# Graded modality

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(Draft of a survey. Comments welcome.)

**Abstract** Obligation, goodness, certainty, likelihood, knowledge, belief, and ability are examples of modal concepts—concepts whose conditions for application depend not only on how the real world is, but also on various non-actual ways that the world could be. *Ought, good, certain, likely, know, believe*, and *be able* are examples of modal words whose meaning is closely related to the modal concepts just mentioned. Is there graded structure underlying (at least some) modal concepts – are they all-or-nothing, or can they come in degrees? To the extent that there is, how should semanticists model the meanings of modal expressions whose meanings make reference to graded concepts? Modals are usually though of as (non-graded) quantificational expressions, but there is also a tradition of treating some modal concepts—such as goodness, likelihood, and obligation—on the analogy of graded concepts like height, weight, and temperature. Recently there has been increased attention to scalar aspects of the meanings of modal expressions, but there are still many interesting open questions about how far modal gradation extends, and how it relates to grammatical gradability of the type studied in the extensive literature on degree expressions.

**Keywords:** modality, gradation, gradability, counterfactuals, concepts

In this chapter I will introduce some of the main empirical questions and theoretical distinctions in the study of the relationship between modality and gradation. Rather than trying to give an exhaustive list of relevant references or to do full justice to each perspective mentioned—an impossible task given the vastness of this terrain—I will try to tell a coherent, historically-oriented story, and I will focus on the topics that I know best. The discussion proceeds as follows: after drawing attention to a crucial distinction between **gradation** of concepts and **gradability** of linguistic items, we will consider the influential tradition of analyzing modal operators in terms of quantification over sets of possible worlds, and its relationship to a parallel tradition of analyzing certain modal concepts in terms of concepts with grade structure, such as degree of goodness or belief. A historical example is used to illustrate the potential for theories of the semantics of gradability—specifically, the boundedness of scales—to illuminate interesting modal concepts. We then turn to a number of examples of analyses of graded modal concepts, organized by conceptual domain: counterfactuals and similarity, gradable and possibly gradable deontic adjectives, and epistemic adjectives and verbs, drawing connections to broader issues in degree semantics along the way, and concluding with a brief discussion of additional gradable modal items such as *want*.[1]

1 There are many worthwhile related topics that are omitted for reasons of space. In addition, important questions around

The key take-home lessons are that there are many linguistically, philosophically, and psychologically interesting connections between modality and gradation, and that much empirical and formal work remains to be done: we have only scratched the surface of this rich and interdisciplinary topic. Progress on this front will require simultaneous attention to compositional semantics, semantic relations among items, and psychological and philosophical work on modality, reasoning, and concepts.

## 1 Graded concepts and gradable expressions

Concepts—mental representations of interesting features of the world, like SQUARE, WEIGHT, RUN, INFINITY, MECHANIC, DOG WITH THREE LEGS—have been the subject of much interest among philosophers and cognitive scientists (see Margolis & Laurence 1999; Murphy 2002). There is no consensus about what precisely concepts are, how they are defined, or where they come from. Here we are interested in the role of concepts in the lexical semantics of natural language expressions, and we can make do with a few simple assumptions. I will assume that concepts are mental representations (rather than mind-independent features of the world); that they can be atomic, or built up as a complex from more basic concepts; and that they ground our interpretations of linguistic expressions, but cannot be identified with the linguistic expressions themselves.

According to the classic "Aristotelian" theory, concepts are defined in terms of necessary and sufficient conditions that determine exactly which things they apply to and which they do not. For instance, SQUARE might be associated with the conditions "two-dimensional figure with four sides of equal length and four equal interior angles". This approach was problematized by Rosch (1978), who observed that even concepts like FRUIT do not seem to be all-or-nothing: experimental participants will rate some things (e.g., apples) to be better examples of fruit than others (e.g., crabapples and tomatoes). Such "prototype effects" are unexpected under the Aristotelian theory, since—whatever the criteria associated with FRUIT are—there would seem to be only two possible answers to the question of whether something satisfies these criteria: "yes" or "no". Subsequent work has corroborated Rosch's basic idea many times over, while also providing a variety of ways to spell out the basic idea of "graded structure" more fully: see Murphy 2002 for discussion [SEE ALSO: Family resemblance, prototypes, and stereotypes] .[2]

However, an analysis in terms of necessary and sufficient conditions remains quite plausible for certain mathematical concepts—SQUARE and PRIME, for example—and logical concepts such as SOME and ALL. The latter case is of special interest in the study of modality. Here we are dealing with the necessary and sufficient conditions for two concepts to be in a certain relation to each other [SEE ALSO: nominal vs. adverbial quantification].

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the relationship between deontic *ought/should* and *must* (von Fintel & Iatridou 2008) are complex and may not be interpretable in terms of a simple scalar relationship between these items, and so are not discussed. I have also not attempted to do justice to the full extent of Angelika Kratzer's broad impact on the study of modality and conditionals. These contributions are covered in greater detail elsewhere in these volumes [SEE ALSO: epistemic modals, weak necessity, counterfactuals, indicative conditionals].

2 The presence of graded structure somewhere in the underlying concept is a prerequisite for vagueness of a linguistic expressions, but it does not entail vagueness: for instance, *more than 6 feet tall* is not vague, even though HEIGHT has graded structure (Kamp & Partee 1995; SEE ALSO: VAGUENESS AND NATURAL LANGUAGE SEMANTICS).

(1) a. The SOME relation holds between two concepts $A$ and $B$ if and only if at least one instance of $A$ is an instance of $B$.

b. The ALL relation holds between two concepts $A$ and $B$ if and only if each instance of $A$ is also an instance of $B$.

There is no gradation here. Either at least one $A$ is a $B$, or no $A$ is a $B$, with no room for intermediate status, and similarly for ALL.[3]

This kind of treatment is not appropriate for concepts that admit of degrees, like TEMPERATURE or WEIGHT. These concepts do more than classify individuals into "yes" and "no" categories: they associate individuals with ordered values. These values are called **degrees**, and a set of degrees with an ordering on them is a **scale**.

Natural languages allow speakers to express concepts using words and phrases. When analyzing words we can ask not only about the structure of the concepts they express, but also about grammatical features of the linguistic means of expressing the concept. In other words, we can ask the question "Does it make sense to think and talk about degrees of concept $X$?"; if the answer is "yes", we can then ask the further question "Does the language we are studying have the grammatical resources to refer implicitly or explicitly to degrees of $X$?". These are subtly, but importantly, different.

Cross-linguistic differences in the expression of degree provide one important motivation for maintaining this distinction carefully. In certain languages, the expressions that make reference to concepts like TEMPERATURE and WEIGHT are not grammatically gradable. However, speakers of these languages clearly understand that these concepts have graded structure. For instance, Bochnak (2014) shows that there is no evidence of grammatical gradability of any kind in the Native American language Washo, even though Washo has verbs corresponding closely to unmodified adjectives in English, like *hot* and *heavy*. These Washo verbs have the same kinds of meanings as English adjectives like *hot* and *heavy*: they are vague, and have context-dependent meanings which vary depending on (for example) whether we are talking about being heavy-for-a-baby, heavy-for-a-suitcase, or heavy-for-a-truck. Washo speakers clearly possess the concept of one thing being heavier than another, and even use overt comparatives when speaking English. It is simply a grammatical fact about their language that it does not allow them to refer explicitly to these degrees. See Beck, Oda & Sugisaki 2004; Beck, Krasikova, Fleischer, Gergel, Hofstetter, Savelsberg, Vanderelst & Villalta 2009; Bochnak 2013 for more on cross-linguistic variation in the expression of degree.

English, however, has expressions that about graded concepts (WEIGHT) and also make these degrees accessible grammatically to operators like *more/-er, very*, and *too*. These expressions—exemplified here by *heavy*—are what I will call "gradable".

(2) a. This truck is heavier than that car.

b. This truck is very/too heavy.

A possible analysis of this interaction is sketched in (3). (4) gives an analysis of the comparative

3 Note that this is not a claim about the English word *all*, which may well be gradable: consider *almost/absolutely all*, etc. (and compare Löbner 1990 on German *alle* "all"). If English *all* is indeed gradable, it does not express the non-graded concept described in (1b), which is conventionally called ALL.

(2a) as an example. (See von Stechow 1984; Kennedy 1997, 2007; Heim 2001 and [SEE ALSO: GRADABLE ADJECTIVES AND DEGREE EXPRESSIONS].)

(3) WEIGHT is a concept which includes at least the following:
   a. a set of degrees $D_{weight}$;
   b. an ordering $\geq$ which determines, for each $d$ and $d' \in D_{weight}$, whether or not $d \geq d'$; and
   c. a function $\mu_{weight}$ which assigns to each individual a unique $d \in D_{weight}$ representing their weight.[4]

We can then analyze comparative sentences with the form of (2a) along the following lines:

(4) For any $x$ and $y$, *x is heavier than y* is true if and only if $\mu_{weight}(x) > \mu_{weight}(y)$,

where "$d > d'$" abbreviates "$d \geq d'$, and NOT: $d' \geq d$". To make the semantics compositional, we need to assign a meaning to *-er*—something like (5).

(5) For any $A$, $x$, and $y$, *x is A-er than y* is true if and only if $\mu_A(x) > \mu_A(y)$, where $\mu_A$ is the measure function associated with $A$.

Note that a concept must have graded structure if there is a gradable expression which refers to degrees of the concept, e.g., by forming comparatives and superlatives or being modified by *very*. Otherwise there would be no degrees for these degree morphemes to act upon. However, we cannot determine conclusively whether a concept has graded structure simply by asking whether its linguistic exponents are gradable: as the vocabulary of Washo illustrates, it is perfectly possible for a non-gradable expression to make reference to a graded concept.

## 2 Graded modality: Some history and examples

Following this line of thought, we can think of the study of "graded modality" as involving two questions. First, we can ask which, if any, modal concepts are graded—which are better analyzed as being like TEMPERATURE and WEIGHT rather than ALL and SQUARE. Second, we can ask whether there are modal expressions that are grammatically gradable. As we will see, there is good reason to answer both questions in the affirmative.

### 2.1 Possible worlds and the ALL/SOME/NONE conception of modal meaning

The modal concepts NECESSITY and POSSIBILITY are usually analyzed as "all" and "some" quantifiers over sets of "possible worlds"—fully-specified ways that the world could be.

(6) a. NECESSARY applies to a sentence $\phi$ if and only if $\phi$ is true in all possible worlds.
   b. POSSIBLE applies to a sentence $\phi$ if and only if $\phi$ is true in some (i.e., at least one) possible world.

4 We are glossing over the important issue of intensionality here. In a more complete theory, WEIGHT would assign a degree $d$ to pairs consisting of a world $w$ and an individual $x$, where $d$ represents how heavy $x$ would be *if* the facts were all as they are in $w$.

This analysis has its roots in the work of Gottfried Wilhelm von Leibniz (1646-1716), who spoke of "possible worlds" and whose analysis of necessary and contingent truth is equivalent to these more modern-looking statements (see Look 2013). Leibniz was concerned with what is now called "absolute" necessity and possibility. For instance, the "necessary truth" of *Two plus two is four* could be analyzed as "In all possible worlds, two plus two is four". For Leibniz, this represents the extreme boundaries of possibility: even God could not create a universe in which basic arithmetical facts are different. The "logical possibility" that the moon is made of green cheese could be analyzed as "In some logically possible world, the moon is made of green cheese". There is no strictly logical reason to reject this bizarre possibility.

The absolute concept of possibility is much weaker than what the English word *possible* is generally used for. If I told someone that it is a possibility that the moon is made of green cheese, he would surely conclude that I am unwell. Likewise, the absolute concept of necessity is too strong for "necessary", which is frequently used to qualify statements expressing (e.g.) strong preferences rather than mathematical or conceptual truths (Rubinstein 2012).

(7) It is necessary that health workers have a basic understanding of the fundamental principles of maintaining a healthy body. (web)

(7) does not suggest that, in all possible worlds, health workers have a basic understanding of the fundamental principles of maintaining a healthy body. Rather, it describes an ideal that should be pursued, even if it is frequently violated in the real world. So, we have to weaken Leibniz's interpretation of NECESSARY somehow if we want to capture the fact that (7) is not patently false.

The modern semantic literature on modal concepts—starting notably with the work of Kripke (1959, 1963) and Hintikka (1962)—has extended Leibniz's ideas in an important way: "all" and "some" in (6) do not really quantify over all possible worlds, but over a smaller set representing some coherent domain of possibilities [SEE ALSO: EPISTEMIC MODALITY; ATTITUDE VERBS]. This enrichment makes the "all/some worlds" approach to modality much more useful for the analysis of a wide range of concepts and linguistic expressions. For example, we might analyze *Mary believes that it's raining* as in (8).

(8) In all worlds *that are compatible with Mary's beliefs*, it is raining.

Similarly, (7) could be relativized to worlds where certain norms are observed.

(9) In all worlds *that conform to the norms of the healthcare profession*, health workers have a basic understanding of the fundamental principles of maintaining a healthy body.

The intuitive falsity of *It is possible that the moon is made of green cheese* can be understood if *possible* is a SOME-quantifier, not over all possible worlds, but over those compatible with what some relevant person or group—say, the speaker—believes.

(10) In some world *that is compatible with the speaker's beliefs*, the moon is made of green cheese.

This interpretation of (10) does a better job of explaining the intuitive bizarreness of *It is possible that the moon is made of green cheese*.

In the wake of Kripke's and Hintikka's influential work, it has become a sort of default analytic choice in formal semantics to treat an enormous variety of modal expressions—of varying syntactic

categories—as picking out SOME/ALL-quantifiers over restricted domains of possible worlds: *may, might, can, should, ought, have to, able, certain/certainly, possible/possibly, necessary/necessarily, believe, think, know, suspect, want, wish*, and many more. Importantly, a formalization along these lines implies that the concept being analyzed is not graded: SOME and ALL are all-or-nothing concepts. However, as we will see in section 3 below, the "some/all worlds" analysis has been developed for certain items in a way that also attends to the graded structure of many modal concepts—most influentially, in the work of Lewis (1973) and Kratzer (1981, 1991, 2012).

### 2.2 Graded modal concepts

The idea that some modal concepts have graded structure has a long history as well. For example, the philosopher John Locke announced in the Introduction to *An Essay Concerning Human Understanding* (1690) that one of the main purposes of the book was to inquire into "the grounds and degrees of belief, opinion, and assent"—taking for granted that these are things that come in degrees. Around the same time, Jacob Bernoulli's *Art of Making Conjectures* (finished 1705, published 1714) proposed a scalar model for forming rational degrees of certainty in light of evidence.

Four centuries earlier, Saint Thomas Aquinas used the assumption that the modal concept of MORAL GOODNESS comes in degrees as a crucial premise in one of his logical deductions of the existence of God in *Summa Theologica* (written 1265-1274).[5]

> Among beings there are some more and some less good, true, noble and the like. But "more" and "less" are predicated of different things, according as they resemble in their different ways something which is the maximum, as a thing is said to be hotter according as it more nearly resembles that which is hottest; so that there is something which is truest, something best, something noblest and, consequently, something which is uttermost being... Now the maximum in any genus is the cause of all in that genus; as fire, which is the maximum heat, is the cause of all hot things. Therefore there must also be something which is to all beings the cause of their being, goodness, and every other perfection; and this we call God.

Aquinas' argument, while probably unconvincing to the modern reader, is linguistically interesting in several ways. It assumes that GOODNESS is a scalar concept, and it explicitly relies on an analogy between this modal concept and the non-modal scalar concept TEMPERATURE. What is more, Aquinas' argument depends on the assumption that the scales resemble each other in that both are **upper-bounded**—both have a fixed maximum element.

This is an issue about what linguists now call **scale structure**, a set of formal properties of degree scales that can be imposed by placing further restrictions on definitions like (3) above, and which have been shown to have semantic and pragmatic consequences. Specifically, the basic graded analysis of MORAL GOODNESS would look something like (11), and the optional upper-bounding

5 The quote is taken from the First part, Question 2 of *The Summa Theologica of St. Thomas Aquinas*, Second and Revised Edition of 1920, translated by Fathers of the English Dominican Province. The full text is available online at http://www.newadvent.org/summa/.

condition would be (12). (TEMPERATURE is a slight variant, with "temperature" replacing "good" and "individual" replacing "proposition".)

(11) MORAL GOODNESS is a concept which includes at least the following:
   a. a set of degrees $D_{good}$;
   b. an ordering $\geq$ which determines, for each $d$ and $d' \in D_{good}$, whether or not $d \geq d'$; and
   c. a function $\mu_{good}$ which assigns to each proposition a unique $d \in D_{good}$ representing its goodness.

(12) In the concept of MORAL GOODNESS, there is a degree $d \in D_{good}$ such that $d \geq d'$ for all $d' \in D_{good}$.

The idea that goodness has graded structure as in (11) appears more or less explicitly in many other places in philosophy, notably utilitarianism, an influential moral theory which enjoins people to maximize the goodness of the consequences of their acts (Mill 1863). Obviously, it only makes sense to "maximize goodness" if goodness is graded.

The graded concepts of goodness and temperature are, as we have already seen, associated with the gradable adjectives *good* and *hot*. If the adjectives expose the degree structure of the associated concepts in a transparent way, we should be able to use linguistic evidence to discern whether the underlying concepts do indeed have fixed maximum degrees. For example, graded concepts may differ in whether their scales have fixed upper and lower bounds. Height clearly has no upper bound—in principle something could keep getting taller forever—but fullness does: there's a limit to how much you can pack into a container before you couldn't possibly get anything else in. This difference has been argued to influence a number of semantic and pragmatic properties of these adjectives, such as which degree modifiers they are able to combine with (Hay, Kennedy & Levin 1999; Rotstein & Winter 2004; Kennedy & McNally 2005; Kennedy 2007) [SEE ALSO: GRADABLE ADJECTIVES AND DEGREE EXPRESSIONS].

The proposed test for upper-boundedness involves whether a gradable adjective expressing the concept can be modified by "maximizing" degree modifiers such as *perfectly*. If an adjective can be modified by *perfectly*, with a maximum-degree reading, this would entail that the graded concept is associated with a fixed maximum degree. For example:

(13) a. The glass is perfectly full.
   b. ? Bill is perfectly tall.

The symbol "?" indicates that the sentence is odd or "infelicitous". These judgments suggest that *full* lives on an upper-bounded scale, while *tall* does not. This seems to accord well with intuition about the concepts of height and fullness (Kennedy & McNally 2005).

Applying this test to *good* and *hot*, we get:

(14) a. ? It is perfectly good to lend money to friends.
   b. ? The weather is perfectly hot today.

(14a) cannot be assigned a maximum-degree interpretation, where it would conflict with the claim that curing cancer is even better than lending money to friends. Similarly, a maximizing reading of

(14b) would entail that it could not possibly be hotter. In the context of the theories of gradation just mentioned, we can take these data points as evidence against Aquinas' claim that goodness and temperature are upper-bounded concepts—and so against the addition of the constraint in (12). This is one example of the potential for the grammar and usage of modal language to illuminate the structure of an interesting modal concept.

## 3 Counterfactuals and deontic gradation

### 3.1 Lewis on counterfactuals

Lewis (1973) pointed out that certain important modal concepts—counterfactuals and deontic comparatives—seem to require more graded structure than "all"/"some"-quantification over sets of worlds could provide. For counterfactual conditionals, Lewis' motivating examples are in (15) (see also [SEE ALSO: COUNTERFACTUALS]).

(15) a. If kangaroos had no tails, they would topple over.
b. If kangaroos had no tails but used crutches, they would not topple over.

An initially plausible analysis of (15a) goes like this. Fix a set of worlds $R$ which are currently relevant to the interpretation of counterfactuals. Then (15a) is true just in case *Kangaroos topple over* is true in all of the worlds in $R$ in which *Kangaroos have no tails* is true.

The problem is that (15a) is intuitively consistent with (15b), but the analysis just sketched predicts otherwise. If all of the $R$-worlds in which kangaroos have no tails are worlds in which they topple over, then all of the $R$-worlds in which kangaroos have no tails *but use crutches* must also be worlds in which they topple over. In schematic form, the analysis predicts that the inference from *If* $\phi$*, would* $\chi$ to *If* $\phi$ *and* $\psi$*, would* $\chi$ should be valid for any sentences $\phi$, $\psi$, and $\chi$. As Lewis observes, this prediction is apparently incorrect.[6]

Lewis proposes that we should continue to interpret counterfactuals as (non-gradable) "all"-quantifiers, but that we should relativize their interpretation to a graded concept of SIMILARITY TO THE ACTUAL WORLD.[7] The idea is that a counterfactual is interpreted as an "all"-quantifier over the worlds that satisfy the antecedent *and are otherwise as similar as possible to the actual world*. In the actual world, kangaroos have tails, and they do not use crutches. So, the possible worlds which are as similar as possible to the actual world, except that kangaroos have no tails, will be ones in which they still do not use crutches. In these worlds, they topple over, and so (15a) is true. However, to interpret (15b) we look at worlds which are even less similar to the actual world: kangaroos have no tails *and* use crutches. In these worlds, they do not topple over. So, (15a) and (15b) can both be true.

Lewis does not model comparative similarity to the actual world using degrees, but with a **qualitative ordering** $\preccurlyeq_w$, where $w_1 \preccurlyeq_w w_2$ means that $w_1$ is at least as similar to the actual world $w$ as $w_2$ is. Lewis imposes several technical conditions on this ordering (including reflexivity,

6 However, see von Fintel 2001 for an argument that the validity of this inference is in fact compatible with the data adopt a dynamic perspective on the interpretation of counterfactuals.

7 This is a simplification intended to prime intuitions. We should really be thinking of a parametrized concept SIMILARITY-TO-$w$, where $w$ is set to the actual world in unembedded contexts [SEE ALSO: attitude verbs].

transitivity, and connectedness) which together enforce features of the concept "at least as similar as" that he finds intuitive. While this kind of gradation appears somewhat different from the degree-based gradation that we discussed above, it is really very closely related: for any ordering satisfying the conditions on Lewis' $\preccurlyeq_w$, we can find a degree-based representation that captures all of the same information and looks just like the ones we used for temperature and weight above. The basic idea is to set up a scale of similarity-to-the-actual-world-$w$, and a function $\mu_{similar\text{-}to\text{-}w}$ which assigns each world a degree on that scale. We then ensure that $\mu_{similar\text{-}to\text{-}w}(w_1) \geq \mu_{similar\text{-}to\text{-}w}(w_2)$ if and only if $w_1 \preccurlyeq_w w_2$.[8]

(16) SIMILARITY TO THE ACTUAL WORLD is a concept which includes at least the following:
   a. a set of degrees $D_{similar\text{-}to\text{-}w}$;
   b. an ordering $\geq$ which determines, for each $d$ and $d' \in D_{similar\text{-}to\text{-}w}$, whether or not $d \geq d'$;
   c. a function $\mu_{similar\text{-}to\text{-}w}$ which assigns to each world a unique $d \in D_{similar\text{-}to\text{-}w}$.
   d. "Centering": A requirement that nothing is more similar to the actual world $w$ than it is to itself: $\mu_{similar\text{-}to\text{-}w}(w) \geq d$ for all $d \in D_{similar\text{-}to\text{-}w}$.

In Lewis' theory, counterfactuals are not gradable, since they universally quantify over a set of worlds. However, the way that these worlds are selected depends crucially on a graded concept of similarity.

### 3.2 Deontic gradation

Building on earlier work relating deontic concepts to a formal notion of **preference order** (von Wright 1963; Rescher 1968), Lewis (1973: §5) suggests a way interpreting graded deontic concepts such as GOODNESS using a semantics closely related to his proposal for counterfatuals. Instead of reading $w_1 \preccurlyeq_w w_2$ as "$w_1$ is at least as similar to $w$ as $w_2$ is", we will read it as "$w_1$ is at least as good as $w_2$, relative to the moral standards that are relevant at $w$". Equivalently, we can rewrite the degree-based representation in (16) but replace "similar to $w$" with "good", ending up with something that is at first glance quite similar to the MORAL GOODNESS scale that we defined in (11) above.

(17) GOODNESS (relative to $w$) is a concept which includes at least the following:
   a. a set of degrees $D_{good\text{-}at\text{-}w}$;
   b. an ordering $\geq$ which determines, for each $d$ and $d' \in D_{good\text{-}at\text{-}w}$, whether or not $d \geq d'$;
   c. a function $\mu_{good\text{-}at\text{-}w}$ which assigns to each world a unique $d \in D_{good\text{-}at\text{-}w}$.

The Centering assumption (16d) does not appear in (17). This condition would not make sense for deontic concepts, since the world that we live in is presumably not the possible world which best conforms to its own moral standards.

8 There is an infinite number of such representations for any $\preccurlyeq_w$ obeying Lewis' conditions. Some care has to be taken in setting up the correspondence formally: see Krantz, Luce, Suppes & Tversky 1971; Klein 1991; Sassoon 2013; Lassiter 2017.

There is an important difference between MORAL GOODNESS (11) and GOODNESS in (17). Above we assumed that MORAL GOODNESS assigns degrees of goodness to **propositions**—sentence-meanings, which we are identifying with sets of possible worlds. This was because, in sentences like *It is better that Bill talks to Mary than it is that he talks to Sue*, *better* relates sentence-meanings. So we should interpret it using a measure function mapping propositions to degrees on a scale.

Since Lewis' GOODNESS measures worlds rather than propositions, we cannot use it directly to interpret *better*-sentences. A common approach, still much used in formal semantics, is to order propositions by considering the relative positions of the **highest-ranked** worlds in each.

(18) $\phi$ *is at least as good as* $\psi$ is true if and only if, for some $w \in \phi$, $\mu_{good}(w) \geq \mu_{good}(w')$ for all $w' \in \psi$.

This derives a notion of goodness for propositions as follows.[9]

(19) $\phi$ *is better than* $\psi$ is true if and only if there is some $w \in \phi$ such that $\mu_{good}(w) > \mu_{good}(w')$ for all $w' \in \psi$.

In effect, we "lift" the graded concept GOODNESS (OF WORLDS) to a graded concept GOODNESS (OF PROPOSITIONS), and use the latter to interpret expressions like *better than*.

### 3.3 Deontic adjectives and gradability

Lewis (1973) also suggests interpreting the deontic adjectives *obligatory* and *permissible* as quantifiers over sets of worlds derived from the un-lifted GOODNESS (OF WORLDS) concept. Adapted for the degree-based version, his proposals are given informally in (20).

(20) a. *It is obligatory that* $\phi$ is true at $w$ if and only if all worlds in $\textbf{BEST}(\mu_{good\text{-}at\text{-}w})$ are worlds in which $\phi$ is true.

b. *It is permissible that* $\phi$ is true at $w$ if and only if at least one world in $\textbf{BEST}(\mu_{good\text{-}at\text{-}w})$ is a world in which $\phi$ is true.

$\textbf{BEST}(\mu_{good\text{-}at\text{-}w})$ returns the set containing all worlds $w'$ whose degree of (world-)goodness is at least as high as that of any other world.

This proposal predicts that *permissible* and *obligatory* should be non-gradable. Even though their meanings are partly determined by a graded concept of (world-)goodness, once we have fixed this ordering there is a determinate fact of the matter about whether all/some of the best worlds make some proposition true, with no room for degrees. We can test whether this prediction is correct using tests discussed in the last section: the availability of degree modification, comparatives, equatives, and superlatives.

(21) a. But we also think it more obligatory to help someone whose pain we perceive than someone whose pain or danger we merely know about at second hand. (Slote 2016: 102)

9 Notes: (1) I am making the "limit assumption" (see Lewis 1973 and much following literature), inessentially, to simplify the presentation. (2) The "highest-ranked" gloss and (19) are only appropriate for theories like Lewis', where the ordering is connected.

b. The most obligatory task is to manage the list and set it according to the items in order to gain good results. (web)

c. It is not - as I understand it - TOTALLY obligatory to be stark naked there all the time, but if you feel so inclined, you can be. (web)

(22) a. Legislation can help by making weaponry less accessible and violence less permissible. (COCA)

b. Humorous X-do-Y exaggerations are the most permissible, because people like being entertained. (Jones 2010: 35)

c. The ad shows the rider with his eyewear UNDER his helmet straps. ... The only time that it is slightly permissible to do this is in winter, when you are forced to wear a skullcap or balaclava. (web)

Taken at face value, these examples suggest that *permissible* and *obligatory* are gradable. If so, Lewis's (1973) proposal to identify them with quantifiers over sets of worlds cannot be right: these quantifiers do not have the necessary graded structure (degree scales or qualitative orderings). An alternative is to suppose that *permissible* and *obligatory* are in fact scalar adjectives whose differences can be attributed to an independently motivated distinction between "minimum" and "maximum" adjectives (Rotstein & Winter 2004; Kennedy & McNally 2005; Kennedy 2007). Specifically, *permissible* could be related to the minimum adjectives (23), whose meaning in the unmodified form can often be paraphrased using "some". Similarly, *obligatory* could be related to the maximum adjectives like *dry* (24), which in their unmodified form can often be paraphrased using the universal "all" or the universal negative "no". Despite being roughly paraphrasable by "some" and "all" or "no", minimum and maximum adjectives lexicalize degrees and are fully gradable, as the (b) and (c) examples indicate.

(23) Minimum adjective *wet*

a. This towel is wet.

= It has a non-zero degree of wetness.

≈ It has some amount of water on it.

b. This towel is wetter than that one. (comparison)

c. This towel is slightly wet. (degree modification)

(24) Maximum adjective *dry*

a. The towel is dry.

= It has a maximal degree of dryness.

≈ It has no amount of water on it.

b. This towel is drier than that one. (comparison)

c. This towel is perfectly dry. (degree modification)

On such an analysis, *permissible* is a gradable adjective whose unmodified form means "having a non-zero degree of permissibility", and *obligatory* is a gradable adjective whose unmodified

means "having a maximal degree of obligation". The intuitive appeal of the "some"/"all" account could then be explained by the fact that these are reasonable paraphrases of the true, degree-based meanings.

This argument is far from conclusive, though. Two points of complication are especially noteworthy here. First, an alternative analysis of *obligatory* might relate it instead to "extreme" adjectives like *scorching* and *gorgeous* (Portner & Rubinstein 2016). Second, sentence-modifying *obligatory* and *permissible* are relatively infrequent in COCA with overt markers of gradability. Part of the explanation of this fact is straightforward: the relevant unmodified constructions (*It is permissible that*, etc.) are already quite infrequent. However, the infrequency of these uses might also provide evidence for a different account in which these items have a non-scalar basic meaning, which can sometimes be coerced into a scalar interpretation (cf. Klecha 2012, 2014). On a theory along these lines, the relative infrequency of modified *permissible* and *obligatory* might reflect some feature of the coercion operation.

### 3.4 Inferential properties of deontic scales

Lewis' graded-but-not-gradable semantics for deontic modals has a second controversial feature: it renders $\phi$ *is better than* $\psi$ true when there is even one $\phi$-world that is better than all worlds in $\psi$, and regardless of the relative likelihood of the various ways for $\phi$ and $\psi$ to be true. Similarly, the analysis predicts that *It is obligatory that Mary leave* is true if all Mary leaves in all of the best possible worlds, regardless of how few or far-fetched these worlds are. This is a feature shared by many systems of deontic logic, some predating Lewis, and also by Kratzer's (1981; 1991) semantics, which has been very influential in linguistic work on modality and more recently in philosophical semantics. Recently several theorists have argued that goodness and obligation are not sensitive merely to the "best" worlds.

For example, Jackson (1991) imagines a doctor who must choose whether to recommend to a patient a safe but mediocre treatment or a risky experimental treatment. His best information suggests that the risky treatment will probably kill the patient, but there is a small chance of a total cure. The safe treatment will definitely work, but will leave the patient with some lasting pain and discomfort. Suppose the doctor judges that

(25) a. the best outcome is one in which the patient is totally cured—which will happen only if she prescribes the risky treatment;
b. the second-best outcome is one in which she prescribed the safe treatment and the patient has lasting pain;
c. the worst outcome is one in which the patient is killed—which will happen only if she prescribes the risky treatment.

If the meaning of *obligatory* and *better* only take into account the goodness of the highest-ranked worlds, it follows that this doctor is also committed to both of the following:

(26) a. It is better to prescribe the risky treatment than to prescribe the safe but mediocre treatment.
b. It is obligatory to prescribe the risky treatment.

This is because, by (25a), the best worlds in which the risky treatment is prescribed are worlds in which the patient is cured. By (25b), these worlds are better than the best worlds in which the mediocre treatment is prescribed. Likewise, since all the best worlds are risky-treatment-and-full-recovery worlds, it would seem to follow that the risky treatment is obligatory. But these consequences seems wrong: it is easy to imagine someone endorsing the judgments in (25) while denying the absurd conclusions in (26). If so, Lewis' account of *better* is in trouble. In fact, Lewis himself acknowledges that the concept that he is describing formally does not accurately represent the concept he is modeling.

> This is not instrumental or intrinsic betterness of any familiar form, but rather *maximax betterness*. Roughly, we are comparing $\phi$-at-its-best with $\psi$-at-its-best, and ignoring the non-best ways for $\phi$ and $\psi$ to hold. (Lewis 1973: 101)

Is there a way to maintain the insights of Lewis' theory while discarding its "maximax" character—that is, while paying attention to non-maximal worlds?

A crucial feature of Jackson's example is that the risky treatment will *probably* kill the patient. After all, if we change the story so that the experimental treatment will probably save the patient—with only a small chance of death—the judgments in (26) seem much less absurd. Pushing on this intuition, Goble (1996); Cariani (2009); Lassiter (2014, 2017) discuss versions of this puzzle involving (variously) *good* or *ought*, and propose to resolve them by using a more complex lifting that takes probabilistic information and non-maximal worlds into account.[10] On their proposal, the scalar concept GOODNESS (OF PROPOSITIONS) is created from the GOODNESS (OF WORLDS) concept by assigning to each proposition a degree of goodness which is a weighted average of the goodness of the worlds in it, where each world receives a weight equal to the probability that it will be true if the proposition occurs. (This is equivalent to the concept of "expected value"/"expected utility" widely used in theories of decision and action.) On this construction, when we consider whether (26a) is true, we must take into account all of the possible outcomes. Even though the best result of the risky treatment is very good, this fact receives very little weight in the final result because it is much more likely that the risky treatment will result in a terrible outcome, death. The safe but mediocre treatment is better, even though its best possible realization is not as good as the best possible outcome of the risky treatment.

Similar objections have been made to Kratzer's (1981; 1991) analysis of *must*, and its extension to *have to, ought*, and *should* by von Fintel & Iatridou (2008) [SEE ALSO: weak necessity]. These theories treat the relevant items, in one way or another, as quantifiers over worlds that are "best" (in one way or another) according to a scalar concept of GOODNESS (OF WORLDS), and the objection to Lewis' theory could be applied here as well. See however Katz, Portner & Rubinstein 2012; Silk 2012; von Fintel 2012 for various efforts to account for this kind of information-sensitivity within the broad framework of Kratzer's theory.

Looking beyond deontic adjectives, some authors have noted evidence for gradability among the deontic auxiliaries *should, ought*, and perhaps *must*. See Lassiter 2011, 2017; Portner & Rubinstein 2016 for data and discussion.

10 See also Jackson 1985; Jackson & Pargetter 1986; Levinson 2003; Finlay 2010; Cariani 2016 for related puzzles and proposals.

## 4 Epistemic gradation and gradability

The existence of graded structure underlying some modal concepts has long been recognized, but the gradability of some modal expressions is a more recent observation. Lewis's (1973) discussion of deontic comparatives, for example, does not draw out the analogies with non-modal comparatives or the possibility of degree modification. Another important question which is not addressed by Hamblin or Lewis is whether gradability in the modal domain extends also to degree modification. Modal degree modification was, to my knowledge, first discussed explicitly by Kratzer (1981: 46-51). Focusing on German, Kratzer's influential article gives a broad and—for the first time—formally unified treatment of a variety of epistemic, deontic, and circumstantial modals. Since Kratzer's analysis of deontic vocabulary is close to Lewis's (1973), I will focus here on her novel treatment of epistemic items and of gradation more generally [SEE ALSO: EPISTEMIC MODALITY].

For instance, Kratzer analyzes the adjective *wahrscheinlich* "probable" and epistemic interpretations of the noun *Möglichkeit* "possibility" and auxiliaries *können* "can". Several of her examples are reproduced in (27).

(27) a. Es kann gut sein, dass der Gauzner-Michl der Mörder war.
It can well be that the G.-M. the murderer was
"There is a good possibility that G.-M. was the murderer."

b. Es besteht aber immer noch eine geringe Möglichkeit, dass der Kastenjakl der
There is however always still a slight possibility that the Kastenjakl the
Mörder war.
murderer was
"There is, however, still a slight possibility that K. was the murderer."

c. Der Gauzner-Michl kann eher der Mörder sein als der Kastenjakl.
The Gauzner-Michl can rather the murderer be than the Kastenjakl.
"G.-M. is more likely to be the murderer than K."

In these examples epistemic *kann* "can" is modified by *gut* "well", and appears to form comparatives using *eher*. Also, the epistemic noun *Möglichkeit* "possibility" can combine with the adjectival modifier *gering* "slight".

Kratzer points out that modeling the distinction between *kann* and *kann gut*—or between *Möglichkeit* and *geringe Möglichkeit*—requires making more distinctions between grades of modality than is possible if the only tools we have at our disposal are "all" and "some"-quantification over a fixed set of worlds. Kratzer proposes to use an "ordering source"—a set of propositions—to draw further distinctions among the relevantly possible worlds, ordering them according to whether one world satisfies all of the ordering-source propositions that another does (and possibly more). She suggests a number of ways to use this ordering to encode grades of possibility that have appropriate logical relations to each other: *kann gut* $\phi$ entails *kann* $\phi$, and so forth.

In addition to drawing attention to the phenomenon of graded modality, Kratzer (1981, 1991) proposes lexical entries that interpret a variety of epistemic and deontic modal expressions with respect to a class of ordered structures defined in terms of a "modal base" and an "ordering source". While stated differently, these structures are—modulo Lewis's (1973) connectedness assumption—

formally equivalent to the similarity orderings that Lewis uses to analyze counterfactuals and certain deontic modals. (This is proved in Lewis 1981.) Kratzer's empirical focus is much broader than Lewis', though. (28) gives two of her definitions in simplified form.

(28) a. *Es kann gut sein, dass* $\phi$ is true if and only if, of the worlds that are maximal in the relevant world-ordering, at least one is a world in which $\phi$ holds.

b. *Es besteht eine geringe Möglichkeit, dass* $\phi$ is true if and only if both

- $\phi$ is in the domain of the world-ordering (i.e., if it is "epistemically accessible"—among the possible worlds being ordered), and
- *Es kann gut sein, dass* $\phi$ is not true.

These proposals could naturally be extended to the English translations of these expressions, as Kratzer (1991) does with some additional modifications. See that paper for many further connections with epistemic auxiliaries, deontic gradation, and conditionals, and see Kratzer 2012 for a thorough update and retrospective.

### 4.1 Compositionality and likelihood

Kratzer's influential analysis of complex graded modal constructions has an important lacuna: it is not compositional. Entire constructions are assigned meanings, and there is no attempt to derive the meanings of the constructions from the meanings of their parts. But we do need a compositional analysis of gradable modal constructions: the meaning of *geringe Möglichkeit* "slight possibility" should be derived from the meanings of its constituent parts *gering* and *Möglichkeit* (Yalcin 2007; Portner 2009). Similarly, *quite* and *more* should have the same meanings in *quite possible* and *more probable* that they do in *quite hungry* and *more important*.

Villalta (2008); Portner (2009) pursue this goal by using Kratzer's rule of Comparative Possibility to induce modal degrees from a world-ordering. Comparative Possibility is the "lifting" rule that we saw when discussing Lewis' semantics earlier: the degree assigned to a proposition is determined by the highest position of any world in the proposition.

(29) Given a world-ordering, assign epistemic degrees to propositions in any way you like as long as the following constraint is observed: $\mu_{epi}(\phi) \geq \mu_{epi}(\psi)$ if and only if, for every world $w_1$ in $\psi$, there is a world $w_2$ in $\phi$ which is ranked at least as high as $w_1$ in the world-ordering.

This gives us the beginnings of a compositional account of complex epistemic constructions. For example, we can treat *slight possibility* as Kratzer (1981) does while maintaining a precise analogy to *slight pain*.

(30) a. $x$ is a pain if and only if the degree of pain associated with $x$ is non-zero.

b. $x$ is a slight pain if $x$ is a pain and the degree of pain associated with $x$ is not great.

(31) a. $\phi$ is a possibility if and only if $\mu_{epi}(\phi)$ is greater than $\mathbf{min}(\mu_{epi})$.

b. $\phi$ is a slight possibility if $\phi$ is a possibility and $\mu_{epi}(\phi)$ is not great.

Epistemic comparisons formed using *kann eher*, or English *more likely*, could be interpreted by analogy to non-modal comparatives like "$a$ is heavier than $b$". (See, however, Herburger & Rubinstein 2014 for a different analysis of *kann eher*.)

(32) a. $\phi$ *is as likely as* $\psi$ is true if and only if $\mu_{epi}(\phi) \geq \mu_{epi}(\psi)$.
b. $\phi$ *is more likely than* $\psi$ is true if and only if $\mu_{epi}(\phi) > \mu_{epi}(\psi)$.

We can also interpret modified forms of *likely*, as in (33).

(33) a. *Bill is three times as heavy as Sam* is true if and only if $\mu_{weight}(\textbf{Bill}) \geq 3 \times \mu_{weight}(\textbf{Sam})$.
b. *Rain is three times as likely as snow* is true if and only if $\mu_{epi}(\textbf{rain}) \geq 3 \times \mu_{epi}(\textbf{snow})$.

The introduction of degrees is a good start on the compositional problem. However, the lifting in (29) leads to another problem (Yalcin 2010; Lassiter 2010, 2015). Suppose that I am about to toss a single fair die. Since the die is fair, all numbers in $\{1,2,3,4,5,6\}$ are equally likely to turn up. So the following sentences are intuitively true:

(34) a. The die is exactly as likely to land on 1 as it is to land on 2. $(\mu_{epi}(1) = \mu_{epi}(2))$
b. The die is exactly as likely to land on 1 as it is to land on 4. $(\mu_{epi}(1) = \mu_{epi}(3))$
c. The die is exactly as likely to land on 1 as it is to land on 6. $(\mu_{epi}(1) = \mu_{epi}(3))$

The problem is that, according to the proposal just sketched, if (34a) and (34b) are both true then (35) must be true as well:

(35) So, the die is exactly as likely to land on 1 as it to land on an even number. $(\mu_{epi}(1) = \mu_{epi}(2 \textbf{ or } 4 \textbf{ or } 6))$

But this conclusion is clearly false: it's clearly more likely that a fair die will land on an even number than it is that it will land on 1. (This is the familiar "best-worlds" problem again: in effect, (29) ranks propositions via the relative positions of their highest-ranked worlds.)

One way to resolve this problem is to model likelihood as **probability** (Rescher 1968; Frank & Kamp 1997; Yalcin 2005, 2007, 2010; Swanson 2006, 2011, 2015; Lassiter 2010, 2011, 2015; Moss 2015). Probabilities are numbers between 0 and 1, inclusive, and a probability measure is a mapping from propositions to probabilities. A tautology has probability 1, a contradiction has probability zero, and the probability of any two mutually exclusive propositions is the sum of their individual probabilities. This analysis correctly predicts that (34) should be invalid, because probabilities of mutually exclusive propositions add up: when $P(\phi), P(\psi)$, and $P(\chi)$ are all equal and non-zero, $P(\psi \textbf{ or } \chi)$ is greater than $P(\phi)$ (in fact, twice as great). It also predicts correctly that (36) should be intuitively true:

(36) The die is exactly three times as likely to land on an even number as it is to land on 1.

This proposal has the consequence that the likelihood scale has a maximum and a minimum element. If $\mu_{epi}$ is a probability measure then a tautology receives the maximum possible degree, and a contradiction receives the minimum possible degree. This seems reasonable: surely nothing could be more likely than a tautology, or less likely than a contradiction. However, recall from §2 that there is a standard test for the existence of a maximal degree on a scale: modification by *perfectly*. Portner (2009) points out that *likely* differs from *full* in that it lacks a maximum-degree reading in these cases.

(37) a. The glass is perfectly full.

b. ? It is perfectly likely that it will snow tomorrow.

(37b) does not mean "Snow is as likely as a tautology", "Snow is absolutely certain", or anything of the sort. The probabilistic approach seem to predict that this reading should be available.

Kennedy & McNally (2005); Kennedy (2007) also use the possibility of modification by *slightly* as a diagnostic for lower-boundedness. On this diagnostic, the probabilistic semantics seems to predict incorrectly that (38b) should be acceptable with a "just greater than zero" meaning.

(38) a. The towel is slightly/barely wet.

b. ? It is slightly/barely likely that it will rain tomorrow.

These data points are very puzzling. Should we conclude that probability is the wrong choice for the scale of *likely/probable* (Portner 2009)? That *likely* and *probable* live on a scale which is exactly like the probability scale except that it excludes the endpoints of 0 and 1, so that the range of $\mu_{epi}$ is $(0,1)$ (Klecha 2012, 2014)? Or that the argument relies on an oversimplified picture of degree modification (Lassiter 2010, 2017)? Each of these proposals can account for these data, but all have drawbacks. Portner's (2009) approach threatens our account of (34)-(36), and would seem to require positing multiple epistemic scales together with additional assumptions to account for the logical connections among items. Lassiter's (2010) account complicates our understanding of standard degree modification tests (though he argues that this move is independently motivated). Klecha's (2012; 2014) proposal may make problematic empirical predictions, as Lassiter (2017) argues.

The theoretical upshot of this debate about the structural aspects of the likelihood scale is still unclear. However, probabilistic treatments of *likely* and *probable* are increasingly common. See also Holliday & Icard (2013) for a detailed investigation of the logic of comparative likelihood which may support the use of a scale slightly weaker than probability, and Lassiter 2015 for an attempt to reconcile this account with the linguistic phenomena.

### 4.2 Further epistemic expressions

Recently a number of authors have discussed the gradable epistemic adjective *clear*, as exemplified in (39).

(39) It is clear that Bill is not feeling well.

Barker & Taranto (2003) identify the "paradox of asserting clarity": an assertion of (39) could be taken to be self-defeating, on the grounds that—if it is indeed clear to all involved—it is a mystery why the speaker would bother saying so. Second, *clear* is clearly a gradable adjective, and specifically a maximum adjective.

(40) a. It is totally clear that Charter does not want to talk to their current customers.

b. [I]t is clearer that Lewis' view is antirealistic with respect to propositions than with respect to properties ... (Plantinga 2003: 222)

Theories of clarity agree that publicly available evidence is relevant: (39) is not true just because the speaker has private evidence not available to the listener. Barker's (2009) explanation of the

Paradox involves the idea that clarity assertions help to negotiate the standard of evidence relevant to the conversation. If (39) is asserted and accepted, then everyone agrees that the evidence in favor of *Bill is not feeling well* is strong enough to justify taking it for granted. The gradability of clarity is then explained by the fact that justification is a graded concept. An alternative due to Wolf & Cohen (2011) explains the gradability of clarity in terms of the probabilistic belief states of ideal reasoners. Crone (2016) combines elements of these proposals into a novel semantics and pragmatics for *clear*, drawing connections with predicates of personal taste and with the literature on awareness in pragmatics (Franke & de Jager 2007; Yalcin 2011). For additional relevant work see Taranto 2006; Bronnikov 2008.

Another gradable epistemic modal that has been studied in some detail is the maximum adjective *certain*. Certainty has interesting connections both to knowledge (Unger 1971; Stanley 2008; Lassiter 2016) and to likelihood (Lassiter 2010, 2011, 2017). The epistemic auxiliary *must* is relevant both because of its semantic connections to gradable epistemics (Lassiter 2018) and because it might display limited gradability (Portner & Rubinstein 2016). There is a debate about whether *possible* is best treated as a "some"-quantifier or as a minimum adjective (cf. *permissible* above). See Lassiter 2010, 2018 for evidence and arguments in favor of this analysis and Klecha 2012, 2014 for counter-arguments. Herburger & Rubinstein (2014) argue along similar lines that the apparent gradability of the German item *möglich* "possible" is illusory.

As we saw in §2.2, the concept "degrees of belief" has a long history. The relationship between belief and the scalar concept of probability has been analyzed in much detail (Kyburg 1961; Leitgeb 2014). However, the question of whether *believe* is a gradable verb does not seem to have been considered. The examples in (41) suggest that a "yes" answer may not be out of the question.

(41) a. What do you <u>believe more</u>, that the CIA killed JFK or that the government did 9/11? (Quora.com discussion topic)

b. I consider myself a feminist and <u>believe very much</u> that many women, like me, excel at entrepreneurship. (web)

Interestingly, *believe* and its near-synonym *think* may differ in this respect, judging by the intuitive oddness of replacing *believe* by *think* in both of these examples. Substituting *know* would also be infelicitous, and indeed Stanley (2005) argues in detail that *know* is not gradable.

## 5 Desire, ability, and beyond

Many other modal expressions are gradable. Some of these have been analyzed as such, and have been neglected here for reasons of space. Many more await detailed treatment. This section gives a quick run-down of some of these items.

*Want, wish, desirable*, and certain other expressions of desire are gradable.

(42) a. Kodak's warranty is an iron clad 1 year. They <u>want more</u> to repair the camera than to buy a new one. (web)

b. Obviously, it is <u>far more desirable</u> for a building to sustain a limited amount of deformation than for it to suffer a complete breakage failure. (web)

c. Do you <u>wish more</u> that unicorns or mermaids existed? (web)

Villalta (2008) discusses gradability of *want* and *wish* (among other embedding verbs and adjectives), proposing that they encode degrees of desire and drawing interesting connections between verbal gradability and the distribution of subjunctive mood. Villalta extracts degrees from the Lewis/Kratzer Comparative Possibility relation as described above (29). Levinson (2003); Lassiter (2011) argue that this move makes incorrect predictions analogous to those discussed involving comparative goodness above: myopic focus on "best" worlds leads to insufficient sensitivity to probabilistic information. These authors propose instead to associate desire predicates with a scale of expected value. Pasternak (2017) offers an interestingly different semantics for *want* which ties the gradability of *want, desire, hate*, and other mental state verbs to the fact that psychological states vary in their intensity.

Ability seems to be a graded concept, and *able* appears to be gradable. Here are two naturally-occurring examples.

(43) a. [T]he programs effectively represent an effort to render other countries more willing and more able to defeat imminent threats to the United States that may emerge from those states. (web)

b. "As a matter of fact, yes," she said defiantly, only slightly able to retain her composure. (web)

c. We were very much able to fight under these conditions, as our enemy would learn, to his own detriment. (Russell 2011: 88)

There is an extensive philosophical literature on ability (see Mandelkern, Schultheis & Boylan 2017 and references therein) and a linguistic literature on related root modals like *can* (Brennan 1993; Hacquard 2006). However, the gradability of *able* and its implications for root modals in general has not been studied in depth.

*Need* is clearly gradable: (e.g., *I need to go home more than I need to talk to Bob*). Lassiter (2011) treats this item (briefly and simplistically) as the extreme counterpart of gradable *want*. Rubinstein (2012) gives an extensive treatment of *need* as an *ALL*-quantifier, but does not consider its gradability. *Necessary* also seems to be gradable (e.g., *How necessary is it to marinade meat before making jerkies?*, a web example from Lassiter 2011). Villalta (2008) analyzes gradable *necessary* as a likelihood operator. Rubinstein (2012) demurs, giving a detailed treatment of this item as a universal teleological modal, but without relating this interpretation to a scalar concept of necessity.

Klecha (2014); Pasternak (2016) give various theories of the gradability of *important*, and Portner & Rubinstein (2016) treat this item along with *crucial, must*, and *should*. There is a large logical and philosophical literature on graded notions of preference with interesting connections to deontic semantics (Hansson 2001), but little or no work relating these ideas to the interpretation of *prefer* and related items. Many more gradable modals remain to be explored: *vital, confident, doubt, suspect, ...*. Investigation of these expressions with simultaneous attention to modal semantics, degree structure, and reasoning potential will hopefully yield valuable insights not only for natural language semantics and philosophy of language, but also for epistemology and the study of reasoning and concepts.

## 6 Conclusion

Much remains to be done at the intersection of theories of gradation and theories of modality, and this work is of potentially great value for linguistics, philosophy, and cognitive science. The linguistic study of graded modality can be informed by a broad range of disciplines: philosophy of language and mind, epistemology, probability-, decision-, and game theory, the psychological study of reasoning and concepts, and artificial intelligence topics such as knowledge representation, inference, and planning, to name just a few. When another survey like this is written in ten or twenty years, I expect that the theoretical and empirical landscape will look rather different, and many of the ideas surveyed here will turn out to be incorrect or too simplistic—though I hope that some will last.

## References


Barker, Chris. 2009. Clarity and the grammar of skepticism. *Mind and Language* 24(3). 253–273.

Barker, Chris & Gina Taranto. 2003. The paradox of asserting clarity. In *Wecol proceedings*, .

Beck, Sigrid, Sveta Krasikova, Daniel Fleischer, Remus Gergel, Stefan Hofstetter, Christiane Savelsberg, John Vanderelst & Elisabeth Villalta. 2009. Crosslinguistic variation in comparison constructions. *Linguistic Variation Yearbook* 9(1). 1–66.

Beck, Sigrid, Toshiko Oda & Koji Sugisaki. 2004. Parametric variation in the semantics of comparison: Japanese vs. English. *Journal of East Asian Linguistics* 13(4). 289–344.

Bochnak, M. Ryan. 2013. *Cross-linguistic variation in the semantics of comparatives*: University of Chicago dissertation.

Bochnak, M. Ryan. 2014. The degree semantics parameter and cross-linguistic variation. To appear in *Semantics & Pragmatics*.

Brennan, Virginia M. 1993. *Root and epistemic modal auxiliary verbs*: University of Massachusetts at Amherst dissertation.

Bronnikov, George. 2008. The paradox of clarity: Defending the missing inference theory. In *Semantics and Linguistic Theory (SALT) 18*, 144–157.

Cariani, Fabrizio. 2009. *The semantics of 'ought' and the unity of modal discourse*: University of California at Berkeley dissertation.

Cariani, Fabrizio. 2016. Deontic modals and probabilities: One theory to rule them all? In Matthew Chrisman & Nate Charlow (eds.), *Deontic modals*, 11–46. Oxford University Press.

Crone, Philip. 2016. Asserting clarity as managing awareness. To appear in *Journal of Semantics*.

Finlay, S. 2010. What ought probably means, and why you can't detach it. *Synthese* 177(1). 67–89.

von Fintel, K. 2012. The best we can (expect to) get? Challenges to the classic semantics for deontic modals. Paper presented at the Central meeting of the American Philosophical Association, February 17. http://mit.edu/fintel/fintel-2012-apa-ought.pdf.

von Fintel, Kai. 2001. Counterfactuals in a dynamic context. In Michael Kenstowicz (ed.), *Ken hale: A life in language*, 123–152. MIT Press.

von Fintel, Kai & Sabine Iatridou. 2008. How to say *ought* in foreign: The composition of weak necessity modals. In Jacqueline Guéron (ed.), *Time and Modality*, 115–141. Springer.

Frank, Anette & Hans Kamp. 1997. On context dependence in modal constructions. In *Semantics and linguistic theory*, vol. 7, 151–168.

Franke, Michael & Tikitu de Jager. 2007. The relevance of awareness. In Paul Dekker Maria Aloni & Floris Roelofsen (eds.), *Sixteenth amsterdam colloquium*, 97–102. ILLC/Department of Philosophy, University of Amsterdam. http://www.illc.uva.nl/AC/AC2007/uploaded_files/proceedings-AC07.pdf.

Goble, Lou. 1996. Utilitarian deontic logic. *Philosophical Studies* 82(3). 317–357.

Hacquard, Valentine. 2006. Aspects of Modality. Ph.D. Thesis, MIT.

Hansson, Sven Ove. 2001. *The structure of values and norms*. Cambridge University Press.

Hay, Jennifer, Chris Kennedy & Beth Levin. 1999. Scalar structure underlies telicity in 'degree achievements'. In *Semantics and Linguistic Theory 9*, 127–144.

Heim, Irene. 2001. Degree operators and scope. In Fery & Sternefeld (eds.), *Audiatur vox sapientiae: A festschrift for Arnim von Stechow*, 214–239. Akademie Verlag.

Herburger, Elena & Aynat Rubinstein. 2014. Is 'more possible' more possible in german? In *Semantics and Linguistic Theory (SALT) 24*, 555–576.

Hintikka, Jaako. 1962. *Knowledge and Belief: An Introduction to the Logic of the Two Notions*. Cornell University Press.

Holliday, Wesley H. & Thomas F. Icard. 2013. On the logic of comparative likelihood. Manuscript, University of California at Berkeley and Stanford University.

Jackson, Frank. 1985. On the semantics and logic of obligation. *Mind* 94(374). 177–195.

Jackson, Frank. 1991. Decision-theoretic consequentialism and the nearest and dearest objection. *Ethics* 101(3). 461–482.

Jackson, Frank & Robert Pargetter. 1986. Oughts, options, and actualism. *The Philosophical Review* 95(2). 233–255.

Jones, Todd. 2010. *What people believe when they say that people believe: Folk sociology and the nature of group intentions*. Lexington Books.

Kamp, Hans & Barbara Partee. 1995. Prototype theory and compositionality. *Cognition* 57(2). 129–191.

Katz, G., P. Portner & A. Rubinstein. 2012. Ordering combination for modal comparison. In Anca Chereches (ed.), *Semantics & Linguistic Theory (SALT) 22*, CLC Publications.

Kennedy, Chris. 1997. *Projecting the adjective: The syntax and semantics of gradability and comparison*: U.C., Santa Cruz dissertation.

Kennedy, Chris. 2007. Vagueness and grammar: The semantics of relative and absolute gradable adjectives. *Linguistics and Philosophy* 30(1). 1–45.

Kennedy, Chris & Louise McNally. 2005. Scale structure, degree modification, and the semantics of gradable predicates. *Language* 81(2). 345–381.

Klecha, Peter. 2012. Positive and conditional semantics for gradable modals. In Anna Chernilovskaya Ana Aguilar Guevara & Rick Nouwen (eds.), *Sinn und bedeutung 16*, vol. 2, 363–376. MIT Press.

Klecha, Peter. 2014. *Bridging the Divide: Scalarity and Modality*: University of Chicago dissertation.

Klein, Ewan. 1991. Comparatives. In Arnim von Stechow & Dieter Wunderlich (eds.), *Semantik:*

*Ein internationales Handbuch der zeitgenössischen Forschung*, 673–691. Walter de Gruyter.
Krantz, David H., R. Duncan Luce, Patrick Suppes & Amos Tversky. 1971. *Foundations of Measurement*. Academic Press.
Kratzer, Angelika. 1981. The notional category of modality. In Eikmeyer & Rieser (eds.), *Words, Worlds, and Contexts*, 38–74. de Gruyter.
Kratzer, Angelika. 1991. Modality. In Arnim von Stechow & Dieter Wunderlich (eds.), *Semantik: Ein internationales Handbuch der zeitgenössischen Forschung*, 639–650. Walter de Gruyter.
Kratzer, Angelika. 2012. *Modality and Conditionals: New and Revised Perspectives*. Oxford University Press.
Kripke, Saul. 1963. Semantical considerations on modal logic. *Acta philosophica fennica* 16(1963). 83–94.
Kripke, Saul A. 1959. A completeness theorem in modal logic. *The journal of symbolic logic* 24(01). 1–14.
Kyburg, Henry. 1961. *Probability and the logic of rational belief*. Wesleyan University Press.
Lassiter, Daniel. 2010. Gradable epistemic modals, probability, and scale structure. In Nan Li & David Lutz (eds.), *Semantics & Linguistic Theory (SALT) 20*, 197–215. CLC Publications.
Lassiter, Daniel. 2011. Measurement and Modality: The Scalar Basis of Modal Semantics. Ph.D. thesis, New York University.
Lassiter, Daniel. 2014. Modality, scale structure, and scalar reasoning. *Pacific Philosophical Quarterly* 95(4). 461–490.
Lassiter, Daniel. 2015. Epistemic comparison, models of uncertainty, and the disjunction puzzle. *Journal of Semantics* 32(4). 649–684.
Lassiter, Daniel. 2016. *Must*, knowledge, and (in)directness. *Natural Language Semantics* 24(2). 117–163.
Lassiter, Daniel. 2017. *Graded Modality*. Oxford University Press.
Lassiter, Daniel. 2018. Talking about (quasi-)higher-order uncertainty. In Cleo Condoravdi & Tracy Holloway King (eds.), *Tokens of meaning: Papers in honor of lauri karttunen*, CSLI Publications.
Leitgeb, Hannes. 2014. The stability theory of belief. To appear in *The Philosophical Review*.
Levinson, D. 2003. Probabilistic model-theoretic semantics for *want*. In Robert B. Young & Yuping Zhou (eds.), *Semantics and Linguistic Theory (SALT) 13*, 222–239. CLC Publications.
Lewis, David. 1973. *Counterfactuals*. Harvard University Press.
Lewis, David. 1981. Ordering semantics and premise semantics for counterfactuals. *Journal of Philosophical Logic* 10(2). 217–234.
Löbner, Sebastian. 1990. *Wahr neben falsch: duale operatoren als die quantoren natürlicher sprache*. Walter de Gruyter.
Look, Brandon C. 2013. Leibniz's modal metaphysics. In Edward N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*, Spring 2013 edn.
Mandelkern, Matt, Ginger Schultheis & David Boylan. 2017. Agentive modals. To appear in *The Philosophical Review*.
Margolis, Eric & Stephen Laurence. 1999. *Concepts: core readings*. MIT Press.
Mill, John Stuart. 1863. *Utilitarianism*. Longmans, Green, Reader, and Dyer.

Moss, Sarah. 2015. On the semantics and pragmatics of epistemic vocabulary. *Semantics and Pragmatics* 8. 1–81.
Murphy, Gregory. 2002. *The big book of concepts*. MIT Press.
Pasternak, Robert. 2016. A conservative theory of gradable modality. In *Semantics and linguistic theory*, vol. 26, 371–390.
Pasternak, Robert. 2017. A lot of hatred and a ton of desire: Intensity in the mereology of mental states. Ms., Stony Brook University. http://semanticsarchive.net/Archive/WVmMzYwO/.
Plantinga, Alvin. 2003. Essays in the metaphysics of modality .
Portner, Paul. 2009. *Modality*. Oxford University Press.
Portner, Paul & Aynat Rubinstein. 2016. Extreme and non-extreme deontic modals. In Matthew Chrisman & Nate Charlow (eds.), *Deontic modals*, 256–282. Oxford University Press.
Rescher, Nicholas. 1968. *Topics in philosophical logic*. Springer.
Rosch, Eleanor. 1978. Principles of categorization. In Eleanor Rosch & Barbara B. Lloyd (eds.), *Cognition and categorization*, 27–48. Lawrence Erlbaum.
Rotstein, Carmen & Yoad Winter. 2004. Total adjectives vs. partial adjectives: Scale structure and higher-order modifiers. *Natural Language Semantics* 12(3). 259–288.
Rubinstein, A. 2012. *Roots of Modality*: University of Massachusetts at Amherst dissertation.
Russell, Steve. 2011. *We got him!: A memoir of the hunt and capture of Saddam Hussein*. Simon and Schuster.
Sassoon, Galit Weidman. 2013. *Vagueness, gradability and typicality: The interpretation of adjectives and nouns*. Brill.
Silk, A. 2012. Modality, weights, and inconsistent premise sets. In Anca Chereches (ed.), *Semantics & Linguistic Theory (SALT) 22*, CLC Publications.
Slote, Michael. 2016. *From enlightenment to receptivity: Rethinking our values*. Oxford University Press.
Stanley, Jason. 2005. *Knowledge and practical interests*. Clarendon Press.
Stanley, Jason. 2008. Knowledge and certainty. *Philosophical Issues* 18(1). 35–57.
von Stechow, Arnim. 1984. Comparing semantic theories of comparison. *Journal of Semantics* 3(1). 1–77.
Swanson, Eric. 2006. Interactions With Context. Ph.D. thesis, MIT.
Swanson, Eric. 2011. How not to theorize about the language of subjective uncertainty.
Swanson, Eric. 2015. The application of constraint semantics to the language of subjective uncertainty. *Journal of Philosophical Logic* 1–26.
Taranto, Gina. 2006. *Discourse adjectives*. Routledge.
Unger, Peter. 1971. A defense of skepticism. *The Philosophical Review* 80(2). 198–219.
Villalta, Elisabeth. 2008. Mood and gradability: An investigation of the subjunctive mood in Spanish. *Linguistics and Philosophy* 31(4). 467–522.
Wolf, Lari & Ariel Cohen. 2011. Clarity as objectivized belief. In *Vagueness and language use*, 165–190. Palgrave Macmillan.
von Wright, Georg Henrik. 1963. *The logic of preference: An essay*. Edinburgh University Press.
Yalcin, Seth. 2005. A puzzle about epistemic modals. *MIT Working Papers in Linguistics* 51. 231–272.

Yalcin, Seth. 2007. Epistemic modals. *Mind* 116(464). 983–1026.
Yalcin, Seth. 2010. Probability operators. *Philosophy Compass* 5(11). 916–937.
Yalcin, Seth. 2011. Nonfactualism about epistemic modality. In A. Egan & B. Weatherson (eds.), *Epistemic modality*, 295–332. Oxford University Press.