# Optimal Power Line Communications Control Policies Using Stochastic Optimization

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***Abstract*— We present a general stochastic optimization framework for periodic systems and apply it to PLC networks with linear and time varying (LPTV) channels. Our method of solution operates online and does not assume prior knowledge about the distribution of channel states, but rather samples the LPTV channel to learn the information necessary to compute optimal control policies. We apply this framework to compute optimal bit allocations (rates) and transmit powers for each transmit period over an AC cycle, subject to requirements on average bit error rates and finite buffer sizes. The resultant policies find bit loadings that maximize throughput, while meeting BER requirements and preventing buffer overflow.**

## I. Introduction

Home networks are gaining market momentum and are the subject of intense technical investigation. Applications include multimedia networking, broadband access, home grid and home control. Proposed technologies to support these in-home applications include power line (PL), phone line (PH), and coaxial cable (CX) and were recently standardized by the ITU as G.hn and by the IEEE as P1901. Power line communications (PLC) [22] in particular offers the benefits of ubiquity and accessibility but is also a challenging medium, reflecting the underlying linear and time varying (LPTV) channel and associated cyclostationary noise [21].

An important challenge of home network design is to optimize network performance within the Quality of Service constraints of heterogeneous applications supported and the capability of the network. This challenge is made more complicated by the dynamic nature of the LPTV channel and PLC environment. Variations in AC loads and other power related effects result in an LPTV channel with a periodic distribution of channel states across AC cycles. Further, as devices such as air conditioners, motors and dimmers are added or deleted from the home power system, the statistical distribution of LPTV channel states and noise can change, requiring different control policies to meet user or system requirements. Unfortunately the distribution of channel states is difficult to specify a-priori, especially with changing loads or devices attached to the network, requiring online methods to optimize the performance of PLC's.

In this paper we present a general stochastic optimization framework for periodic systems and apply it to a PLC network. We present an online approach for finding a solution to problems posed in this framework. This approach does not assume prior knowledge about the distribution of channel states, but rather samples the LPTV channel to learn the information necessary to compute optimal control policies. Optimal control policies are functions or algorithms that allocate PLC network resources in response to changing channel conditions or other changes in the network environment. They are dynamic and reallocate resources as channel conditions change; As devices are attached or removed from the power system the approach adapts by computing new control policies.

We apply this general framework to compute optimal bit allocations (rates) and transmit powers for each transmit period over an AC cycle, subject to requirements on average bit error rates (BER) and finite buffer sizes. The resultant policies find bit loadings that maximize throughput, while meeting BER requirements and preventing buffer overflow. We present numerical simulations that illustrate the approach and that demonstrate the performance benefits of this approach.

The remainder of this paper is organized as follows: Section II describes the PLC system model and Section III the PLC problem and associated stochastic optimization model. Section IV describes a general method of solution. Section V specializes these results to the PLC problem. Section VI is a numerical example and describes the online behavior of this approach. Section VII summarizes our results and describes additional research steps.

## II. Model

We consider a discrete time model where each AC cycle is divided into $C$ transmit periods. The distribution of channel states is not assumed known but is assumed to be periodic with period $C$. We assume nodes communicate using a TDMA protocol, such as the protocol described in the G.hn standard, so that only a single transmitter transmits at a time. We assume that a transmitter transmits for $t = 1, ..., N \leq C$ time periods. Each transmitter can sample the channel state $G_t$ for any time slot it is scheduled to transmit. For clarity, we assume a single frequency band; The model (and solution approach) can be extended to multiple orthogonal frequency bands such as those used in OFDM systems.

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MQAM modulation is used in each transmit period with constellation size $2^{k_t}$, where $k_t$ is the number of bits at time $t$. The PLC transmitters can adapt the number of bits transmitted at each time period as a function of the channel state $k_t = k_t(G_t)$. At time $t$, a symbol is transmitted at power level $s_t$. Similarly, each transmitter can adapt its power as a function of the channel state $s_t = s_t(G_t)$. The functions $k_t(G_t)$ and $s_t(G_t)$ are called the rate and power policies for the transmitter.

We use the following formula for approximating BER from [1]:

$$BER_t(\gamma_t) \approx 0.2 \exp\left(\frac{-1.6\ s_t\ \gamma_t}{2^{k_t}-1}\right) \tag{1}$$

where $\gamma_t$ is the received signal to noise ratio at time $t$. This BER approximation is tight to within 1 dB for $k \geq 1$ [1].

A transmitter maintains a finite size transmission buffer of $M$ bits. Bits arrive at rate $I_t$ and are transmitted at rate $k_t$. The buffer backlog of untransmitted bits at the beginning of time slot $t+1$ is

$$y_{t+1} = [y_t - k_t + I_t]^+ , \tag{2}$$

where $[\ ]^+$ is the positivity operator. Backlog in excess of $M$ bits is dropped, so $y_{t+1}$ is constrained to a maximum of $M$ bits.

To streamline notation, vectors are written in bold face. For example, we write $\mathbf{k}(G)$ and $\mathbf{s}(G)$ for the vector of rate and power control policies and $\mathbf{y}$ for the vector of system states.

## III. Power Line Communication

In this section we formulate the problem of finding optimal policies for a PLC system. In Section IV we generalize the problem and describe a general online method to compute optimal control policies. In Section V we specialize these general results to the the PLC system described in this Section. The formulation results in a stochastic optimization problem, the solution to which is the set of optimal control policies. Policies are functions or algorithms that allocate PLC resources in response to changes in channel conditions. Policies are optimal in the sense that they extremize a performance metric, constrained by user and system requirements. In general policies are difficult to express analytically and computational techniques are often used. Since the distribution of channel states is periodic, only a single cycle needs to be considered in describing the problem.

The performance metric we consider is spectral efficiency. The idea is to find policies that maximize spectral efficiency subject to constraints selected by users or the system operator and constraints imposed by the hardware on the system. The system operator constraints are constraints on average transmitter power and average BER over the $N$ periods the transmitter is in operation each cycle. We impose two hardware constraints. Specifically we assume that there is a maximum constellation size that can be transmitted, and that transmitter buffers are limited to $M$ bits. The framework for developing optimal policies is quite general, and hence can typically incorporate additional constraints. Additional average constraints are readily incorporated into the formulation as are additional hardware constraints that must hold at every time instant. When the performance metric and constraints are convex, then the solution will yield optimal control policies. When this is not the case, then the solution may only be a locally optimal policy.

Formally the problem of finding optimal control policies can be expressed as the stochastic optimization problem

$$\underset{\mathbf{s}(G),\mathbf{k}(G),\mathbf{y}}{\text{Maximize:}} \quad \mathbf{E}\left[\sum_{t=1}^{N} k_t(G_t)\right], \tag{3}$$

$$\text{Subject to :} \quad \mathbf{E}\left[\sum_{t=1}^{N} s_t(G_t)\right] \leq NS_{avg} \tag{4}$$

$$\mathbf{E}\left[\sum_{t=1}^{N} BER_t(G_t, s_t(G_t))\right] \leq NBER_{avg} \tag{5}$$

$$y_{t+1} = [y_t - k_t + I_t]^+ \quad t = 1, ..., N-1 \tag{6}$$

$$y_t \leq M \quad t = 1, ..., N \tag{7}$$

$$1 \leq k_t(G_t) \leq k_{max}, \quad t = 1, 2, ..., N, \tag{8}$$

where $S_{\text{avg}}$ is the average power budget for each transmission, $BER_{\text{avg}}$ is the average tolerable bit error rate and $2^{k_{\max}}$ is the maximum constellation size that we can be transmitted. The optimization is over the control policies $\{k_t(G_t)\}$ and $\{s_t(G_t)\}$ and over the variables $\{y_t\}$. Equation (3) is the average transmission rate each cycle. The expectation is over the $N$ channel states $\{G_t\}_{t=1}^{N}$ used by the transmitter each cycle. Equation (4) constrains the average transmitter power over the $N$ periods to $S_{\text{avg}}$. The optimal control policies allocate power over both the $N$ time periods, but also, the set of possible channel states. Likewise equation (5) constrains the average BER rate to $BER_{\text{avg}}$.

Hardware constraints are represented in equations (6) - (8). Equations (6) - (7) capture the evolution of backlog in the system and constrain it to be less than $M$ bits in every period. The purpose of this constraint is to prevent buffer overflow by adjusting the transmitter rate $k_t$ to manage this buffer. We assume throughout this paper that it possible for the system to support a transmission rate of $I_t$ under the power and BER constraints, and that this quantity is known. Equation (8) reflects the physical limitation on transmitter constellation size.

The optimal policies balance the tradeoff between higher rate and worsened BER, in some instances increasing power without increasing transmitter rate to reduce BER.

There are several challenges to solving this stochastic optimization problem. First, the distribution of channel states is unknown, and second, the problem is difficult to solve analytically. In the next section we describe a general online technique that iteratively finds the optimal control policies from samples of the LPTV channel.

## IV. Generalized Framework and Method of Solution

In this section we generalize the PLC problem of Subsection III and describe an online learning algorithm for finding optimal control policies.

### A. General Framework

The PLC problem is of the more general form

$$\underset{\mathbf{a}(G),\mathbf{y}}{\text{Maximize:}} \quad \mathbf{E}\left[f_0\left(\sum_{t=1}^{N} k_t\right)\right], \tag{9}$$

$$\text{Subject to :} \quad \mathbf{E}[g(\mathbf{y},\mathbf{a}(G)),G] \leq 0 \tag{10}$$

$$y_{t+1} = f_1(y_t, a_t(G_t), G_t) \tag{11}$$

$$(y_t, a_t(G_t)) \in \mathbf{H} \quad t = 1, ..., C \tag{12}$$

where $\mathbf{y}$ is the vector of $\{y_t\}$ and $\mathbf{a}(G)$ is the vector$\{a_t(G_t)\}$. The problem considers $t = 1, ..., N \leq C$ time periods. The functions $f_0$, $f_1$ and $g$ are assumed to be continuously differentiable, with $f_0$ concave and strictly increasing and $f_1$ and $g$ strictly convex. The convexity assumptions assure globally optimal control policies. The variables are the control policies $\{a_t(G_t)\}$ and the state variables $\{y_t\}$. The set $\mathbf{H}$ is convex.

Equation (3) is of the form of (9), and equations (4) and (5) are of the form (10). The expectation averages over the distribution of $G$ and as a consequence the optimal control policies $\mathbf{a}(G)$. Equation (6) is of the form (11) and represents the state evolution of the system over the $C$ time periods. The future state $y_{t+1}$ is determined by the current state $y_t$, the value of the current control policy $a_t(G)$, and the random variable $G_t$. Equations (7) and (8) are of the form (12) and represent bounds on the control policies and system state.

### B. General Method of Solution

Even when the distribution of $\{G_t\}$ is unknown, problems of the type described in IV-A can sometimes be iteratively solved using techniques based on stochastic approximation [17], [18]. These techniques compute averages by sampling the underlying random sequence $\{G_t\}$ and under suitable conditions on the unknown distribution of $G_t$ and smoothness conditions on the functions, computes converging estimates of the optimal control policies. The approach computes a new estimate of the optimal control policies every cycle. Convergence occurs at an exponential rate [18]. We assume $\{G_t\}$ are IID, although this condition can be relaxed. To streamline notation we will write $\mathbf{a}(G)$ for the vector of control policies and $\mathbf{y}$ for the vector of system states.

Our approach is based on forming the Lagrangian for the PLC problem of Section IV-A

$$L(\mathbf{a}(G),\lambda) = \mathbf{E}\left[f_0\left(\sum_{t=1}^{N} a_t(G_t)\right) - \lambda g(y_t, \mathbf{a}(G))\right], \tag{13}$$

where $\lambda \geq 0$ is the Lagrange multiplier associated with constraint (10), the expectation is over $\{G_t\}_{t=1}^{C}$, and the control policy $a_t(G_t)$ is a function of $G_t$.

At optimality the optimal control policy $\mathbf{a}^*(G)$ and the optimal Lagrange multiplier $\lambda^*$ satisfy the saddle point condition such that for $\mathbf{a}(G)$ and positive $\lambda$,

$$L(\mathbf{a}^*(G),\lambda) \geq L(\mathbf{a}^*(G),\lambda^*) \geq L(\mathbf{a}(G),\lambda^*). \tag{14}$$

The value of $\lambda^*$, given the functional forms of $f_0$, $f_1$ and $g$, is determined by the distribution of $\{G_t\}$. As this distribution changes the value of $\lambda^*$ will change. Our approach iteratively generates estimates that converge to the optimal pair $(\mathbf{a}^m(G), \lambda^m) \to (\mathbf{a}^*(G), \lambda^*)$, where $m$ is our iteration index. In this way our approach learns the the optimal value of $(\mathbf{a}^*(G), \lambda^*)$

We use different but coupled, techniques to find $\mathbf{a}^m(G)$ and $\lambda^m$. For notational compactness, we write $\mathbf{a}^m$ for $\mathbf{a}^m(G)$ in what follows. The technique used to find $\mathbf{a}^m$ is based on recursively finding the gradient to (13) and using the Robbins-Monroe algorithm [17] to generate updates. Suppose the operations of expectation and differentiation can be interchanged, define $\overline{d}(\mathbf{a},\lambda) = (\overline{d}(a_t,\lambda), t \leq N)$ by

$$\overline{d}(a_t,\lambda) \overset{def}{=} \frac{\partial L}{\partial a_t} = \mathbf{E}\left[\frac{\partial f_0\left(\sum_{t=1}^{N} a_t\right)}{\partial a_t} - \lambda \frac{\partial g(y,\mathbf{a})}{\partial a_t}\right] \tag{15}$$

$$= \mathbf{E}\left[\frac{\partial f_0}{\partial a_t} - \lambda \sum_{j=1}^{N} \frac{\partial g(y,\mathbf{a})}{\partial y_j}\frac{\partial y_j}{\partial a_t} - \lambda \frac{\partial g(y,\mathbf{a})}{\partial a_t}\right] \tag{16}$$

$$\overset{def}{=} \mathbf{E}\left[d(a_t,\lambda,G)\right] \tag{17}$$

Note that $d(a_t,\lambda,G)$ is an unbiased estimator of $\overline{d}(a_t,\lambda)$.

Define $\dot{Y}_t^i(\mathbf{a}) = \partial y_t(\mathbf{a})/\partial a_i$. Then based on the state evolution equation (11), we can calculate this derivative recursively by:

$$\dot{Y}_{t+1}^i(\mathbf{a}) = f'_{1y}(y_t, a_t, G_t)\dot{Y}_t^i(\mathbf{a}) + \frac{\partial}{\partial a_i} f(y_t, a_t, g_t). \tag{18}$$

At each iteration $m$, the randomness (e.g. the channel) is sampled and the Robbins Monroe formula used to update the control policies

$$a_t^{m+1} = \prod_H \left[a_t^m + \varepsilon^m d(a_t, \lambda^m, G)^{m+1}\right] \tag{19}$$

where $\{\varepsilon^m\}_{m=1}^{\infty}$ is an arbitrary non-summable but square summable sequence of diminishing real valued series, $\prod_H(\mathbf{a})$ is the closest point of $\mathbf{a}$ to the set $H$ and we define $d(a_m, \lambda, G^m)$ as (17) evaluated at iteration $m$. At $m = 0$ we start with an arbitrary random action set $\mathbf{a}^0$, and a random initial Lagrange multiplier $\lambda^0$. Note, $a_t^{m+1}$ is a function of $\lambda^m$. The Lagrange multiplier is in turn a function of the distribution of $G$. It is in this way that the optimal control policies are functions of of $G$ and its distribution.

The computation of $\lambda^m$ also uses the Robbins-Monroe formula, but is based on a subgradient to (14). It can be shown [23] that the constraint $g(y_t, \mathbf{a}(G))$ is a stochastic subgradient to (14). The update equation becomes

$$\lambda^{m+1} = \left[\lambda^m + \varepsilon^m g(y_t^m, \mathbf{a}^m(G))\right]^+. \tag{20}$$

This update for the Lagrange multiplier is trying to minimize (13) with respect to $\lambda$. But because $g(y_t, \mathbf{a}(G))$ is a stochastic subgradient it points in the proper direction only on average. The effect of the term $\varepsilon^m$ is to "smooth-out" the variations in the stochastic subgradient. The $m+1$ estimate of the Lagrange multiplier $\lambda^*$ is a function of $\mathbf{a}^m(G)$, thus coupling the pair $(\mathbf{a}^m(G), \lambda^m)$.

## V. PLC Algorithm

In this section we specialize the general method of solution results from Section IV-B to the specific PLC problem described in Section III.

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**PLC Algorithm**

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**(Initialization)**

Initialize the Lagrange multipliers and action set with random positive numbers for $m = 0$.

**(Primal step)**

Update the powers, rates and buffer states from (19)

$$s_t^{m+1} = \prod_{[0,\infty]} \left[ s_t^m + \varepsilon^m \left( \lambda_s - \lambda_b \frac{\partial BER_t}{\partial s_t} \right) \right] \quad (21)$$

$$k_t^{m+1} = \prod_{[1,k_{max}]} \left[ k_t^m + \varepsilon^m \left( 1 - \lambda_b^m \frac{\partial BER_t}{\partial k_t} - \sum_{i=1}^{N} \lambda_{y,i}^m \frac{\partial y_i}{\partial k_t} \right) \right] \quad (22)$$

$$y_t^{m+1} = \prod_{[0,M]} \left[ y_t^m - \varepsilon^m \lambda_{y,t}^m \right] \quad (23)$$

**(Price update)**

Update the Lagrange multipliers from (20):

$$\lambda_s^{m+1} = \prod_{[0,\infty]} \left[ \lambda_s^m + \varepsilon^m \left( \sum_{t=1}^{N} s_t - N S_{avg} \right) \right] \quad (24)$$

$$\lambda_b^{m+1} = \prod_{[0,\infty]} \left[ \lambda_b^m + \varepsilon^m \left( \sum_{t=1}^{N} BER_t - N BER_{avg} \right) \right] \quad (25)$$

$$\lambda_{y,t}^{m+1} = \prod_{[0,\infty]} \left[ \lambda_{y,t}^m + \varepsilon^m \left( y_{t,m} - M \right) \right] \quad (26)$$

**(Repeat)**

Let $m := m + 1$ and go to the primal step until convergence.

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Note that in this algorithm in the rate update formula (22) we can use the recursion (18) in order to compute the terms $\frac{\partial y_i}{\partial k_t}$ efficiently. Using this recursion, one can check that $\frac{\partial y_i}{\partial k_t}$ is zero for all $i \leq t$ and also:

$$\frac{\partial y_i}{\partial k_t} = \begin{cases} -1 & \text{if } i > t \text{ and } y_j \neq 0 \text{ for all } j \in [t+1, i] \text{ ;} \\ 0 & \text{otherwise;} \end{cases} \quad (27)$$

The intuition behind (27) is that the buffer size at the $i^{th}$ time slot, $y_i$, is independent of the rates of transmission, $k_t$, in next time slots $t > i$ and has a minus one derivative w.r.t previous time slots until the buffer becomes empty. This fact can be deduced from applying (2) in recursion (18).

This algorithm needs a training time for finding the optimum Lagrange multipliers (prices). After the convergence of prices, the algorithm has learned the necessary information that it needs from the statistics of the channel and thereafter those prices can be treated as constants. Since the convergence of the algorithm occurs in an exponential rate this training time for learning the characteristics of the channel is practical.

## VI. Simulation and Control Policy Behavior

In this section we illustrate the performance of a PLC system using the optimal control policies described in Section V. We also compare the performance of the PLC system, over the same channel realizations, when transmitter power and transmission BER are held constant. In both cases we assume an LPTV channel.

We simulate several AC power cycles divided into $C = 50$ time periods. For concreteness we let the transmitter transmit every period $N = C$. The channel gains $G_t$ vary randomly but with a periodic distribution over the $C$ time periods. They are constructed by adding zero mean Gaussian random variables to a periodic sequence of channel gains. Figure 1 illustrates the channel. Both the continuous and sampled channel gains are shown.

Figure 2 illustrates the performance of the optimal policies. The figure shows the channel condition $G_t$, the rates $k_t$ allocated to the different channel conditions , the associated transmitter power $s_t$, and the average buffer backlog $y_t$. The input data rate $I_t$ is 1.75. When buffer backlog is small and channel conditions are nominal, transmitter rates are roughly constant and the transmitter power varies approximately inversely to the channel condition. This occurs as the optimal control policies allocate power to maintain the BER floor and hold rate constant. When backlog is small and channel conditions are poor, the policies are forced to decrease transmitter rate to make more power available to maintain the BER floor. In both cases, the optimal control policies allocate power to maintain the BER floor and if necessary at the expense of throughput.

When buffer backlog is near its allowed maximum $M$, the behavior of the control policies changes. Power is first allocated to increase the transmitter rate to prevent buffer overflow. This is seen in time periods 25-30, where power and rate are increased to reduce the buffer backlog. This behavior is true whether the channel conditions are good or

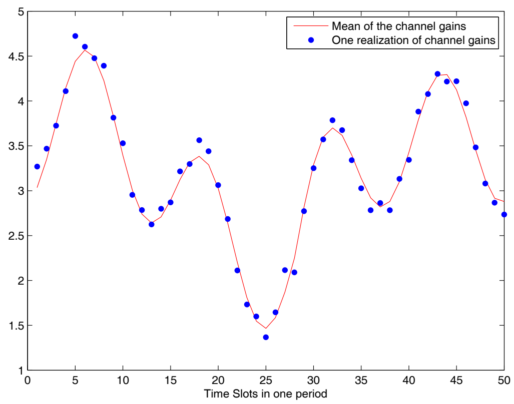


Fig. 1. Channel gains, mean and one realization

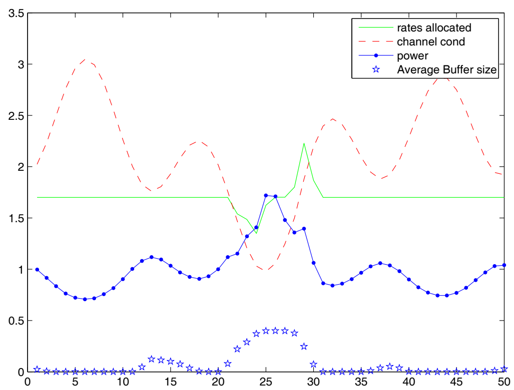


Fig. 2. PLC optimal performancwe

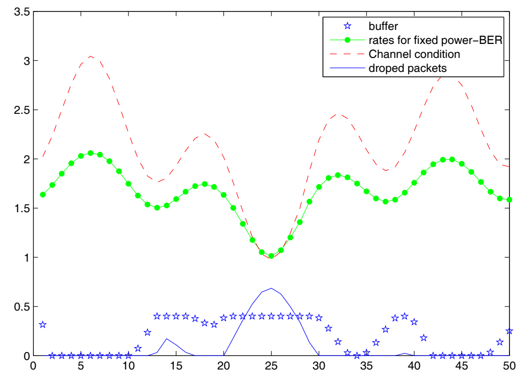


Fig. 3. Fixed power and BER performance

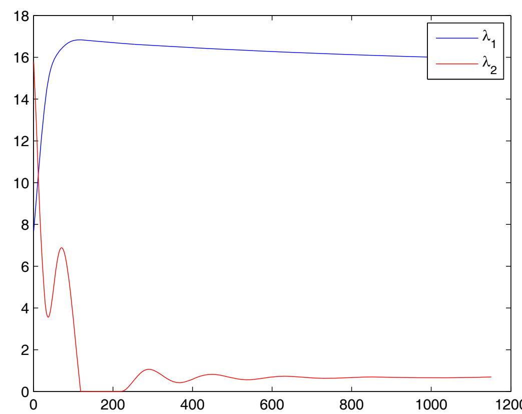


Fig. 4. Lagrange multipliers for the average power and BER constraints

poor. Note, the average transmitter rate is set by the policies allocating power over the set of possible channel states.

Figure 3 illustrates the behavior of the PLC system when we choose non-optimal control policies. Here transmitter power and BER are fixed at $S_{\text{avg}}$ and $BER_{\text{avg}}$. As can be seen, the transmitter rate now varies proportionately with the channel state and independently of the buffer backlog. As a consequence in periods of poor channel conditions, the buffer backlog increases and eventually overflows. The figure depicts the lost packets as occurring at the three occasions transmitter rate is less than the arrival rate of new packets to be sent.

Figure 4 shows the convergence behavior of the approach. The Lagrange multipliers are shown for each iteration of the algorithm. After approximately four hundred iterations the Lagrange multipliers are constant and the optimal control policies have been attained.

## VII. Conclusions

We have presented a general stochastic optimization framework for periodic systems and applied it to a PLC network. Our method of solution operates online and does not assume prior knowledge about the distribution of channel states, but rather samples the LPTV channel to learn the information necessary to compute optimal control policies. We applied this general framework to compute optimal bit allocations (rates) and transmit powers for each transmit period over an AC cycle, subject to requirements on average bit error rates and finite buffer sizes. The resultant policies find bit loadings that maximize throughput, while meeting BER requirements and preventing buffer overflow. We presented numerical simulations that illustrate the approach and that demonstrate the performance benefits of this method.

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