Stochastic Optimization for Variable Rate Applications with Time-Varying Statistics

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***Abstract*—A scheme to provide Quality of Service for buffered variable rate applications is presented that is largely indiscriminate to channel distributions, indeed will track changing statistics, and is effectively spectrum optimal. The solution is based on the framework of Full Recourse Optimization with Expected Constraints where the optimal solution is learned through updates of the Lagrange multiplier. The convergence speed and nearness to optimality are found, and the buffer stability probabilities are met. Analysis and simulations are provided to validate the scheme's performance.**

I. Introduction

In wireless systems, mobile users experience a dynamic environment that makes reliable transmission difficult. As mobile application demands grow more diverse, cross-layer optimization has been suggested as a method to maximize efficiency while meeting the end user's application requirements. One of the general objectives is to maximize the spectral efficiency for the given bandwidth while minimizing any service interruption to the user, thus maximizing both the possible number of users supported and maintaining Quality of Service (QoS).

In many applications, mobile users utilize a buffer to minimize service disruption and hedge against deep fades. To satisfy this user's QoS requirement, the buffer occupancy cannot underflow, or risks service interruption. In [1], a cross-layer transmission scheme was introduced that utilized a buffer state feedback bit to vary the average power and maintain buffer stability. However, the scheme required knowledge of the channel distribution.

In mobile scenarios, the complete channel statistics are largely unknown and time-varying. To maintain continual QoS support, solving a single optimization problem and using the solution for a significant duration seems sub-optimal and impractical. Rather, the optimal solution is likely to be varying, suggesting an adaptive algorithm that is largely indiscriminate to channel statistics, yet guarantees optimality, is desirable. The proposed scheme is an adaptive stochastic optimization method based on ideas of Wireless Network Utility Maximization [2].

The literature for real-time and video traffic includes ideas from variable rate stream encoding [3], [4], and dynamic programming solutions [5], [6]. In [7], a cross-layer utility maximization framework is posed to balance rate maximization and fairness, but does not support QoS guarantees. In [8], buffer stability is examined, however statistical distributions are required.

In this paper, a scheme is introduced for buffered, variable rate applications that achieves nearly optimal spectral efficiency and does not require *a priori* knowledge of the channel distribution. By utilizing the stochastic method Full Recourse Optimization with Expected Constraints (FROEC), the algorithm tracks the optimal solution even as channel statistics change. The paper investigates the convergence and steady-state properties of FROEC, and determines the power policy required to meet specified buffer stability probabilities.

II. System Model

Consider a user terminal running an application that draws a random number of r_n bits from a buffer at time step n. Moreover, let the incoming rate, determined by the transmitter, be R_n. Then the buffer occupancy progression is given as,

$$\begin{aligned} b_n &= b_{n-1} + R_n - r_n \\ &= b_0 + \sum_{i=1}^{n} R_i - \sum_{i=1}^{n} r_i \end{aligned} \tag{1}$$

Where b_0 is the initial buffer occupancy that is assumed to be set via a pre-loading scheme. This paper assumes a flow model, hence non-integer number of bits.

The application traffic r_n is modeled as a nominal rate plus an independent random variation with unknown distribution. The nominal rate is R^{app}, and the variance is σ^2_{app}.

Channel State Information (CSI) given as γ_n are assumed to be independent and known at the transmitter, however the fading distribution $p(\gamma)$ is unknown and can be slowly time varying.

III. Proposed Scheme

The goal of the proposed scheme is to transmit a minimum number of bits within a finite-time horizon and be spectrally optimal. Specifically, the scheme determines the power policy P_n that minimizes the average transmit power $\mathbb{E}[P]$ while guaranteeing a minimum number of bits have been transmitted. As the user is expected to have a buffer, this criterion is formulated to ensure the instantaneous buffer level b_n does not underflow with ϵ probability in the worst-case time step n,

$$\max_n \text{pr}(b_n \leq 0) \leq \epsilon \tag{2}$$

A. Full Recourse Optimization with Expected Constraints

The proposed scheme is a power minimization problem subject to an average rate target R^{trgt}, and it is shown to solve this problem optimally.

$$\begin{array}{ll} \min_{P\geq 0} & \mathbb{E}[P_n] \\ \text{subject to:} & \mathbb{E}[\log_2(1+\gamma_n P_n)] = R^{\text{trgt}} \end{array} \tag{3}$$

Where determining R^{trgt} is the key to meeting the ϵ criterion and is found in Section III-C.

When the fading statistics are known, e.g. Rayleigh, Rician, Nakagami, this problem is solved by using the fading distribution and determining the Lagrange multiplier or the water-level. This leads to the well known water-filling power policy [9].

However, if the fading distribution $p(\gamma)$ is unknown, the framework of FROEC is used to determine the optimal power policy. The solution is a three-step algorithm with negligible complexity. The first step is to estimate the channel at time step n, and solve the optimization problem using an initial Lagrange multiplier $\lambda_n = \lambda_0$. The solution is a closed form expression,

$$P_n = \left[\lambda_n - \frac{1}{\gamma_n}\right]^+ \tag{4}$$

This implies the instantaneous rate,

$$R_n = \log_2(1+\gamma_n P_n) \tag{5}$$

The second step is calculating the stochastic subgradients,

$$\delta g_n = R^{\text{trgt}} - R_n \tag{6}$$

And finally, the Lagrange multiplier is updated,

$$\lambda_{n+1} = \lambda_n + \Delta_n \delta g_n \tag{7}$$

Where Δ_n is the step-size and dictates convergence speed and optimality as explained in Section III-B.

B. Convergence and Optimality

The optimality of this scheme relies on the convergence of λ_n to λ^*, the optimal Lagrange multiplier. As an adaptive algorithm, the convergence speed, tracking properties, and nearness to optimality are dictated by the step size Δ_n. For example, by decreasing the step size as $\Delta_n = \Delta/n$, the solution converges to the optimal but will not track changing statistics. To ensure λ_n will adapt to a new value if the channel statistics change, a fixed step-size is required, hence this scheme assumes $\Delta_n = \Delta$. By using a fixed step-size, the solution does not converge exactly to the optimal value, rather it converges in probability to the optimal solution and is given as [10],

$$Pr[||\lambda_n - \lambda^*|| \geq \alpha|\lambda_0] \leq A_1(\Delta) + A_2(\lambda_0)\exp(-h(\Delta)n) \tag{8}$$

the term $A_1(\Delta)$ is a constant whose magnitude is a function of the step-size, and is the random cloud about the optimal solution in steady-state. The second term is the initial transient response, whose effect decreases exponentially fast in time. In steady-state, the deviation $z_n = \lambda_n - \lambda^*$ is shown to be i.i.d. $\sim \mathcal{N}(0, \sigma_z^2)$ [10, §8.4], where σ_z^2 is determined below. In essence, a small step-size takes longer to converge, but is closer to the optimal solution. A sample-path of λ_n is given in Figure 1.

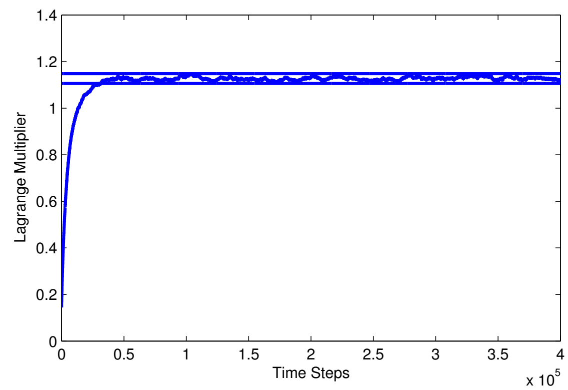

Fig. 1. λ^n converges in probability exponentially fast, $3\sigma_z$ confidence interval given.

To determine σ_z^2, a result from stochastic approximation is required. Given an iterative expression of the form [10, Eq (8.2.1)],

$$\lambda_{n+1} = \lambda_n + \Delta[h(\lambda_n) - M_n] \tag{9}$$

where $h(\lambda_n)$ is a deterministic continuously differentiable function, and M_n is a Martingale Difference Sequence (MDS), i.e., a sequence $\{X_1, X_2, \ldots, X_{n+1}\}$ is an MDS if $\mathbb{E}[X_{n+1}|X_1, \ldots, X_n] = 0$. Then, the following result holds [10, Eq (8.4.1)] in steady-state,

$$2\nabla h(\lambda^*)\Delta\sigma_z^2 + \Delta^2\sigma_M^2 = 0 \tag{10}$$

Where σ_M^2 is the variance of the MDS. In order to express Eqn (7) in the form of Eqn (9), let $\zeta_n = \log_2(\gamma_n)$ and $\zeta_0^n = \log_2(1/\lambda_n)$, then from Eqn (4) and Eqn (5), the instantaneous rate is,

$$R_n = \begin{cases} \zeta_n - \zeta_0^n, & \zeta_n \geq \zeta_0^n \\ 0, & \zeta_n < \zeta_0^n \end{cases} \tag{11}$$

To satisfy Eqn (9), R_n must be written as a differentiable function, ergo consider the new random variable,

$$\tilde{\zeta}_n = \begin{cases} \zeta_n, & \zeta_n \geq \zeta_0^n \\ \zeta_0^n, & \zeta_n < \zeta_0^n \end{cases} \tag{12}$$

So that, $R_n = \tilde{\zeta}_n - \zeta_0^n$

Now, the update of λ_n is expressed as a continuously differentiable function and a MDS,

$$\begin{aligned} \lambda_{n+1} &= \lambda_n + \Delta(R^{\text{trgt}} - R_n) \\ &= \lambda_n + \Delta(R^{\text{trgt}} - \tilde{\zeta}_n + \zeta_0^n + (\mathbb{E}\tilde{\zeta} - \mathbb{E}\tilde{\zeta})) \\ &= \lambda_n + \Delta(R^{\text{trgt}} + \zeta_0^n - \mathbb{E}\tilde{\zeta} - M_n) \end{aligned} \tag{13}$$

From above, $h(\lambda_n) = R^{\text{trgt}} + \zeta_0^n - \mathbb{E}\tilde{\zeta}$ and $M_n = \tilde{\zeta}_n - \mathbb{E}\tilde{\zeta}$. To solve Eqn (10), $\nabla h(\lambda^*)$ is given as,

$$\nabla h(\lambda^*) = -\frac{1}{\lambda^*}\log_2(e) \tag{14}$$

And $\sigma_M^2 = \tilde{\sigma}_\zeta^2$, i.e. the variance of $\tilde{\zeta}$. This leads us to the sampling noise variance,

$$\sigma_z^2 = \Delta\tilde{\sigma}_\zeta^2 \lambda^* \frac{c_1}{2} \tag{15}$$

where $c_1 = \ln(2)$.

However, a useful bound on σ_z^2 is in terms of σ_ζ^2 not $\tilde{\sigma}_\zeta^2$, because σ_ζ^2 is a only a function of the channel, while $\tilde{\sigma}_\zeta^2$ relies on the power scheme, i.e. cutoff power in poor SNR.

To show $\sigma_\zeta^2 \geq \tilde{\sigma}_\zeta^2$, consider shifting both p.d.f's $\hat{p}(\zeta) = p(\zeta - \zeta_0^*)$ and $\hat{\tilde{p}}(\tilde{\zeta}) = \tilde{p}(\tilde{\zeta} - \zeta_0^*)$ where $\zeta_0^* = \log_2(1/\lambda^*)$. Shifting the p.d.f.'s does not change the variance of either random variable. Then,

$$\begin{aligned}
\sigma_\zeta^2 &= \int_{-\infty}^{\infty} \zeta^2 \hat{p}(\zeta) d\zeta - \left(\int_{-\infty}^{\infty} \zeta \hat{p}(\zeta) d\zeta\right)^2 \\
&\geq^{(a)} \int_{0}^{\infty} \zeta^2 \hat{p}(\zeta) d\zeta - \left(\int_{-\infty}^{\infty} \zeta \hat{p}(\zeta) d\zeta\right)^2 \\
&\geq^{(b)} \int_{0}^{\infty} \zeta^2 \hat{p}(\zeta) d\zeta - \left(\int_{0}^{\infty} \zeta \hat{p}(\zeta) d\zeta\right)^2 \\
&= \mathbb{E}\tilde{\zeta}^2 - (\mathbb{E}\tilde{\zeta})^2 = \tilde{\sigma}_\zeta^2
\end{aligned} \tag{16}$$

(a) integrating less positive terms, (b) subtracting a larger value, true for $\zeta_0^* \leq \frac{\mathbb{E}\log_2(\gamma) + \mathbb{E}_{\gamma_0}\log_2(\gamma)}{2}$. In essence, this bound is appropriate in moderate to high SNRs. Therefore, the variance is bounded as $\sigma_z^2 \leq \Delta\sigma_\zeta^2 \lambda^* \frac{c_1}{2}$.

Hence, the FROEC approach with a fixed step-size solves the optimal Lagrange multiplier within the uncertainty of z_n. And z_n is modeled as $\mathcal{N}(0, \Delta\sigma_\zeta^2 \lambda^* \frac{c_1}{2})$.

C. Rate Target

The rate target R^{trgt} necessary to meet the buffer probability guarantee ϵ is determined in a similar manner to [1], in which case the distribution $p(\gamma)$ was assumed to be known hence $\sigma_z^2 = 0$, and the traffic was constant hence $\sigma_{\text{app}}^2 = 0$.

Two assumptions are made: first, the buffer size is large compared with the application's rate requirement for a given time step:

$$N = B_{\text{th}}/R^{\text{app}} > 10 \tag{17}$$

This assumption is realistic as buffer size is typically much larger than the number of bits an application uses in a scheduling instance. The second assumption is the initial buffer state can be set through an initial buffer loading transmission scheme $b_0 = B_{\text{th}}$ (e.g. the user's application does not begin drawing bits until the buffer state crosses the threshold).

The buffer underflow guarantee is,

$$\epsilon \geq 1 - \min_{n:n\geq N} \text{pr}\left(\sum_{i=1}^{n} R_i + B_{\text{th}} - \sum_{i=1}^{n} r_i > 0\right) \tag{18}$$

The rates entering the buffer during n time steps is given as,

$$\begin{aligned}
\sum_{i=1}^{n} R_i &= \sum_{i=1}^{n} \tilde{\zeta}_i - \zeta_0^i \\
&\approx^{(a)} \sum_{i=1}^{n} \tilde{\zeta}_i + \log_2(\lambda^* + z_i) \\
&= \sum_{i=1}^{n} \tilde{\zeta}_i + \sum_{i=1}^{n} \log_2(\lambda^*) + \sum_{i=1}^{n} \log_2(1 + \frac{z_i}{\lambda^*}) \\
&\approx^{(b)} \sum_{i=1}^{n} \tilde{\zeta}_i + \sum_{i=1}^{n} \log_2(\lambda^*) + \frac{1}{c_1\lambda^*}\sum_{i=1}^{n} z_i \\
&=^{(c)} W_n + n\log_2(\lambda^*)
\end{aligned} \tag{19}$$

(a) Follows from Section III-B, (b) Assuming the step-size is chosen sufficiently small to ensure $z_i/\lambda^* << 1$, using the small angle approximation $\log_2(1+x) \approx x\log_2(e)$. (c) The Central Limit Theorem because the buffer size is assumed large in Eqn (17), and in steady-state,the R_i's maintain the same statistics and are independent and identically distributed. Therefore, W_n is modeled as a Gaussian random variable. It follows that $W_n = \sum_{i=1}^{n}\left(\tilde{\zeta}_i + \frac{1}{c_1\lambda^*} z_i\right)$, and hence $W_n \sim \mathcal{N}\left(n\mathbb{E}[\tilde{\zeta}], n\sigma_\zeta^2\left[1 + \frac{1}{2c_1\lambda^*}\Delta\right]\right)$.

The bits draining from the buffer during n time steps is given as $Z_n = \sum_{i=1}^{n} r_i$. Again, using the CLT, Z_n is modeled as a Gaussian random variable $Z_n \sim \mathcal{N}\left(nR^{\text{app}}, n\sigma_{\text{app}}^2\right)$.

To simplify notation, consider,

$$R^{\text{trgt}} = \log_2(\lambda^*) + \mathbb{E}[\tilde{\zeta}] \tag{20}$$

and,

$$\sigma_{\text{tot}} = \sigma_\zeta\left(1 + \frac{1}{2c_1\lambda^*}\Delta\right) + \sigma_{\text{app}} \tag{21}$$

Then, to meet the stability criterion in Eqn (18),

$$\epsilon = \max_{n:n\geq N} Q\left(\frac{B_{\text{th}} + nR^{\text{trgt}} - nR^{\text{app}}}{\sqrt{n}\sigma_{\text{tot}}}\right) \tag{22}$$

The argument of the Q-function is given as $f(n)$, then using decreasing-monotonicity of the Q-function and convexity of $f(n)$, N^* is found that minimizes $f(n)$. Therefore,

$$\begin{aligned}
N^* :&= \arg\min_n \frac{B_{\text{th}} + nR^{\text{trgt}} - nR^{\text{app}}}{\sqrt{n}\sigma_{\text{tot}}} \\
&= \frac{B_{\text{th}}}{R^{\text{trgt}} - R^{\text{app}}}
\end{aligned} \tag{23}$$

Applying N^*, the time step that maximizes the underflow probability, into Eqn (22), the underflow guarantee is

$$\epsilon = Q\left(\frac{2}{\sigma_{\text{tot}}}\sqrt{B_{\text{th}}\left(R^{\text{trgt}} - R^{\text{app}}\right)}\right) \tag{24}$$

This leads to the target rate that ensures the worst-case probability of underflow is less than ϵ, $R^{\text{trgt}} = R^{\text{app}} + \Delta R$, where

$$\Delta R = \frac{\sigma^2_{\text{tot}} \left[Q^{-1}(\epsilon)\right]^2}{4B_{\text{th}}} \tag{25}$$

This expression has commonality with the derivation in [1], and provides a relationship between the initial buffer level, the QoS guarantee, and the uncertainty in the system. As intuition suggests, more uncertainty requires a larger rate deviation to maintain buffer stability. In this case, the variance is a function of the channel variation σ^2_ζ, the step-size Δ, and the traffic variation σ^2_{app}.

IV. Numerical Results: Buffer Stability Scheme

In this section, the numerical results are presented by implementing the proposed scheme for both an underflow guarantee *and* an overflow guarantee. Rather than guaranteeing a minimum number of bits is being transmitted, this section implements a window guarantee, i.e. the transmitted bits is above the minimum number, and below a maximum number to ensure that a buffer does not underflow or overflow. The scheme is implemented with two rate targets, R^{trgt}_+ and R^{trgt}_-, and depending on the instantaneous buffer level b_n switches between the solutions of the two rate targets, as given in the state diagram Figure 2.

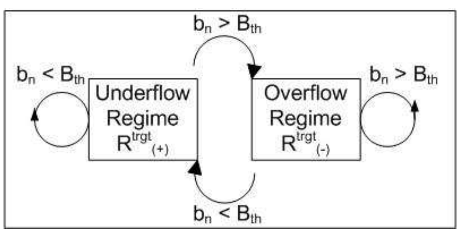

Fig. 2. Switching scheme based on instantaneous buffer level.

A sample path of the Lagrange multipliers used in this switching scheme is given in Figure 3.

In Figure 4, the proposed scheme is compared to single-level water filling (SWF), a spectrally optimal transmission scheme for an average rate target. As is clear, the proposed scheme is able to maintain buffer stability and keep the buffer levels within a finite window, while SWF cannot.

A. Power Efficiency in Rayleigh Fading

The proposed algorithm is stable and optimal regardless of fading distribution, and indeed, does not need to know the distribution *a priori*. However, power/spectral efficiency is a function of the fading distribution, hence to gain insight into efficiency, the result given in Eqn (26) assumes Rayleigh fading. In this case, the efficiency of the proposed scheme is well approximated as [1],

$$\overline{P} \approx \overline{P^{opt}} \left[1 + \frac{(\ln 2)^2}{2} \Delta R^2\right] \tag{26}$$

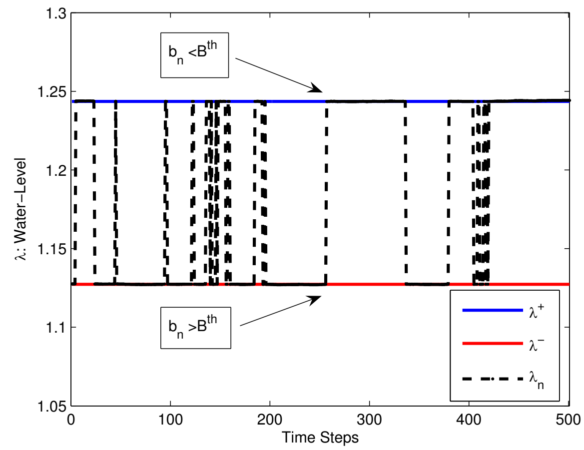

Fig. 3. Proposed scheme switches between solutions of two different optimization problems based on the instantaneous buffer level.

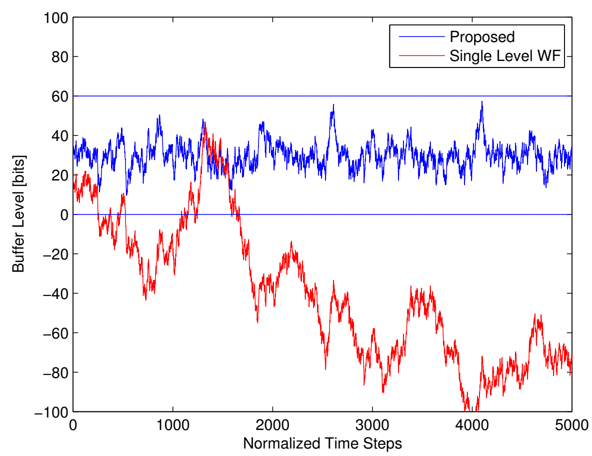

Fig. 4. Proposed scheme provides buffer stability, while single-level water-filling does not provide overflow/underflow guarantees.

B. Spectral Efficiency

The spectral efficiency of the proposed scheme is compared to the SWF, optimal truncated channel inversion (TCI) [9], and the Buffer State Information (BSI) scheme [1]. Note that SWF, TCI, and BSI require the channel distribution $p(\gamma)$, while the proposed scheme does not. SWF maintains the same long-term average rate, but does not guarantee buffer stability. Nonetheless, SWF serves as an upper-bound for the optimal scheme.

The QoS parameter $\epsilon = 5 \times 10^{-3}$. Spectral efficiency is shown in two fading distributions: Rayleigh and Nakagami($m = 2$) fading. To make comparisons with Shannon capacity, the applications average rate requirement R^{app} was arbitrarily set to the ergodic capacity C, and then only the normalized buffer size $N = B_{\text{th}}/R^{\text{app}}$, needs to be specified.

The spectral efficiency curves measure the efficiency of a transmission scheme normalized by the allocated bandwidth.

As can be seen, the proposed scheme has effectively the same spectral efficiency as SWF and BSI over a wide range of SNR values, implying near optimal spectral efficiency. In Figure 5 and Figure 6, $N = 10$, the minimum value that satisfies assumption Eqn (17). At higher N, the proposed scheme converges to the efficiency of SWF.

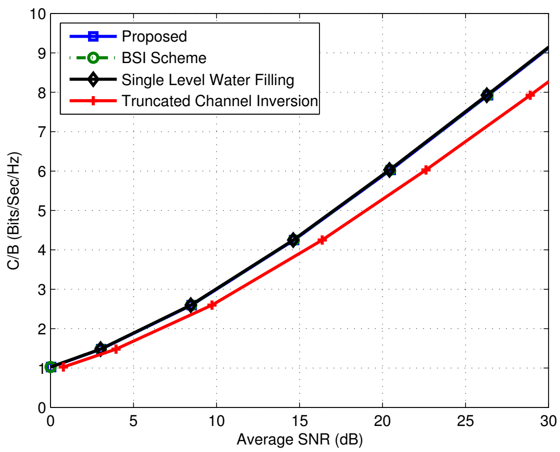

Fig. 5. Spectral efficiency for Rayleigh Fading.

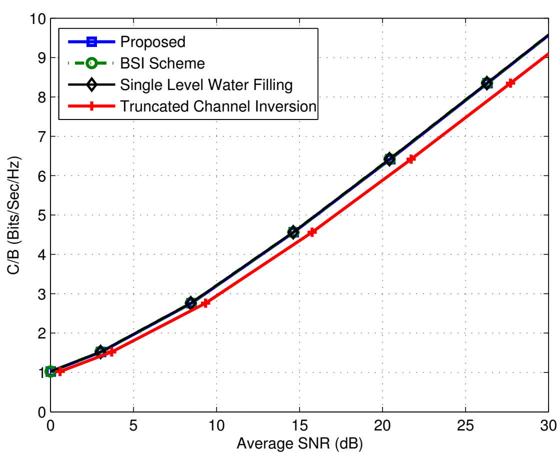

Fig. 6. Spectral efficiency for Nakagami-m Fading (m=2).

V. Summary

A scheme is proposed that ensures buffer stability, does not require knowledge of the channel statistics, and is effectively spectrum optimal. By using ideas from stochastic optimization, the algorithm learns and tracks the optimal solution, and the convergence speed and nearness to optimality are given. The scheme's performance is compared to the well-known water-filling and truncated channel-inversion transmission schemes, where only proposed scheme is able to maintain buffer stability *and* have nearly optimal spectral efficiency. Analysis and simulations are provided as validation.

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