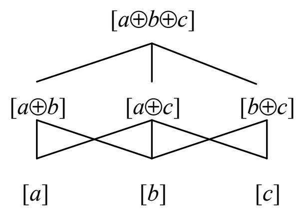
(18) becomes important once we assign algebraic structures as denotations to types of objects, since not all algebraic operations are defined for all structures. The prediction is that a sentence will be semantically unacceptable, even if it can be derived syntactically, if computing it requires performing an operation on a structure for which the operation is not defined. S&Z illustrate this claim with the verb *behave*:

[1] We might think to appeal to the fact that comparative clauses are islands to extraction in order to account for the missing readings in (16), but this would give the wrong results: it would rule out (16a) instead of (16b).

(19) a. How did John behave?
b. *How didn't John behave?
c. How did everyone behave?
i. For each person, tell me: how did he behave?
ii. *What was the behavior exhibited by everyone?

Behave requires a complement that denotes a manner. S&Z suggest that manners denote in a free join semilattice, as Landman [15] does for masses.

(20) Free join semilattice



A noteworthy property of (20) is that it is closed under union, but not under complement or intersection. For instance, the union (technically, join) of $[a]$ with $[b \oplus c]$ is $[a \oplus b \oplus c]$, but the intersection (meet) of $[a]$ with $[b \oplus c]$ is not defined. The linguistic relevance of this observation is that it corresponds to our intuitions of appropriate answers to questions about behavior. In S&Z's example, suppose that three people displayed the following behaviors:

(21) John behaved *kindly and stupidly.*
Mary behaved *rudely and stupidly.*
Jim behaved *loudly and stupidly.*

If someone were to ask: "How did everyone behave?", interpreted with how taking wide scope as in (19c-ii), it would not be sufficient to answer "stupidly". The explanation for this, according to S&Z, is that computing the answer to this question on the relevant reading would require intersecting the manners in which John, Mary and Jim behaved, but intersection is not defined on (20). This, then, is an example of when "an extracted phrase should take scope over some intervener but is unable to". Similarly, (19b) is unacceptable because complement is not defined on (20).

Extending this account to amounts is slightly trickier, since amounts seem to come in two forms. In the first, which S&Z label "counting-conscious", *wh*-expressions are able to take scope over universal quantifiers. S&Z imagine a situation in which a swimming team is allowed to take a break when everyone has swum 50 laps. In this situation it would be possible to ask:

(22) [At least] How many laps has every swimmer covered by now?

If the number of laps covered by the slowest swimmer is a possible answer, then counting-conscious amount expressions had better denote in a structure in which intersection is defined. [2] The lattice in (23) – essentially the structure normally assumed for all degrees – seems to be an appropriate choice.

(23) Lattice

Intersection and union are defined in this structure, though complement is not. This analysis predicts correctly that *how many/much* should be able to take scope over existential quantification but not negation.

(24) a. How many laps has at least one swimmer covered by now?
[Answer: the number of laps covered by the fastest swimmer.]
b. *How many laps hasn't John covered by now?

3.2 Extending the Account to Comparatives

Many authors assume that amounts always denote in (23) or some other structure $\langle D, \leq \rangle$ for $D \subseteq \mathbb{R}$. The problem we began with — why can't *Every girl is less tall than John* mean "The shortest girl is shorter than John"? — relied tacitly on this assumption, which entails that it should be possible to intersect sets of degrees.

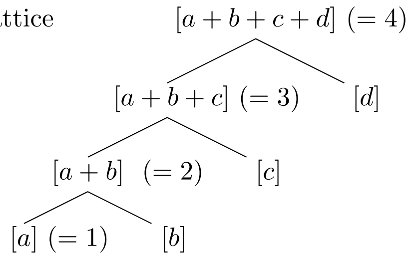
I would like to suggest an alternative: heights and similar amounts do not denote in (23), but in a poorer structure for which intersection is not defined, as S&Z claim for island-sensitive amount *wh*-expressions. As S&Z note, such a structure is motivated already by the existence of non-counting-conscious amount *wh*-expressions which are sensitive to a wider variety of interveners than *how many* was in the examples in (22) and (24). This is clear, for example, with amounts that are not associated with canonical measures:

(25) How much pain did every student endure?
a. "For every student, how much pain did he/she endure?"
b. * "What is the greatest amount of pain s.t. every student endured that much, i.e. how much was endured by the one who endured the least?"

The unacceptability of (25b) is surprising given that the degree expression is able to take wide scope in the overtly similar (22). S&Z argue that, unless counting is involved, amount expressions denote in a join semilattice:

[2] As Anna Szabolcsi (p.c.) suggests, the use of canonical measures like *laps, kilos, meters* may help bring out this reading.

(26) Join semilattice



(26) should be seen as a structure collecting arbitrary unit-sized bits of stuff, abstracting away from their real-world identity, like adding cups of milk to a recipe (S&Z pp.247-8). An important formal property of (26) is that "if p is a proper part of q, there is some part of q (the witness) that does not overlap with p" (p.247). As a result, intersection is not defined unless the objects intersected are identical. S&Z claim that this fact is sufficient to explain the unavailability of (25b), since the heights of the various students, being elements of (26), cannot be intersected.

This explanation for the unavailability of reading (25b) relies on a quite general proposal about the structure of amounts. As a result, it predicts that amount-denoting expressions should show similar behavior wherever they appear in natural language, and not only in *wh*-expressions. The similarities between amount-denoting *wh*-expressions and comparatives, then, are explained in a most straightforward way: certain operations are not defined on amount-denoting expressions because of the algebraic structure of their denotations, regardless of the other details of the expressions they are embedded in. So, returning to (9),

(27) Every girl is less angry than Larry is.
Scope-splitting: *less* > *every girl* > *d-tall*
$\mathbf{max}(\lambda d.\mathbf{angry}(\text{Larry})(d)) > \mathbf{max}(\lambda d.\forall x[\mathbf{girl}(x) \rightarrow \mathbf{angry}(x)(d)])$
* "Larry's max degree of anger exceeds the greatest degree to which every girl is angry (i.e., he is angrier than the least angry girl)."

This interpretation is not available because, in the normal case, $\mathbf{max}(\lambda d.\forall x[\mathbf{girl}(x) \rightarrow \mathbf{tall}(x)(d)])$ is undefined. I conclude that the puzzle described by the Heim-Kennedy constraint was not a problem about the scope of a particular type of operator, but was generated by incorrect assumptions about the nature of amounts. Amounts are not simply points on a scale, but elements of (26). This proposal is independently motivated in S&Z, and it explains the restrictions captured in HK as well as other similarities between comparatives and weak islands.

A small but important exception to this generalization is that, since $A \cap A = A$ for any A, intersections **are** defined on (26) in the special case in which all individuals in the domain of quantification are mapped to the same point of the join semilattice. The prediction, then, is that the scope-splitting readings of (25) and (27) should emerge just in case it is assumed that all individuals have the property in question to the *same* degree. And indeed, S&Z note that there is a third reading of sentences like (25) which presupposes that the girls are all equally angry (see also Abrusán [1] for discussion).[3] Though S&Z do not make this connection, it seems that their theory makes another correct prediction in this domain: universal quantifiers can intervene when this presupposition is appropriate.[4]

3.3 Maximum Readings of Existential Quantifiers and Their Kin

The crucial difference between join semilattices (26) and chains (23) is that the latter is closed under intersection while the former is not. However, both are closed under union, which corresponds to existential quantification. Here the predictions of the present theory diverge from those of the LF-constraint in (10): on our theory, existential quantifiers and other quantifiers which are computed using only unions should be acceptable as interveners in both comparatives and amount *wh*-questions. (10), in contrast, forbids *all* quantificational DPs from intervening scopally.

In fact we have already seen an example which shows a quantificational intervener of this type: the only available reading of (24) is one which requests the number of laps finished by the *fastest* swimmer. These readings are also available with amount *wh*-questions, as (28) shows.

(28) How tall is at least one boy in your class?
[Answer: 6 feet.]

This reading does not seem to be available with the quantifier *some*. However, this is probably due to the fact that, for independent reasons, *some* in (29) can only have a choice reading.

(29) How tall is some professor?

a. "Pick a professor and tell me: How tall is (s)he?"
b. # "How tall is the tallest professor?"

The readings in (24) and (28) may be more robust than the corresponding reading of (29b) because *at least*, in contrast to *some*, does not support choice readings (cf. Groenendijk & Stokhof [6]).

Maximum readings also appear with *a NP* when the NP is focused:

(30) [The strongest girl lifted 80 kg.]

OK, but how much did a BOY lift?

[Answer: the amount that the strongest boy lifted.]

Since our account emphasizes the similarities between comparatives and *wh*-questions, we expect that similar readings should exist also in comparatives. These readings are indeed attested: for instance, Heim [9, p.223] notes this example.

[3] We cannot tell whether the comparative in (27) has this reading, since it is truth-conditionally equivalent to the direct scope reading.

[4] Thanks to an anonymous reviewer for emphasizing the importance of the reading which contains this "very demanding presupposition", and to Roberto Zamparelli for pointing out that this is an unexpected and welcome prediction of S&Z's account.

(31) Jaffrey is closer to an airport than it is to a train station.

This is true iff the *closest* airport to Jaffrey is closer than the *closest* train station.

An additional naturally occurring example which seems to display a maximum reading with an existential quantifier in a comparative complement, viz.:

(32) "I made the Yankee hat more famous than a Yankee can."
(Jay-Z, "Empire State of Mind", *The Blueprint 2*, Roc Nation, 2009)

From the context of this song, the artist is clearly not saying that he made the Yankees hat more famous than some particular Yankee (baseball player) can, or more than a typical Yankee can, but more than *any* Yankee can.

Finally, if *any* and *ever* in contexts such as the following has existential semantics, as claimed in Kadmon & Landman [10] and many others, then (37a) and (37b) also involve intervention by an existential quantifier:

(33) a. Larry endured more pain than any professor did.
b. My brother is angrier than I ever was.

Maximum readings, then, are well-attested in both amount comparatives and amount *wh*-questions. This supports the present theory in treating quantifier scope in comparatives and *wh*-questions in the same way, and is incompatible with the LF constraint (10) proposed by Heim.

4 Conjunction and Disjunction in the Comparative Complement

Noting an ambiguity similar to that in (3) – where a sentence with *or* in the comparative complement is equivalent on one reading to the same sentence with *and* replacing *or* – Schwarzschild & Wilkinson [23] suggest:

> *or* in these examples may in fact be a negative polarity item ... which has a conjunctive interpretation in this context, in the way that negative polarity *any* or *ever* seem to have universal interpretations in the comparative.

The difference between ordinary *or* meaning '$\vee$' and NPI *or* meaning '$\wedge$', then, is a matter of lexical ambiguity.[5] This is a possible analysis, but I think that it

[5] This claim is important to Schwarzschild & Wilkinson [23] because their semantics for comparatives prevents scope ambiguities of the type considered here from being computed, so that they have no choice but to treat *or* as ambiguous. However, Schwarzschild [22] notes that the earlier proposal in Schwarzschild & Wilkinson [23] wrongly predicts that there should be no scope ambiguities of the type considered here, and proposes an enrichment that is compatible with the analysis of the ambiguity of sentences with *or* and the non-ambiguity of the corresponding sentences with *and* suggested here. See section (6.1) for details.

would be desirable to derive the ambiguity of (3) without positing two meanings of *or*, in line with Grice's [5] Modified Occam's Razor ("Senses are not to be multiplied beyond necessity"). The present theory yields an explanation of this fact, and of why similar constructions with *and* are unambiguous.

The only available reading of the sentence in (4) is the one in (34a), where (34) is treated as an elliptical variant of (35).

(34) John is richer than his father was and his son will be.

a. $\mathbf{max}(\lambda d[\mathbf{rich}(d)(father)]) < \mathbf{max}(\lambda d[\mathbf{rich}(d)(John)]) \wedge \mathbf{max}(\lambda d[\mathbf{rich}(d)(son)]) < \mathbf{max}(\lambda d[\mathbf{rich}(d)(John)])$

(35) John is richer than his father was and he is richer than his son will be.

Why can't (34) be read as in (36), which ought to mean that John is richer than the *poorer* of his father and his son?

(36) $\mathbf{max}(\lambda d[\mathbf{rich}(d)(father) \wedge \mathbf{rich}(d)(son)]) < \mathbf{max}(\lambda d[\mathbf{rich}(d)(John)])$

The theory we have adopted offers an explanation: because conjunction, like universal quantification, relies on the operation of intersection, (36) is not available for the same reason that *Every girl is less angry than Larry* doesn't mean that the least angry girl is less angry than Larry. Computing (36) would require taking the intersection of the degrees of wealth of John's father and his son; but this is not possible, because this operation is not defined for amounts of wealth.

In contrast, the same sentence with *or* instead of *and* is ambiguous because the maximum reading (37a) can be computed using only unions, which are defined in a join semilattice. (37b) is the alternative reading on which (37) is elliptical, like the only available reading of (34).

(37) John is richer than his father was or his son will be.

a. $\mathbf{max}(\lambda d[\mathbf{rich}(d)(father) \vee \mathbf{rich}(d)(son)]) < \mathbf{max}(\lambda d[\mathbf{rich}(d)(John)])$
["He is richer than both."]

b. $\mathbf{max}(\lambda d[\mathbf{rich}(d)(father) < \mathbf{rich}(d)(John)]) \vee \mathbf{max}(\lambda d[\mathbf{rich}(d)(John)])$
["He is richer than one or the other, but I don't remember which."]

Thus it is not necessary to treat *or* as lexically ambiguous: the issue is one of scope.

5 Modals and Intensional Verbs

On S&Z's theory, existential quantifiers are able to intervene with amount expressions because join semilattices are closed under unions. This produces the "maximum" readings that we have seen. The analysis predicts, correctly, that (38) is ambiguous, a type of case discussed at length in Heim [9].

(38) (This draft is 10 pages.) The paper is allowed to be exactly 5 pages longer than that. [9, p.224]

a. *allowed > exactly 5 pages -er > that-long*
$\exists w \in Acc : \mathbf{max}(\lambda d : \mathbf{long}_w(p,d)) = 15pp$
"In some accessible world, the paper is exactly 15 pages long, i.e. it may be that long and possibly longer"

b. *exactly 5 pages -er > allowed > that-long*
$\mathbf{max}(\lambda d[\exists w \in Acc : \mathbf{long}_w(p,d)) = 15pp$
"The max length of the paper in any accessible world, i.e. its maximum allowable length, is 15 pages"

I am not entirely certain whether the corresponding *wh*-question is ambiguous: some speakers think it is, and others do not. The robust reading is (39b), which involves scope-splitting; the questionable reading is the choice reading (39a).

(39) How long is the paper allowed to be?

a. *allowed > how long > that-long*
? "Pick an accessible world and tell me: what is the length of the paper is that world?"
[Answer: "For example, it could be 17 pages long."]

b. *how long > allowed > that-long*
"What is the max length of the paper in any accessible world, i.e. its maximum permissible length?"
[Answer: "20 pages – no more will be accepted."]

If the answer given to (39a) is not possible, this may again be related to restrictions on the availability of choice readings, or possibly *allowed* does not have a quantificational semantics at all (as I will suggest for *required*).

So far, so good. However, since intersection is undefined with amount expressions, S&Z and the current proposal seem to predict that this ambiguity should be absent with universal modals, so that neither (40b) nor (41b) should be possible.

(40) (This draft is 10 pages.) The paper is required to be exactly 5 pages longer than that. [9, p.224]

a. *required > exactly 5 pages -er > that-long*
$\forall w \in Acc : \mathbf{max}(\lambda d : \mathbf{long}_w(p,d)) = 15pp$
"In every world, the paper is exactly 15 pages long"

b. *exactly 5 pages -er > required > that-long*
$max(\lambda d[\forall w \in Acc : \mathbf{long}_w(p,d)]) = 15pp$
"The max common length of the paper in all accessible worlds, i.e. its length in the world in which it is shortest, is 15 pages"

(41) How long is the paper required to be?

a. *required > how long > that-long*
"What is the length s.t. in every accessible world, the paper is exactly that long?"

b. *how long > required > that-long*
"What is the max common length of the paper in all accessible worlds, i.e. its length in the world in which it is shortest?"

But (40b) and (41b) *are* possible readings, and in fact are probably the most robust interpretations of (40) and (41).

S&Z suggest briefly that modals and intensional verbs may be acceptable interveners because they do not involve Boolean operations. I am not sure precisely what they have in mind, but we should consider the possibility that the issue with (40)-(41) is not a problem about the interaction between degree operators and universal quantification, but one about the analysis of modals and intensional verbs.

For instance, suppose that we were to treat modals not as quantifiers but as degree words, essentially as minimum- and maximum-standard adjectives as discussed in Kennedy & McNally [13] and Kennedy [12]. This analysis is motivated on independent grounds in Lassiter [17].[6] For reasons of space the theory will not be described in detail, but – on one possible implementation of the analysis for *allowed* and *required* – its predictions are these: only the (b) readings of (40)-(41) should be present, and it is (40a) and (41a) that need explanation. In the case of (40), at least, the (a) entails the (b) reading, and so we may suppose that only (40b) is generated, and its meaning is "Fifteen pages is the minimum requirement".[7] On such an analysis, (40a) is not a different reading of (40) but merely a special case of (40b), where we have further (e.g., Gricean) reasons to believe that that the minimum is also a maximum.[8]

The advantage of such an analysis, from the present perspective, is that it explains another stipulative aspect of Heim's proposed LF-constraint: why is the restriction limited to quantificational *DPs*? If the proposal I have gestured at in this section is correct, we have an answer: "universal" modals are immune to the prohibition against taking intersections with amounts because they are not really quantifiers. That is, computing them does not involve universal quantification over worlds, but simply checking that the degree (of probability, obligation, etc.) of a set of worlds lies above a particular (relatively high) threshold.

Even if this particular suggestion turns out to be incorrect, the difference between universal quantifiers and universal modals noted by S&Z and Heim

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[6] A proposal due to van Rooij [27] and Levinson [18] seems to make similar predictions for *want*. This may account for the fact that *want*, like *require*, is a scope-splitting verb.

[7] The suggestion made here also recalls a puzzle noted by Nouwen [19] about minimum requirements and what Nouwen calls "existential needs". *Need* is another scope-splitting verb, as it happens. I suspect that Nouwen's problem, and the solution, is the same as in the modal scope-splitting cases discussed here.

[8] An anonymous reviewer notes that this account does not extend from *required* to *should*, citing the following puzzling data also discussed in Heim [9]:

(i) Jack drove faster than he was required to.

(ii) Jack drove faster than he should have.

If the law requires driving at speeds between 45 mph and 70 mph, (i) is naturally interpreted as saying that Jack drove more than 45 mph, but (ii) says that he drove more than 70 mph. There are various possible analyses of these facts from the current perspective, including treating *required* as a minimum-standard degree expression, but *should* as a relative-standard expression (like *tall* or *happy*).

is an unexplained problem for all available theories of comparatives and weak islands, and not just the proposal made here. The general point of this section is simply that the setup of this problem relies on one particular theory of modals which may well turn out to be incorrect. Fleshing out a complete alternative is unfortunately beyond the scope of this paper, however.

6 Comparison with Related Proposals

In this section I compare the proposal made here with two influential proposals in the recent literature. The general conclusion is that the problem in (1) is not resolved by these modifications to the semantics of comparatives and/or *wh*-questions, and that a separate account is needed. However, the line of thought pursued here is essentially compatible with these proposals as well.

6.1 Schwarzschild & Wilkinson

An influential proposal due to Schwarzschild & Wilkinson [23] argues that comparatives have a semantics based on intervals rather than points. They show that, on this assumption, it is possible to derive apparent wide scope of quantifiers in the comparative complement without allowing QR out of the comparative complement. Since the latter would violate the general prohibition against extraction from a comparative complement, this result is welcome. However, it is also empirically problematic, and attempts to cope lead back to a solution of the type given here.

Schwarzschild & Wilkinson give the comparative sentence in (42a) a denotation that is roughly paraphrased in (42b):

(42) a. Larry is taller than every girl is.
b. The smallest interval containing Larry's height lies above the largest interval containing the heights of all the girls and nothing else.

(42b) will be true just in case, for every girl, Larry is taller than she is. As a result, the undesired "shortest-girl" reading is not generated.

Schwarzschild [22] acknowledges, though, that the proposal in Schwarzschild & Wilkinson [23] is too restrictive: it predicts that there should *never* be scope ambiguities between *more/-er/less* and quantifiers in the comparative complement. We have already seen a number of examples where such ambiguities are attested, involving existential quantification and its ilk (in (32)) and existential and (perhaps) universal modals in (38) and (40). In order to maintain the interval-based analysis, Schwarzschild [22] introduces a point-to-interval operator π which can appear in various places in the comparative complement (see also Heim [8] for a closely related proposal and much discussion). In this way, Schwarzschild derives the ambiguities discussed here without raising the QP out of the comparative clause.

The important thing to note, for our purposes, is that while Schwarzschild & Wilkinson [23] present a semantics on which the problems discussed here do not arise, Schwarzschild's [22] modification re-introduces scope ambiguities in comparatives in order to deal with the restricted set of cases where they do arise. This modification is valuable because it explains how these ambiguities arise despite the islandhood of the comparative clause; however, as Heim [8, pp.15-16] points out, in order to "prevent massive overgeneration of unattested readings, we must make sure that π never moves over a DP-quantifier, an adverb of quantification, or for that matter, an epistemic modal or attitude verb". This is essentially Heim's [9] proposed LF-constraint (10) re-stated in terms of the scope of π.

So we are back to square one: the interval-based account, though it has the important virtue of explaining apparent island-violating QR out of the comparative complement, does not explain the core puzzle that we are interested in, why (42a) lacks a "shortest-girl" reading. So the current proposal, or something else which does this job, is still needed in order to explain why (42a) does not (on the high-π reading) have the "shortest-girl" reading.[9,10]

6.2 Fox & Hackl

Next we turn to an influential proposal by Fox & Hackl [4]. I show that the theory advocated here is not in direct competition with Fox & Hackl's theory, but that there are some complications in integrating the two approaches which may cause difficulty for Fox & Hackl's.

Fox and Hackl argue that amount-denoting expressions always denote on a dense scale, effectively the lattice in (23) with the added stipulation that, for any two degrees, there is always a degree that falls between them. The most interesting data from the current perspective are in (43) and (44):

(43) a. How fast are we not allowed to drive?
b. *How fast are we allowed not to drive?

(44) a. How fast are we required not to drive?
b. *How fast are we not required to drive?

The contrasts in (43) and (44) are surprising from S&Z's perspective: on their assumptions, there is no maximal degree d such that you are not allowed to drive d-fast, and yet (23a) is fully acceptable. In addition, (43a) and (44a) do not ask for maxima but for minima (the least degree which is unacceptably fast, i.e. the speed limit). Fox and Hackl show that the minimality readings of (43a) and (44a), and the ungrammaticality of (43b) and (44b) follow if we assume (following [3] and [2]) that *wh*-questions do not ask for a maximal answer but for a maximally informative answer, defined as follows:

[9] Although intervals are usually assumed to be subsets of the reals – and so fit naturally with totally ordered structures like (22) – there is no barrier in principle to defining an interval-based degree semantics for partially ordered domains. Of course, some issues of detail may well arise in the implementation.

[10] A similar point holds for Abrusán [1]: her interval-based semantics does well with negative and manner islands, but does not account for quantificational interveners.

(45) The maximally informative answer to a question is the true answer which entails all other true answers to the question.

Fox & Hackl show that, on this definition, upward monotonic degree questions ask for a maximum, since if John's maximum height is 6 feet, this entails that he is 5 feet tall, and so on for all other true answers. However, downward entailing degree questions ask for a minimum, since if we are not allowed to drive 70 mph, we are not allowed to drive 71 mph, etc.

This is not as deep a problem for the present theory as it may appear. S&Z assume that *wh*-questions look for a maximal answer, but it is unproblematic simply to modify their theory so that *wh*-questions look for a maximally informative answer. Likewise, we can just as easily stipulate that a join semilattice (26) is dense as we can stipulate that a number line (23) is dense; this maneuver would replicate Fox & Hackl's result about minima in downward entailing contexts. In this way it is possible simply to combine S&Z's theory with Fox & Hackl's. In fact, this is probably independently necessary for Fox & Hackl, since their assumption that amounts always denote in (23) fails to predict the core data of the present paper: the fact that *How tall is every girl?* and *Every girl is less tall than John* lack a "shortest-girl" reading. I conclude that the two theories are compatible, but basically independent.

Finally, note that the maximal informativity hypothesis in (45), whatever its merit in *wh*-questions and other environments discussed by Fox & Hackl, is not appropriate for comparatives: here it appears that we need simple maximality.[11]

(46) a. How fast are you not allowed to drive?
b. *You're driving faster than you're not allowed to.

A simple extension of the maximal informativity hypothesis to comparatives would predict that (46b) should mean "You are exceeding the speed limit". In contrast, the maximality-based account predicts that (46b) is unacceptable, since there is no maximal speed which is not allowed. This appears to be the correct prediction. However, it is worth noting that, because of the asymmetry between (43) and (46), combining the two theories in the way suggested here effectively means giving up the claim that the comparative is a type of *wh*-operator. Since this idea has much syntactic and semantic support, it is probably worth looking for an alternative explanation of (43) and (44) that does not involve adopting the proposal in (45).

7 Conclusion

To sum up, the traditional approach on which amounts are arranged on a scale of degrees fails to explain why the constraint in (10) should hold. However, the numerous similarities between limitations on comparatives and amount-denoting *wh*-questions with quantifiers suggest that these phenomena should be explained by a single theory. S&Z's semantic account of weak islands predicts, to a large

[11] Thanks to a PLC reviewer for bringing the contrast in (46) to my attention.

extent, where quantifier intervention is possible and where it is not. The crucial insight is that intervention effects are due to the kinds of operations that quantifiers need to perform, and not merely the structural configuration of various scope-taking elements. S&Z's theory also predicts correctly that narrow-scope conjunction is impossible in amount comparatives, but narrow-scope disjunction is possible. To be sure, important puzzles remain; but the algebraic approach to comparative scope offers a promising explanation for a range of phenomena that have not been previously treated in a unified fashion.

Furthermore, if S&Z's theory turns out to be incomplete, all is not lost. The most important lesson of the present paper, I believe, is not that S&Z's specific theory of weak islands is correct — as we have seen, there are certainly empirical and technical challenges[12] — but rather that weak island phenomena are not specific to *wh*-questions. In fact, we should probably think of the phenomena summarized by the Heim-Kennedy constraint as *comparative weak islands*. However the theory of weak islands progresses, evidence from comparatives will need to play a crucial role in its development — and vice versa.[13]

References

1. Abrusán, M.: Contradiction and Grammar. PhD thesis, MIT (2007)
2. Beck, S., Rullmann, H.: A flexible approach to exhaustivity in questions. Natural Language Semantics 7(3), 249–298 (1999)
3. Dayal, V.: Locality in wh-quantification. Kluwer, Dordrecht (1996)
4. Fox, D., Hackl, M.: The universal density of measurement. Linguistics and Philosophy 29(5), 537–586 (2006)
5. Grice, H.P.: Studies in the Way of Words. Harvard University Press, Cambridge (1989)
6. Groenendijk, J., Stokhof, M.: Studies in the Semantics of Questions and the Pragmatics of Answers. PhD thesis, University of Amsterdam (1984)
7. Hackl, M.: Comparative quantifiers. PhD thesis, MIT (2000)
8. Heim, I.: Remarks on comparative clauses as generalized quantifiers. Ms. MIT, Cambridge (2006)
9. Heim, I.: Degree operators and scope. In: Fery, Sternefeld (eds.) Audiatur Vox Sapientiae: A Festschrift for Arnim von Stechow. Akademie Verlag, Berlin (2001)
10. Kadmon, N., Landman, F.: Any. Linguistics and philosophy 16(4), 353–422 (1993)

[12] In particular, S&Z do not give a compositional implementation of their proposal. I do not see any very deep difficulties in doing so, although, as Anna Szabolcsi (p.c.) points out, treating amounts and counters differently in semantic terms despite their similar (or possibly identical) syntax might seem unattractive to some. Thanks to Anna Szabolcsi and an anonymous ESSLLI reviewer for pointing out the need for this note.

[13] Thanks to Chris Barker, Anna Szabolcsi, Arnim von Stechow, Emmanuel Chemla, Yoad Winter, Lucas Champillon, Rick Nouwen, Roberto Zamparelli, Roger Schwarzschild, several anonymous reviewers, and audiences at the 33rd Penn Linguistics Colloquium and the 2009 ESSLLI Student Session for helpful discussion and advice. An earlier version of this paper appeared as Lassiter [16].

11. Kennedy, C.: Projecting the adjective: The syntax and semantics of gradability and comparison. PhD thesis, U.C., Santa Cruz (1997)
12. Kennedy, C.: Vagueness and grammar: The semantics of relative and absolute gradable adjectives. Linguistics and Philosophy 30(1), 1–45 (2007)
13. Kennedy, C., McNally, L.: Scale structure, degree modification, and the semantics of gradable predicates. Language 81(2), 345–381 (2005)
14. Kroch, A.: Amount quantification, referentiality, and long wh-movement. Ms., University of Pennsylvania (1989)
15. Landman, F.: Structures for Semantics. Kluwer, Dordrecht (1991)
16. Lassiter, D.: Explaining a restriction on the scope of the comparative operator. University of Pennsylvania Working Papers in Linguistics, 16(1) (2010)
17. Lassiter, D.: Gradable Epistemic Modals, Probability, and Scale Structure. In: Proceedings from Semantics and Linguistic Theory XX (to appear, 2010)
18. Levinson, D.: Probabilistic Model-theoretic Semantics for *want*. In: Proceedings from Semantics and Linguistic Theory XIII (2003)
19. Nouwen, R.: Two puzzles about requirements. In: Proceedings of the 17th Amsterdam Colloquium, pp. 326–334 (2009)
20. Rizzi, L.: Relativized Minimality. MIT Press, Cambridge (1990)
21. Rullmann, H.: Maximality in the Semantics of wh-constructions. PhD thesis, University of Massachusetts, Amherst (1995)
22. Schwarzschild, R.: Scope-splitting in the comparative. Handout from MIT colloquium (2004), `http://www.rci.rutgers.edu/~tapuz/MIT04.pdf`
23. Schwarzschild, R., Wilkinson, K.: Quantifiers in comparatives: A semantics of degree based on intervals. Natural Language Semantics 10(1), 1–41 (2002)
24. Szabolcsi, A.: Strong vs. weak islands. Blackwell Companion to Syntax 4, 479–531 (2006)
25. Szabolcsi, A., Zwarts, F.: Weak islands and an algebraic semantics for scope taking. Natural Language Semantics 1(3), 235–284 (1993)
26. Takahashi, S.: More than two quantifiers. Natural Language Semantics 14(1), 57–101 (2006)
27. van Rooij, R.: Some analyses of pro-attitudes. In: de Swart, H. (ed.) Logic, Game Theory, and Social Choice. Tilburg University Press (1999)
28. von Stechow, A.: Comparing semantic theories of comparison. Journal of Semantics 3(1), 1–77 (1984)