Joan Robinson on "History versus Equilibrium"

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Abstract

Joan Robinson's endeavor to expose a lacuna between "history" and "equilibrium" constitutes a significant plank in her overall critique of orthodox economic theory. It also *goes beyond* the hornet's nest she successfully exposed in her celebrated critique of the treatment of "capital" in neoclassical theory. I argue in this paper that, instead of a closed circle of equilibrium relations, universally applicable and independent of time and place, Robinson sought to develop a system of analysis that is more eclectic and open to history in a very definite sense. Indeed, the hard core of her work is really an attempt to outline an alternative approach to economics that provides a way of understanding economic history in all its richness and diversity. She herself did not manage to advance this approach to any significant extent. Much work has been done since then that strongly bears out her concerns, advances her effort, and serves to clarify and give deeper insights into the nature of the problems she posed. These problems remain till today the most challenging in economic theory.

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The long wrangle about 'measuring capital' has been a great deal of fuss over a secondary question. The real source of trouble is the confusion between comparisons of equilibrium positions and the history of a process of accumulation. (Robinson, 1978, p. 135)

A model applicable to actual history has to be capable of getting out of equilibrium; indeed, it must normally not be in it. (Robinson, 1962, p. 25)

To construct models that cannot be applied is merely an idle amusement. It is only by interpreting history, including the present in history, that economics can aspire to be a serious subject. (Robinson, 1980, p. 90)

1. Introduction

In her well-known essay entitled "History versus Equilibrium" (1978, Ch. 12), Joan Robinson discusses the uses of the concept of equilibrium in economic theory and its relevance to analysis and understanding of the actual historical process of accumulation in the capitalist economy. A sharp distinction, indeed an opposition, is drawn between "history" and "equilibrium". It is a recurrent theme that runs throughout her later writings. In these writings, she expressed a great deal of skepticism of the historical relevance of the equilibrium concept in its various manifestations and mounted a trenchant critique of the orthodox economic theories associated with it.¹ It is a subtly layered argument that she made, which had the potential of being mistakenly interpreted as a blanket dismissal of what others have called "equilibrium economics". She sought to counter this effect in a spirited defence of "The Relevance of Economic Theory" (1973, Ch. 12) addressed, in part, to "the radical economists ... in American universities."

Though critical of the concept and uses of equilibrium, Robinson was not a "Luddite". She was too diligent and penetrating an analyst to dismiss the advantages, albeit recognized to be quite limited, of using the equilibrium concept as a tool for analytical purposes. She herself used the device to great effect in her own work. She viewed it, at times, as a "thought experiment", useful for solving "analytical puzzles", even to the point of recognizing a "perverse pleasure" in this practice (1956, p. 147, n. 3). She also thought: "It is useful for eliminating contradictions and pointing towards causal relations that will have to be taken into account in interpreting history." (1980, p. 90).

¹ One of the earliest written statements of her views on this subject is in the 1953 "Lecture delivered at Oxford by a Cambridge Economist" (1973, Ch. 27). Among her Cambridge colleagues, Kaldor was perhaps the first to go public with a systematic critique of the use of the equilibrium concept in orthodox theory (see, for instance, his two papers of 1934 in Kaldor (1960, Chs. 1 & 2)). Robinson acknowledged that her "ideas were formed in a long series of debates with … Kaldor … though he did not always approve the use to which I put them." (1956 p. vii). For a discussion of some implications of Kaldor's stated views on "equilibrium economics", see Harris (1991). I am prepared to argue that Robinson's critique is ultimately deeper and more far-reaching, and her practice more consistent, than that of Kaldor, but that will have to be the subject of another paper.

Moreover, she was fully convinced of the power of abstraction in economic analysis: "A model which took account of all the variegation of reality would be of no more use than a map at the scale of one to one." (1962, p. 33). At the same time, she insisted on the historical specificity of the analytical problem at hand, hence the need to develop relevant and realistic economic theory, without seeking to minimize the inherent problems involved in so doing. The object of analysis, she continued to insist, is the capitalist economy, in which "the capitalist rules of the game" constitute the defining order of things, and this reality imposes requirements on the analysis in order for it to be meaningful and relevant.²

Robinson's endeavor to expose a lacuna between "history" and equilibrium" constitutes a significant plank in her overall critique of orthodox economic theory. It also *goes beyond* the hornet's nest she successfully exposed in her celebrated critique (1978, Ch. 8) of the treatment of "capital" in neoclassical theory. Indeed, it is evident that she considered the latter a secondary matter, as in the first quote cited above. To be properly understood, this endeavor has to be seen, in my judgement, as *subsidiary to* a larger and more positively oriented effort. That is: her "long struggle to escape" from the confines of the (neoclassical) intellectual tradition of Marshall, Wicksell and Walras, in order to advance the project of the Keynesian revolution, with the aid of insights gained from a critical reading of Marx, towards a "theory of the dynamic development of capitalism."³ She took seriously, and as a life-long commitment, the task of carrying on this effort, readily acknowledged the analytical difficulties involved, and offered significant clues on how to proceed.

I argue in this paper that, instead of a closed circle of equilibrium relations, universally applicable and independent of time and place, Robinson sought to develop a system of analysis that is more eclectic and open to history in a very definite sense. Indeed, the hard core of her work is really an attempt, brilliantly executed, and without resort to mathematical wizardry, to outline an alternative approach to economics that provides a way of understanding history in all its richness and diversity. She herself did not manage to advance this approach to any significant extent. Much work has been done since then that strongly bears out her concerns, advances her effort, and serves to clarify

 $^{^2}$ She spent the first six chapters of her *magnum opus* (1956) elaborating in great detail the meaning of this presumption as the starting point of the full-fledged analysis presented there. In the follow-up work (1962), she starts out with a sharp distinction between two types of economic theory: one appropriate to a society of independent property owners each specialized to producing particular products, the other a hierarchical class-society of property owners and workers.

³ To identify the crucial turning point in her intellectual development, I would locate it between the (1942) *Essay on Marxian Economics* and the (1952a) *Rate of Interest and Other Essays*. By the time of (1952a) and the brief note (1952b) on "the Model of an Expanding Economy", it became evident that the positive effort of reconstruction had already begun. The route that she followed up to that point in the process is carefully laid out in the "Acknowledgements and Disclaimers" of (1952a) and, in the "Introduction", she makes the especially revealing comment: "I offer the argument at this primitive stage as an agenda for discussion, rather than as a completed piece of analysis. Having grown up swaddled in equilibrium theory I find my muscles soft, and to venture into dynamic problems induces a tendency to vertigo. Since there may be others in like case, I feel it is worth the attempt to clear up some very simple problems, in the hope that our heads may grow stronger as we go on." *The Economics of Imperfect Competition* (1933), celebrated for starting an earlier revolution *within* neoclassical economic theory, represents the period of being "swaddled in equilibrium theory", as she later attested in frank self-criticism (1960, pp. 222-45).

and give deeper insights into the nature of the problems she posed. These problems remain till today the most challenging in economic theory.

2. The Canonical Neoclassical Model of Accumulation

In order to provide a specific analytical context and meaning for Robinson's concerns, it is useful to consider the construction that has long been taken to represent the essentials of the neoclassical theory of capital accumulation.⁴ The central idea is that of accumulation as a dual process of "deepening" the structure of capital and of capital-"widening", which is held in check by rentiers, impatient to consume their income, who must be compensated for the cost of foregoing present consumption. The core principles of this construction were originally put together by Wicksell.⁵ Hence, it is often referred to as a "Wicksell Process" (Robinson, 1962, pp. 102-3, 132-5). It has been passed down by many different routes, with added complications, to modern day practitioners.⁶ A special case of it, its most recent vintage, is found in the well-known neoclassical model of economic growth (Solow, 1956; Swan, 1956; Meade, 1961) which launched a vast industry of theoretical and empirical research.⁷ Samuelson (1962, 1966) sought to give it a firm theoretical foundation in a "parable" that was subsequently undercut by the theorems on "reswitching" and "capital-reversal" in capital theory. Yet, oddly enough, it continues to occupy a central place in numerous scholarly efforts to explain actual historical processes.

To simplify the exposition, the idea of "capital deepening" is illustrated in Figure 1. In order not to test the patience of the careful Robinsonian reader, note immediately that, in this story, capital is rigorously assumed to be a single homogeneous entity, such as corn which can be used as seed or consumed directly.⁸ In the right-hand quadrant, the

⁷ Robinson preferred to classify these later contributions as "neo-neoclassical" (1973, p. 147), in order to distinguish them from the earlier neoclassical contributions of Wicksell, Walras, Marshall, and Pigou.

⁸ As a stand-in for fixed capital, the analogy of corn may be thought to break down on realistic grounds, since in reality corn appears more as circulating capital that is fully consumed in production and replaced at

⁴ A more detailed elaboration of this construction and dissection of its underlying properties is presented in Harris (1978, Ch. 9; 1980; 1981).

⁵ Robinson's preoccupation and uneasiness with the pre-existing formulation by Wicksell, whom she considered among her "progenitors", was undoubtedly a major factor determining the course of the analysis presented in *The Accumulation of Capital*, 1956. In an appendix (p. 396), she praised Wicksell, perhaps too generously, for providing "the key to the whole theory of accumulation and of the determination of wages and profits." At the same time, she found already in Wicksell the same source of trouble as she later criticized in the work of others: "The main difficulty presented by Wicksell's analysis is that he seems to be discussing in the same breath a comparison between static states with different quantities of capital and a process of accumulation going on through time" (p. 397).

⁶ The Fisherian tradition represents a different line of descent (from Irving Fisher, 1907, 1930) within the same neoclassical family. In its original version with a multiplicity of physical capital goods, it is not capable of easy interpretation as a process of accumulation (Fisher did present a one-commodity version, but only as a "first approximation"). In its modern version, as general equilibrium with the interaction of "time preference" and "dated commodities" (the Arrow-Debreu theory), it has never been shown to be other than a purely formal apparatus of thought, despite the efforts of some researchers to design and empirically implement so-called "computable general equilibrium models".



Figure1. Capital Deepening

curve MPK is the marginal product of capital, derived from a well-defined production function (assume it is twice differentiable and subject to constant-returns-to-scale) relating corn as input, combined with labor in continuously variable proportions, to corn as output, viewed in the aggregate at the level of the economy as a whole. It is negatively sloped because of diminishing returns to the variable factor. The total supply of labor is taken as given in the background. The production function represents the state of technical knowledge, which is assumed to be given. Every combination of capital and labor or capital-labor ratio is considered to represent a different technique of production (or "capital intensity" or "degree of mechanization"), involving the substitution of capital for labor, that would be chosen at the corresponding rate of profit, r, measured on the vertical axis. The lower the rate of profit, the greater the quantity of capital relative to labor that the profit-maximizing representative firm would employ.⁹ Thus, MPK represents the demand curve for capital as a stock.

the harvest. But corn can in fact be stored. The early neoclassicals chose a variety of vegetable types to represent capital, such as trees in the forest, maturing wine, or the "Crusonia" plant, where the passage of time or the "period of production" becomes the essential capital-theoretic element. Any entity will do, for present purposes, as long as it preserves the monotonic inverse relationship in MPK. Robinson (1973, p. 147) deployed for this purpose, the imaginary entity called "leets", as a play on the entity "steel" used by Meade (1961) in his construction of the neoclassical theory. Other names suggested by her and other writers include Meccano sets, ectoplasm, putty, and jelly.

⁹ This particular specification is usually associated with the name of J. B. Clark. Wicksell, for his part, had wisely noted reasons why the quantity of capital, measured as an aggregate of values, could be positively related to the rate of profit (subsequently called the "Wicksell Effect").

In the left-hand quadrant, MEI is the marginal efficiency of investment, a flow concept. It represents the rate of profit that the firm expects to get from different amounts of current output committed to investment, *given the stock of capital already invested and the technique in use*. There is one MEI for every level of the capital stock. It is negatively sloped, reflecting the condition of diminishing returns, assumed applicable to investment as it is to the stock of capital.¹⁰ A minimum interest rate, i*, is required to induce saving by the representative saver in the form of investment loans. The amount of saving is assumed to be an increasing function S(i) of the interest rate on loans, because of increasing cost (the cost of "waiting") to the saver from foregoing current consumption.

Now, let the initial stock of capital in existence at the first "moment", time 0, be given at the level K_0 . Then there exists an equilibrium in the "stock" market, at (K_0 , r_0), in which the available stock of capital is fully utilized because it meets an equal demand at the profit rate r_0 . Simultaneously, in the loan market (which is here conterminous with the output market), there exists an equilibrium, (S_0 , i_0), in which the demand for investment loans represented by MEI₀ meets an equivalent supply of saving at the interest rate i_0 . The profit rate on existing capital exceeds the interest rate on loans because the expected yield on the marginal investment, which governs what the profit-maximizing capital-owner is willing to pay for loans, is lower than the productivity of existing capital due to diminishing returns, and the saver is thrifty enough to be willing to supply savings at that interest rate. In the background, the demand for labor at the associated level of output and capital/labor ratio exactly matches the available labor-supply because the real wage rate exactly equals the marginal product of labor at that level of employment.

Call this initial moment a "short period". Then, the foregoing is proof that there *exists* a "short-period equilibrium" corresponding to the given conditions of that moment. By this is meant that there is full consistency between the plans of all market participants and, hence, those plans can be implemented. A distinctive feature is that all markets clear: there is no excess demand or excess supply. Furthermore, this equilibrium is *unique*, in that there is one and only one such solution.

Insofar as the investment plans at time 0 are actually implemented, then this economy enters the next period, time 1, with a larger capital stock, say $K_1 = K_0 + S_0$.¹¹ In the new situation, a similar outcome as at time 0 is repeated. The (greater) quantity of available capital is absorbed through an increase in capital intensity at a (lower) rate of profit, r_1 . At this rate of profit, the (lower) demand for investment loans finds an equal supply of savings at (S_1 , i_1). In the background, full employment of the constant labor supply occurs at a (higher) wage rate corresponding to the (higher) marginal product of labor associated with the (higher) capital intensity of production. These results describe a short-period equilibrium corresponding to the given conditions of time 1.

¹⁰ This assumption is a highly problematical feature of the analysis, even in the context of this simple model, for reasons that need not delay us here but are explained at length in Harris (1981, pp. 367-70).

¹¹ It must be assumed that this transition, brought about by the activity of investment itself, is effected during some interval of time that allows for the implementation of investment plans. In reality, this would depend on the physical character of the capital good itself, on transportation costs, and other complications. Furthermore, investment plans may not be realized. However, in the special conditions of this case, implementation of investment plans are always realized.

In the same vein, this analysis can be extended to subsequent periods, time 2, 3, 4, ... *ad infinitum*. It is found, then, that there exists a definite sequence of short-period equilibria, propelled by the activity of investment. This sequence terminates in a particular equilibrium with the following unique characteristics. The capital stock K* yields a rate of profit just equal to i*. At that profit rate, the capital owner is willing to forego investment since the yield on investment at the margin of existing capital just covers the interest cost of a loan. At that interest rate, the saver is indifferent to saving because the reward for foregoing consumption at the margin just covers the cost of waiting. In the labor market, there is full employment of the constant labor supply at the market-clearing wage rate.

Since the activity of investment has come to a halt, the equilibrium solution (K*, i*) constitutes a terminal point in the sequence. With zero investment, a constant labor supply, and consistency of plans in all markets, this equilibrium is permanently sustainable with all activities operating at a constant level. It therefore constitutes a *stationary state* (Robinson called it "Kingdom Come" (1971a, p. 9)). It can be easily proved that there exists such an equilibrium and it is unique. To distinguish it from the antecedent points in the sequence, call this a *long-period equilibrium*. According to standard neoclassical practice, a further proof would be required to show that it is a *stable* equilibrium. That would constitute an additional step in the analysis, requiring specification of the economic behavior of the representative agents in the different markets, that is to say, how they respond to the circumstance of markets being out of equilibrium and whether or not those responses would lead, on an appropriate time scale, to the eventual achievement of equilibrium.



Figure 2. Capital Widening

This analysis may also be readily extended to deal with technical change and growth of labor supply. In either case, the effect is to shift out the MPK curve. This is

illustrated in Figure 2, where the accumulation process, conceived now as a process of "capital-widening", may be represented in the following terms. Starting from K_0 and MPK₀, a once-over shift to MPK₁ raises the rate of profit above $r^* = i^*$. This induces a new round of saving and investment and pushes the economy to a new long-run equilibrium at K₁. Similarly, a subsequent shift to MPK₂ propels the economy to a new equilibrium at K₂. Continuing in this way, as further doses of technical change and/or labor-supply growth occur, the economy traces out a path of expansion at a rate limited only by the pace of the twin forces of technical change and labor-supply growth. Observing only the long-period equilibrium positions (and ignoring the posited transition between them), it will be found that the rate of profit remains constant along this path and equal to the rate of interest. The trend of the wage rate depends on the specific relation between three forces. Labor-supply growth, by itself, lowers the wage rate due to diminishing returns to the labor input. Increase of capital by itself, and technical change, by increasing labor productivity, raise the wage rate.

Finally, following further the logic of this conception of an accumulation process leads to the special construction represented by modern neoclassical growth theory, the case of a *steady state*, an equilibrium path along which growth occurs at a constant rate equal to the growth rate of both labor supply and technical knowledge.¹² Such a path may exhibit different features depending on the assumed "bias" or "neutrality" of technical change, which is essentially a matter of specifying the shape of the production function. In order to preserve certain selected constancies on the path, technical change must be of a certain type, i.e. "Harrod neutral". Robinson referred to this particular case as a *golden age*. In an inventive take-off from this benchmark, she constructed, for the purpose of her own analysis, many other types of "ages" of growth, each with its own characteristic features (Robinson, 1956, 1962).

The neoclassical construction presents a simple and attractive "story". It is useful to lay bare what that story is. Evidently, it conveys a striking image of the accumulation process as the "history" of a smooth and inevitable progression (convergence) towards an equilibrium that, even when disturbed by the supposedly exogenous factors of technical change and population growth, is essentially self-perpetuating. It is sometimes presented as a heuristic device, or a "parable", not intended to be taken literally. Nevertheless, despite such reservations, it has been subjected to widespread adaptation and use as an explanatory device to explain actual historical trends in growth and development, and to provide policy prescriptions, in many different empirical settings.

Robinson regarded the neoclassical construction as wholly inadequate for such purposes; in fact she flatly rejected its use in empirical work.¹³ The reasons for her

¹² Steady-state growth paths are a larger class of so-called "quasi-stationary equilibria" of which the stationary state is a special case with growth rate equal to zero.

¹³ See, for instance, Robinson (1973, pp. 152-4, 167-73). She found empirical confirmation for her position on this issue in the studies of Phelps Brown (1957) and Fisher (1969, 1971). Rymes (1971) provided firm support in an elaborate analysis of the conceptual problems involved in empirical application of the neoclassical construction. However, it should be noted that the main emphasis of Robinson's critique was a focus on the internal logic and theoretical underpinnings of the neoclassical construction. She argued, in reference to this and other models: "These models are all too much simplified and too highly integrated for it to be possible to confront them with evidence from reality. At this stage, they must be judged on the a priori plausibility of their assumptions." (1962, p. 87). For a vigorous defense of this

objection, complex and subtle in the details, are spelled out over many writings. Some of these are examined in the next sections.

3. The Measurement of Capital

The idea of capital as "corn" is an obvious simplification, with a distinguished parentage, that is readily admissible as a first approximation. It was so used, and effectively so, by Ricardo in his discussion of the determination of the rate of profit in a simple agrarian economy (closed to international trade) and the prospect that the accumulation process would be driven to an end in the stationary state.¹⁴ One "source of trouble", for both Ricardo's conception and that of neoclassical theory, comes in attempting to generalize the claims based on this simple idea to a more complex, hence more "realistic" context, in which capital consists of many capital goods differing in their physical specification, age, durability, industry of application, and other dimensions of their use value.¹⁵

In making the transition to this more complex world, and attempting to maintain the essential elements of the neoclassical construction, one must immediately confront a problem that Robinson posed, i.e. the problem of "measuring capital". Specifically, what is the scalar measure of "capital", consisting of heterogeneous capital goods and not just "corn", that is supposed to express the "capital intensity of production", such as to be consistent with a key element of the neoclassical construction, namely, the presumption that (a) a lower rate of profit is uniquely associated with a more capital intensive production method which yields a lower marginal product of capital and, hence, (b) that the profit rate is to be considered in some meaningful sense as ascribable to the (marginal) productivity of capital?

For Robinson, this problem is, at heart, a matter of the logic of comparing different equilibrium positions, hence of comparative equilibrium analysis. Hence, it is most meaningfully discussed in the context of long-period equilibrium positions (stationary states), with a given state of technical knowledge, where the amount of capital is fully adjusted to the technique of production that is appropriate to the prevailing rate of

¹⁵ "We must be careful not to make a simplification in such a way that the model falls to pieces when it is removed". (Robinson, 1962, p. 33).

methodology of appraisal of concepts and theories in economics, see Nooteboom (1986). Some useful clarification is also provided by Maki (1989).

¹⁴ The significance and limitations of Ricardo's conception of capital as "corn" were laid out in lucid and transparent form by Sraffa and Dobb in their introduction to Ricardo's *Principles*. Robinson, who was intimately familiar with this insight from the time it appeared in 1951, was able to seize upon it as "a gleam of light" in constructing her own analysis (1978, p. xvii). She granted the limited usefulness and relevance of Ricardo's conception in the context of his time ("... he was applying what he believed to be a realistic analysis of the actual situation to problems of policy. His stationary state was not an equilibrium, but an awful warning." (1980, p. 81)) and sought to separate Ricardo's usage from what later became neoclassical orthodoxy ("... it was not right to throw him into the same box as Pigou in timeless equilibrium", *ibid.*, p. 81).

profit, so that "full equilibrium" exists with zero net investment.¹⁶ By appropriate modifications, this analysis can also be done in the context of steady-state growth paths.

It is now known, partly as a result of the extended debate and theoretical analysis stirred up by Robinson's provocative question that, in general, there is no choice of any economically meaningful index of capital that would confirm the validity of the neoclassical presumption, except under very special conditions. One such special case, corresponding to that of Marx's "equal organic composition of capital" in all producing sectors, is formally equivalent to a "one-commodity model" and therefore reverts to the case of capital as "corr".¹⁷

The neoclassical presumption is invalidated as soon as a transition is made to consider the case of just two differentiated capital-goods where it turns out that, if measured by an index of equilibrium prices in terms of a chosen numeraire, there is no necessary monotonic inverse relation between capital intensity and rate of profit (Garegnani, 1970). The resulting relation depends on the combination of three specific elements, identifiable as a *price effect*, a *composition effect*, and a *substitution effect* (Harris, 1973). With any number of heterogeneous capital goods, these problems are compounded. Thus, the general possibility of what was known previously as a "Wicksell Effect" is confirmed. Furthermore, and this is a deeper point shown in a wide-ranging debate to have far reaching implications no less damaging to neoclassical presumptions, it turns out that there may be "reswitching of techniques": the same technical method of production may recur at different levels of the profit rate (Symposium, 1966).

In the course of working out her own answer (1956) to the problem of "measuring capital", Robinson successfully exposed some of the crucial elements of this problem by setting up what she called a "book of blueprints" to characterize the heterogeneity of capital goods and technical methods (alpha, beta, gamma, etc.) representing "the spectrum of techniques" associated with a "given state of technical knowledge". She constructed, in this context, the interesting device of a "productivity curve" for ordering the different techniques and showed that it mimicked the neoclassical idea of a "well-behaved" production function but only under the strict assumption that all profitable techniques are evaluated at the *same* rate of profit, thus emphasizing the necessity of a valuation index.¹⁸ In addition, she hit upon the possibility that reswitching of techniques

¹⁶ As Robinson put it, "Each set of thriftiness conditions has its appropriate stationary state." This can be seen in Figure 1 where, by extension, there would be a different equilibrium K^* for every level of the interest rate i* representing the zero-saving rate of interest or minimum supply price of saving. Logically, one must think of these equilibria as different "islands" with no communication whatsoever between them.

¹⁷ For the record, it is worth pointing out here, that in a much earlier incarnation, that of Robinson (1938), she had come upon the problem of measuring capital from a different direction, namely, that of seeking to provide a logically consistent method for classification of inventions associated with a process of technical change. The analysis that she offered, though rightly celebrated at the time as a seminal contribution, may be said to founder on the very same capital-valuation difficulty that she later encountered in the neoclassical theory, requiring for consistency the assumption of a one-commodity model. When, at a later date, she turned to a re-examination of this paper, she readily admitted that "This note belongs to a period when ... I was in a great state of confusion about the meaning of a 'quantity of capital'." (Robinson, 1971b, p. 52).

¹⁸ Robinson (1956, p. 412). This device is further elaborated and corrected as a "profitability curve" in Harris (1973, 110-111).

could occur but unwisely considered it at the time to be a mere "curiosum", "not of great importance," and "rather unlikely that cases of this kind should be common." ¹⁹

It was the work of Sraffa (1960) and of those who later elaborated and extended the scheme of analysis that he developed, which provided the fundamental key to understanding the intricate relationships involved, *as far as concerns the comparison of long-period equilibrium positions with a complex structure of production*. Much clarity has been produced by pursuing this line of analysis, serving to illuminate a broad range of issues in economic theory.

In the subsequent evolution of her own work, Robinson evidently relied very much on the insights gained from Sraffa's contribution and from the ongoing body of work derived from it. However, it is also abundantly clear, already in *The Accumulation of Capital*, and before, as well as in subsequent work, that she wanted to push beyond comparative equilibrium analysis as such towards the larger goal of analyzing the process of capital accumulation which she regarded as the central feature of the historical process of capitalist development. For that purpose, she considered not only the neoclassical construction to be wholly inadequate but also *the method of comparative equilibrium analysis* itself to be too confining in the scope of the issues addressed as well as intrinsically incapable of dealing with what she considered the primary question.²⁰ It is in this context that one must confront, she insisted, the role of "time" in the analysis of equilibrium and the potential for "confusion" that it creates.

In this connection, it is worth noting that, though Robinson heaped high praise on Sraffa's model for its "beauty and ingenuity" (1980, p. x), she was forthright in pointing out what she considered to be its limitations: "There is a great deal to be learned from this model, particularly in a negative direction. … But as the basis for analysis in a positive direction there is a difficulty about the specification of Sraffa's model in terms of logical time. … *This problem arises because there is no causality in Sraffa's system.* … *if we are to introduce decisions into the model, we must introduce time*" (emphasis added) (1980, pp. 88-9).²¹

4. Logical and Historical Time

A more basic "source of trouble" in the use of equilibrium analysis, Robinson argued, lies in a necessary distinction between logical and historical time. It is a distinction based on a substantive difference that is suppressed when the historical process of accumulation is interpreted as a movement from one equilibrium position to another, a sequence of equilibrium positions, or a progression along an equilibrium path.

¹⁹ Robinson (1956, pp. 109-10). She later pointed out that her attribution of this discovery to Ruth Cohen was "a private joke" (1973, p. 145).

²⁰ "The comparison of different economies with the same technical possibilities and different rates of profit is an exercise in pure economic logic, without application to reality". (Robinson, 1962, p. 33).

²¹ The Sraffian model has been used to characterize a process of technical change as a sequence of longperiod positions (Schefold, 1976; 1979; 1980). It was elegantly developed by Pasinetti (1981) to display the properties of a process of "structural change" in a special kind of full-employment growth-equilibrium called a "natural economic system". The peculiarities and limitations of this conception were reviewed in Harris (1982).

Recognition of this distinction makes it illegitimate, hence unacceptable, to draw any direct inference from the analysis of equilibrium existing in logical time to be applied as an explanation of events taking place in historical time, let alone events in real time. Logical time is, in a sense, anti-historical.

For clarification of this issue, let us return to the case of the neoclassical construction illustrated in Figures 1 and 2. Consistent with the logic of this construction, time is a dimension that exhibits the following distinctive properties.

- 1. In proving the existence and uniqueness of equilibrium, no reliance is placed on time as such. It is seemingly absent from the analysis. It is, nevertheless, there, in the background. "Time may be conceived to lie at right angles to the page." (Robinson, 1962, p. 22). It would necessarily come into play in the adjustment to equilibrium and the transition from one equilibrium to another.
- 2. Time is the dimension that separates the short period and the long period.
- 3. In the *short period*, only certain things "happen": the stock of capital, labor supply, and state of technical knowledge are fixed, demand for capital adjusts to the given supply of capital, labor demand adjusts to the given supply of labor, saving and investment adjust to each other. There is "movement" over time through a determinate sequence of short periods so as to bring into existence the conditions of the long period. Thus, in the *long period* (stationary state), other things "happen": supply and demand for capital are "fully" adjusted to each other, labor supply and technical knowledge remain fixed. It follows that we should properly distinguish a third "period" in which everything "happens": as on the steady-state path or golden age in which accumulation of capital, labor supply growth, and technical change all occur with regularity and are "fully" adjusted to each other. Accordingly, we could call this the *secular period*, with its own time scale, different from those of the short and long period.
- 4. Time is divisible into finite components. Each short period is necessarily of limited duration in time because it is about to be upset by the implementation of investment plans. Similarly, the sequence of short periods leading to the stationary state has an end-point in finite time.²² In contrast, the stationary state itself has no end point, it continues indefinitely in time. Likewise, the steady state (or golden age) goes on forever. And since, as Robinson argues, the stationary state and the golden age, if either exists, must have always existed in past time, then time goes from -∞ to +∞. However, there is a distinction between them, if not much of a difference. In the stationary state, nothing changes. Therefore, it could just as well be conceived as a timeless equilibrium. In the golden age, proportions do not change, only the scale of the economy and at a constant proportional rate, which is a reflection of the assumed linearity in the structure of the economy. Thus, but for the change in scale, the golden age could easily be collapsed into the timeless equilibrium of the stationary state.
- 5. Time is infinitely divisible. It is therefore possible, in principle, to conceive of things happening at an infinitesimal instant of time. Furthermore, within the logic of this particular construction, there is nothing to preclude all relevant actions (in the short

 $^{^{22}}$ This is not strictly necessary to the logic of the case. Using the logic of the calculus, the stationary state is the asymptote as time goes to infinity.

period) from occurring simultaneously at an instant of time.²³ Production is assembled in an instant to produce an output, corn, which by definition can be indifferently consumed or saved and invested. The state of technology is fully known at any instant, as is the supply of factors.

- 6. The "future" is always like the "past" and known with certainty. Therefore, decisions taken "today" in anticipation of future events are always confirmed by future events. Expectations are always fulfilled and plans realized.
- 7. Time is reversible: it is possible to go both forward and backward in time. For instance, in Figure 1, inasmuch as it is logically tenable to posit a sequence of short periods going forward in time, through positive amounts of investment, from K₀ towards the stationary state level K*, it would be equally tenable to posit a sequence going backward in time from some level K_t > K* towards K*, through consumption (negative investment) of accumulated capital. Similarly, in Figure 2, technical "progress" (assumed to be of the "disembodied" form) can be made to go both forward and backward.

The preceding properties together constitute what Robinson called *logical time*. They underpin the neoclassical construction of the accumulation process elaborated above.

In contrast, *historical time* is based on an appeal to properties that are drawn from the reality of actual experience. The following are some of the properties that Robinson emphasized.

- 1. In historical time, "Today is a break [moment] in time between an unknown future and an irrevocable past." (Robinson, 1962, p. 26).
- 2. The past is embodied in the current situation and limits the range of actions that can be taken to bring about an adjustment to changing circumstances. In this respect, the economy is, so to speak, "locked into" the initial conditions existing today and inherited from the past. This property is grounded, in part, in the historical reality of an industrial society (as distinct from a society of "corn" producers) characterized by a complex division of labor, and succinctly expressed in the idea of heterogeneity of capital goods. It means, specifically, that capital goods exist in a fixed form embodying the existing state of technical knowledge, are adapted to specific uses, in most cases are not directly transferable without cost to other uses, cannot be directly consumed, and require a time-intensive process of investment (scrapping) for expansion (reduction) of the existing stock. It is grounded also in the reality of the "labor force" as conscious beings, differentiated in age and other physical and social characteristics, endowed with acquired skills and knowledge, specialized to different spheres of production, and capable of acting in organized groups (e.g. trade unions) to defend their positions (wages, hours, "benefits") gained in the past.²⁴

²³ Alternatively, it would be possible to add a dose of "realism" by the simple specialization of making things happen in discrete time and allowing for the existence of lags in adjustment. But this would substantively alter the conditions of the problem and could produce results inconsistent with the logic of this construction.

²⁴ "The fossils embedded in the stock of capital (and in the supply of labour trained to various occupations or settled in various districts) destroy the possibility of perfectly smooth development." (Robinson, 1952a, p. 125).

- 3. The technical conditions of production ("technology") may change over time as a result of innovation or "technical progress". But, the techniques that exist today are the result of changes that have occurred in the past and decisions taken in the past that have brought them into existence today in anticipation of what today was likely to be. They are reflected in the age or vintage of different outfits of capital goods existing "today". Because of a turbulent past, different techniques may coexist in time. Then, the range of techniques actually in use "today" can be varied by moving existing outfits ("old" vintages, "fossils") in the stock of capital goods "in and out of mothballs". Adoption ("choice") of "new" techniques is a matter of production of, and investment in, new vintages that incorporate the new techniques.
- 4. Production itself is a time-intensive activity (the "production run") with different durations in time (long or short "runs").
- 5. Time is not reversible.²⁵ Knowledge gained cannot be lost.²⁶ Production, once completed, cannot be undone: goods produced for sale must be sold or else inventoried ("put to stock").
- 6. Decisions and actions are taken "today" in the light of beliefs ("expectations" or "guesses") about their future consequences and the consequences of the decisions and actions of others. The future is intrinsically unknowable, "not even knowledge of probability distributions". Decisions are "rational" in the sense that they are based on existing knowledge and the projection of such knowledge into the future.
- 7. Acquisition of knowledge is an activity ("learning", "experience", "search") that takes place along a time-space dimension and involves real costs. The computational costs involved in sifting the information gathered in this way makes it prohibitive, if not impossible, for any single individual to acquire "full" knowledge. Hence, knowledge is always "imperfect": "the full information required to make a correct choice can never be available". (Robinson, 1980, p. 8).
- 8. "Since all individual choices are based upon more or less independent and inaccurate judgements about what outcomes will be, it is impossible that they should be consistent with each other." They "may turn out later to have been mistaken". Hence, the assumption of 'perfect foresight' "has no point of contact with empirical reality." (Robinson, 1980, pp. 8, 89).
- 9. Decisions and actions by individuals or collective institutions that involve human agency (firms, trade unions, government) are imbued with "inertia", entailing that there is "stickiness" or "lags" in adjustment to changing circumstances.
- 10. "Money" (cash on hand, finance, and credit) and the institutions which support it are strictly necessary requirements of economic activity taking place in historical time.

²⁵ "... time goes only one way; there is no going back to correct a mistake; an equilibrium cannot be reached by a process of trial and error." (Robinson, 1980, p. 8).

²⁶ "Marshall was aware of the difficulty. He drew a long-period supply curve going forward through time, with economies of scale and learning by doing. At any date that had once been reached, he conceived that there was a curve running backwards showing lower costs than on the forward curve because economies that have once been achieved would not be lost if demand were to shrink so that output had to be reduced. But this device raises more problems than it solves." (Robinson, 1980, p. 88). She is here skeptical of Marshall's treatment for a reason she had given elsewhere: "The reason is that he somehow boiled the effect of technical progress going on through time into the movement down his supply curve". (Robinson, 1952a, p. 151).

It is the means of payment for effecting transactions today ("medium of exchange"), including the exchange of labor time, and the vehicle for carrying wealth in "liquid" form over time ("store of value").

Robinson's argument is that these properties of historical time stand opposed, in their implications, to the properties of logical time. (A direct contrast of the two sets of properties as related to relevant economic variables is summarized in Table 1.) In her view, that would make the neoclassical construction, based as it is on logical time, devoid of any explanatory significance in dealing with the actual history of accumulation.

	Properties	
Economic Variable	Logical Time	Historical Time
1. Directionality of time	Reversibility	Irreversibility
2. Time intensity of action	Instantaneous	Discreteness; lags; inertia
3. Expectations	Self-realizing; correct foresight	Falsifiable; future unknowable
4. Information/Knowledge	Complete, free, symmetric	Imperfect, costly, local learning
5. Capital goods	Substitutability	Specificity; lumpiness
6. Investment	Elastic	Inertia; driven by animal spirits
7. Technical change	Disembodied	Embodied; path-dependent
8. Money/Finance	Barter; passive money; complete	Active money; liquidity
	futures market	preference; incomplete markets

 Table 1.
 Logical versus Historical Time

5. A Causal-Historical Model of Accumulation

I interpret Robinson's basic proposition as follows.²⁷ There are two distinct and mutually exclusive conceptions of the accumulation process. They may be represented in the simplest formal terms as the following two dynamic processes:

Case 1. $dx/dt = F_x^e(x(t)), -\infty \le t \le +\infty; x^e = equilibrium point.$

Case 2. $dx/dt = F_x^0(x(t)), \quad 0 \le t \le +\infty; x^0 = initial condition.$

Case 1 is an *equilibrium process* in which the function governing the movement in time generates a unique and stable equilibrium solution x^e and is invariant both to the starting point and to the path of movement to equilibrium.

Case 2 is an *historical process* in which the function governing movement along any path is uniquely dependent on the initial condition or state variable x^0 and, for full generality, may be considered to shift as experience builds up along a given path.

Only in a very special, unique, and hypothetical state, which she calls the state of *tranquility* (Robinson, 1956, pp. 59, 66-67), could these two cases be made to collapse

²⁷ The interpretation which follows was earlier sketched out in Harris (1991).

into each other. In reality, however, the observed historical process is generally in a state of *turbulence*, not one of tranquility. In conditions of turbulence, disturbing events ("surprises") occur that are not predictable with certainty (they are "unpredetermined"). They are the "stuff" out of which actual history is made. If and when such an event occurs, it makes the actual, realized, position of the economy different from what it was projected to be in the prior process leading up to that point in time. The actual position represents the "initial condition" to which the economy must then adapt in a forward-looking process. "What will happen next?" is an open question. That reflects the essential "openness" of actual history, on which a properly constructed economic model of the process may be applied to throw some light. For that purpose, the model of an equilibrium process.

Robinson's answer to the question of "what will happen next?" would then run along the following lines.

If all past history had been one of equilibrium, then one may infer that any perturbation which occurs here and now would set into operation forces that cause the perturbation to cancel itself out and bring about a return to equilibrium. The economic system would then be self-correcting, at least for small perturbations. It is quite another thing, however, if history has never been anywhere near equilibrium. It would be illegitimate then to claim that, starting from today, there will come into play a process of getting to equilibrium. The system could, and would likely, wander off into the unknown without ever achieving equilibrium.

A mathematician would correctly reply that, from the standpoint of an abstract analysis of stability, these two cases are not qualitatively different. But, for the social theorist and historian, there is a world of difference between them. Specifically, the difference is that, in the one case, the properties of equilibrium have already been learned in history and can confidently be expected to persist. In the other case, there can be no necessary presumption that a real process of learning, which is in general a path dependent process, will lead to an equilibrium, if any exists and whether it is unique or not. (Harris, 1991, p. 98).

It is worth adding here another key feature of her analysis that is often missed. Specifically, she conceives of the economy at every moment, in the short period and the long period, as *an under-determined system*. In formal terms, there are more variables, at least one, than there are equations to solve the model. Consequently, even if an equilibrium can be shown to exist, there is always something about to happen that will upset any tendency to equilibrium. This is the essential "open" feature of a truly historical process. Throughout all of her analysis, it appears that the investment decision is that loose variable. It is made to hang on historically contingent factors expressed in "the animal spirits" of the capitalist investors.²⁹

Because of the emphasis placed on expectations, it is sometimes said that her analysis allows a "subjective" element to "rule the roost". Or, to use another metaphor, she allows the tail of expectations to wag the capitalist dog. I believe that this criticism is

²⁸ "The most important consequence of a troubled past lies in its influence on expectations." (Robinson, 1952a, p. 125). "A world in which expectations are liable to be falsified cannot be described by the simple equations of the equilibrium path". (Robinson, 1962, p. 25)

²⁹ "To attempt to account for what makes the propensity to accumulate high or low we must look into historical, political and psychological characteristics of an economy." (Robinson, 1962, p. 37).

based on a misunderstanding.³⁰ In her analysis, it is capitalist investment decisions that rule the roost. But, though driven by the heat of the competitive struggle, they are intrinsically fraught with "fundamental uncertainty". In the short run, they are subject to inertia (they adapt slowly relative to other "fast" adaptors) and in the long run they may vary within a wide range dependent on specific historical circumstances.³¹ Therefore, "history" always has to be brought into the analysis.

Now, as Robinson asserted, "There is one point on which all schools of thought can agree – that the actual process of capitalist accumulation goes on through historical time." (1980, p. 74). On the other hand, much of economic analysis, including some of her own, is set out in terms of equilibrium states – in particular, the steady state – based on logical time. Therefore, the crucial problem ("the *real* source of trouble") for economic analysis, she argued, is the following:

In a steady state all events are predetermined. Anything that happens 'today' is fully determined by the past, including expectations about 'today' that were held in the past. It is precisely those expectations, confidently held, which are now reflected in the various stocks in existence in the appropriate configuration 'today'.

To discuss the effects of change in any element in our story, we must break this link between the past and the future and treat 'today' as a gap between the two in which unpredetermined events may occur. This is necessary to set the analysis in historical, not logical, time. (Robinson, 1980, p. 74)

To fill in the story of a movement towards equilibrium, a complicated dynamic process must be specified and to specify a process that will actually reach equilibrium is by no means a simple matter. (Robinson, 1980, p. 87).

The now common practice for dealing with this problem, among economists of different schools of thought, is to resort to using the tools of *stability analysis*.³² To my knowledge, Robinson did not offer a systematic appraisal of this practice. From her

³⁰ For her defense of her position, see Robinson (1980, p. 128).

³¹ She discussed at length issues involved in the specification of an investment function, reviewing critically various alternatives proposed by others (1952a, pp. 159-164; 1956, Chs. 21-22; 1962, pp. 78-87). She opted for a specification of the rate of accumulation as a function of the expected rate of profit in her own formulation of a model of accumulation (1962, pp. 46-51). She was elsewhere mostly concerned to emphasize the variety of factors and limits acting upon the "desire to accumulate" in different phases of growth (booms, recessions) and over the long haul (with technical progress, natural-resource scarcity, population growth), drawing a sharp distinction between investment in fixed capital, working capital, and inventories.

³² Samuelson (1947), in the bloom of youth, had earlier sought to resolve this problem by deploying his "correspondence principle", a theorem concerning the formal correspondence between comparative statics and dynamics, to support the hypothesis that equilibrium states not capable of persisting over time are less likely to be found in reality than those which are. After all, "How many times has the reader seen an egg standing upon its end?" he asked (p. 5). That proposition was obviously a slippery slope, based on employing the principles of mechanics to sweep under the rug a substantive problem of social and historical analysis. That it is a dubious proposition was pointed out by Arrow and Hahn (1971, pp. 320-321). But in 1975, in the context of the "reswitching" debate, Robinson was not about to let him get away with it, while standing her own ground on this issue. In his response to her, which she justly considered to be "patronizing" (1980, p. 138), he was evidently not prepared to accept or admit the weakness in his own position. Mirowski (1989) has documented well how and when economics got off the track by bringing the principles of mechanics into the mainstream of the discipline.

cursory remarks on the subject (for instance, 1980, Chs. 4 & 7), one could infer that she was skeptical of its capacity to uncover the deeper layers of the problem (for example, the process of technical change) or to yield results having great generality or directly applicable to empirical reality. She reckoned that, quite apart from the obvious difficulty of managing the complex relations among the (quantifiable) economic variables and finding results capable of meaningful interpretation, mathematics was incapable of giving full play to the (non-quantifiable) human and social factors involved.

She was, however, not averse to the use of mathematical modeling as such, if designed to eliminate errors in "thinking" and to "clear the logical ground for a discussion of real issues involved in the analysis of capitalist accumulation." (Robinson and Bhaduri, 1980). She thought that much insight could be gained in this way, by proceeding in the step-by-step manner of successive approximation: "To sort out the analysis of this turbulent scene involves the whole of economics and … we must approach it bit by bit" (Robinson, 1980, p. 22).

However, she rejected "pseudo-causal models" (1962, pp. 23-29). An acceptable economic model had to be, what I would call, *causal-historical*, in the sense that the initial conditions and behavioral relations are fully specified and plausible and, in particular, expectations are specified to satisfy the properties of historical time: "the economy follows the path because the expectations and behaviour reactions of its inhabitants are causing it to do so" (1962, p. 26). Her argument suggests that, under those conditions, she might be prepared to accept the idea of *local stability* of an equilibrium, if one exists. This is strictly because, in that case, the economy is presumed to have been "near enough" to equilibrium in the past so that the inhabitants have come to learn from experience the properties of that equilibrium and may confidently act as if they expect it to continue. By the same token, she regarded the question of *global stability* (i.e. when "the actual position [is] appreciably off the prescribed path") as "a nonsense question to ask" (1962, p. 24). Presumably, she would also have rejected the idea of a "rational expectations equilibrium" as wholly inconsistent with historical time.

She did not examine the possibility that there might be *multiple equilibria*, which we now know to be generally the case, not a special case. In some respects, that would have made her argument for a causal-historical model even stronger. An example of this possibility is constructed in Figure 3, which allows for the existence of regions of increasing returns in the production function (instead of generalized diminishing returns) in an otherwise standard model of steady-state growth. In this example, there exists multiple equilibria, some stable (at b and d), others unstable (at a and c). Evidently, the relevant dynamic outcome depends on the historical starting point of the economy. Starting below some minimum level of k leads to a degenerate outcome.

To be fair to her, it must also be noted that she did make, at various times, a concerted effort to get the neoclassical construction to stand up in historical time. She concluded that it could not be made to work in this context (1956, Notes, Diagrams; 1960, Part II; 1962, pp. 57-8, 102-3, 132-5; 1971a, pp. 14-15). As for the more elaborate inter-temporal theory of general equilibrium, she confessed that: "I have never been able to make that theory stand up long enough to knock it down." (1980, p. 128). Her skepticism, on both counts, was actually confirmed early on by the work of neoclassical theorists who showed that, under the usual neoclassical behavioral rules and with

heterogeneity of capital goods, the neoclassical long-run equilibrium is unstable (see Hahn, 1966; Shell and Stiglitz, 1967; Kurz, 1968).



Figure 3. Multiple Growth-Equilibria with a Non-Linear Technology

6. The Robinsonian Model of Accumulation

The core of Robinson's own analysis of the accumulation process in historical time is presented in three major texts (1952a, 1956, and 1962). The analysis is laid out mostly in literary terms. It is tightly reasoned, and dense with details that are brought in and thrown out at will. This manner of exposition may prove to be off-putting to the impatient reader brought up on using flashy mathematical tools. But patience in following out the argument and managing the flow of details will be amply rewarded with some precious gems and colorful vignettes.

The analysis typically begins with, or assumes in the background, the conditions of the *short period*. But this is not the market-clearing short-period of the neoclassical model. Instead, it is the short-period of Keynes, updated to remove what she often indicates to be the deficiencies of Keynes' own analysis. In the Robinsonian short period, initial conditions matter, and they matter decisively in terms of the size and composition of the capital stock and state of technology inherited from the past, which cannot be altered except within narrow limits, and the posited expectations of the future expressed in the level of investment plans. Investment demand, which determines aggregate demand and income through the multiplier, is subject to inertia: it is the "slow" adaptor relative to other variables. Besides, it is bounded from above by the size and composition of the capital stock in the capital-goods-producing sector. Saving adjusts to the level of planned investment through changes in the level and distribution of income, not through changes in the level of the interest rate as in the neoclassical model. In the labor market, the real wage is also a "slow" adaptor, because labor-market institutions are subject to hysteresis, giving rise to "stickiness" of money wages. Likewise, firms (large enough to have a say in the output market) are "slow" to raise prices in a "weak" market (and to lower prices may "spoil" the market by disturbing the existing conditions of competition). *The short-period equilibrium* consistent with these conditions is necessarily one of underutilized capacity and unemployed labor. It is also an *under-determined equilibrium*, which is about to be upset by the implementation of investment plans.

The *long period* is not a length of time. Like the short period, it is a process. It is distinguished from the short period by the specific adjustments which take place in each process.³³ The long period is the process of the working out of investment plans through changes in the level and composition of the capital stock in accordance with the state of long-term expectations and through interaction with the other factors that can plausibly be assumed to adjust as the inherited stock of capital is (gradually) changed to accommodate the changing level and composition of demand.

For the analysis of accumulation, she sets up a neat and simple model in which the central focus is on "the double-sided relationship between the rate of profit and the rate of accumulation" (1962, pp. 46-51). She shows the possible existence of an equilibrium in which the desired and actual rate of accumulation coincide because the rate of profit generated by that rate of accumulation is just what is required to maintain it. The underlying idea is derived from a combination of the Keynesian principles of effective demand and the multiplier extended to the long-period context of growth in productive capacity, with Kalecki's "widow's cruse" principle that allows firms to gain (or lose) profits depending on the state of demand and their ability to adjust mark-ups in response to market conditions. A key element is that firms' investment plans are influenced by the expected rate of profit and expectations depend on actual current experience.

This model represents the stripped-down ("bare bones") version of Robinson's analysis of accumulation.³⁴ She puts it through its paces by examining the conditions under which an equilibrium may or may not exist and, if it exists, whether or not it is likely to be sustained. Again, the crucial feature is that it is an under-determined equilibrium. And again, this is essentially because of the role of investment: it cannot be

³³ She emphasizes that, in order to start up the long-period analysis, the initial conditions must be appropriately specified. In particular, it must be assumed that the economy is already in short-period equilibrium (1962, p. 46). In this sense, the short-period is embedded in the long period. Given the logic of this starting-point, it would naturally lead to the question of how, if at all, the long-period process of accumulation could eliminate a condition that necessarily exists at its start, i.e. the condition of underutilized capacity and unemployed labor. A striking outcome of Robinson's analysis is to confirm that this condition is fully consistent with long-period equilibrium. Therein lies, one might infer, the power of a causal-historical analysis.

³⁴ Following the same lines as Robinson and Kalecki, some elements of this model and of Kalecki's were picked up and assembled in Harris (1974; 1978, Chs, 8 & 10) so as to provide an extended analysis of the short-period and long-period possibilities. A further extension was later developed by Rowthorn (1982) and subsequently by Marglin (1984), Dutt (1984), and Marglin and Bhaduri (1991). This framework of analysis has since come to represent the standard for what is now called Post-Keynesian growth theory.

pinned down to satisfy the requirements of equilibrium. Though the *rate of accumulation* may be more or less sensitive to variations in the rate of profit, the *accumulation function* depends on conditions that are outside the model, i.e. on "animal spirits". She shows that various dynamic disequilibrium outcomes are possible and analyzes their implications. They depend on "internal contradictions" that may make it impossible to find an equilibrium solution. Or, if an equilibrium exists, it is about to be upset by "disturbances" from outside. Equilibrium is therefore decidedly not a "natural state", nor is it permanently sustainable in the specific context of the ongoing turbulence of history.

On the foundations of this basic model, Robinson proceeds to construct, by careful layering of details that are brought in and thrown out of the analysis, numerous dynamic scenarios of equilibrium and disequilibrium, distinguished by the specific internal contradictions which characterize each of them and/or the specific disturbances which bring them about. She gives each its own name as a particular "age" of accumulation. Of those arising from internal contradictions, one interesting case is that of a *bastard golden age*. It is characterized by persistent unemployment, which is therefore a condition that is as much likely to come about in the long period as in the short period. Another is the case of an *inflation barrier*.³⁵ There are many other cases, representing a rich array of possible dynamic paths that an economy might take. Each deserves close scrutiny as the characterization of conditions that, it is presumed, could come about in actual history. In this context, the golden age emerges as a singular case in which it happens, by accident, that all the requirements of equilibrium are satisfied and full equilibrium prevails without internal contradictions or outside disturbances. It is the mirror image of the neoclassical steady-state path with full employment.

Robinson regarded the golden age as a "mythical state of affairs not likely to obtain in any actual economy" (1956, p. 99), or "nothing more than a piece of simple arithmetic" (1952a, p. 96). In the context of her own analysis of its properties, she independently proved a "neo-neo-classical theorem" (1962, Ch. IV & Appendix 2), showing that consumption is maximized if the rate of accumulation is equal to the profit rate in the golden-age economy. Oddly enough, when neoclassical growth theorists later rediscovered this theorem in the neoclassical growth model, it was celebrated as the "golden rule of accumulation" and interpreted as a prescription for economic policy in a capitalist economy.

7. Towards Dynamic Economics (or History Vindicates Joan Robinson)

One of the more interesting turns in the history of economic thought is the trend that has taken place since Robinson wrote, away from an earlier and highly restrictive form of economic dynamics towards a "truly" dynamic economic analysis. This particular turn has many different branches and is not yet fully explored nor documented. But it would not be premature to say that it has fully vindicated some of the basic concerns of Robinson while taking economic dynamics into deeper realms. The irony in this history is that the mathematicians have come around (unknowingly in some cases) to validating, in their own way, some of the ideas embedded in Robinson's "untidy" literary style, which she discovered entirely without the use of mathematics. Also, it shows that she was far ahead of her time.

³⁵ A model incorporating this intriguing case was explicitly formulated and analyzed in Harris (1967).

The earlier analysis of economic dynamics was focused on linear models. Widely used in the analysis of business cycles, these models were subject to well-recognized limitations that restricted their interpretive value (Pasinetti, 1960). They yielded very limited forms of motion consisting of: explosive growth, oscillations that are explosive, damped or of uniform amplitude, and smooth convergence. To get meaningful "turning points" consistent with observed business cycles, arbitrary "ceilings" and "floors" (a kind of "exogenous" non-linearity) had to be imposed on the endogenously generated dynamics. Worst of all, the cycle could only be made consistent with capital accumulation by imposing an exogenous growth rate of output, hence "separating the cycle from the trend".

Robinson was rightly skeptical of the business cycle models. Yet, she welcomed the development, within this genre, of the different approaches of Harrod and Kalecki towards "a theory of economic dynamics", viewing them as a positive move to carry forward the Keynesian revolution through "generalizing" Keynes' *General Theory*. Her own efforts during this period were designed to further advance this new wave of economic theorizing. Against the background of these developments, she could not help but view the emergence of the "neo-neoclassical" growth models, explicitly designed to counter the meaning of Harrod's "dynamic equation" by reviving the "Wicksell process", as a backward step. Hence, her use of the term "neo-neoclassical" to describe them.

She did not witness the more recent emergence of neoclassical models of "endogenous" growth. She would undoubtedly have regarded them as a hybrid of selfcontradictory principles and behavioral rules: i.e. introduction of the historically relevant idea of increasing returns based on scale economies, investment in R&D, and education of the labor force, in combination with the neoclassical presumptions of marginal productivity pricing of factors of production, marginal-cost pricing of output, passive adjustment of investment to the "natural growth rate", and correct foresight.

In contrast, the recent trend towards dynamic analysis of "complex" systems is based on non-linearities and evolutionary principles, that capture explicitly the role of initial conditions, increasing returns, learning, cumulative feedback effects, inertia, hysteresis, natural selection, and other properties of "historical time" that are deeply relevant to an understanding of the real economy in motion (Anderson *et al.*, 1988; Arthur, 1994). This type of dynamic analysis has been shown to yield complex forms of motion, stable and unstable, including limit cycles, path dependence, and chaos, with solutions consisting of multiple equilibria, a single equilibrium, or no equilibrium at all. The rich variety of dynamic paths and outcomes found in these analytical studies is definitely consistent with and supportive of the patterns of economic dynamics that Robinson sketched out or suggested in her work.

It turns out, for instance, that a simple specification of Ricardo's "corn model", which has had a long life as an equilibrium model since the time of Ricardo, is capable of yielding chaotic behavior (Bhaduri and Harris, 1987). This result undermines the long-standing presumption of a necessary convergence to a stationary state in this model.

Analyses of the neoclassical growth model in this new framework of economic dynamics, even with "corn" as capital good, or with more general technologies, have found a similar pattern of complex dynamics (see, Cass and Shell, 1983; Benhabib and Nishimura, 1985; Grandmont, 1985; Boldrin and Montrucchio, 1986). These results undermine the presumption of a necessary convergence to the steady-state in this model.

Various studies of the "cross-dual dynamics" of output and prices in a Sraffa-type model of competition (reported in Semmler, 1986, and earlier by Medio, 1978) have found unstable trajectories, instead of "gravitation" to a long-period equilibrium, even in the most congenial case of expectations tied down to knowing "in advance" the equilibrium vectors of output and prices (a type of correct foresight). These results leave in doubt the "causality" of the Sraffian equilibrium construct, thus confirming Robinson's stated intuition regarding this issue.³⁶ These and other studies reported in the same volume and elsewhere (Semmler, 1989) also demonstrated a wide range of dynamic possibilities that can be obtained, using sophisticated tools of mathematics and computer simulation, in the study of different aspects of economic behavior (some grounded in historical time, others in purely logical time), at both the micro- and macro-levels of the capitalist economy.

The study of technical change, long recognized as a crucial feature of the accumulation process beginning with the early contributions of Marx, became dormant after the initial bunching of innovations around Schumpeter's contributions. Robinson's early system for classification of inventions had barely touched the surface of this problem and in her later work she did not add much. Analysis of this problem has recently received new life from the integration of new theoretical models and analytical tools with a vast accumulation of empirical studies.

There are significant gaps, missing links, and other shortcomings in the work that has been done so far, for instance, as concerns the specification of institutional structure, labor market interactions affecting money wages and prices, price- and outputdetermination in an oligopolistic setting, investment behavior of firms, the role of financial markets, and the formation of expectations. Thus, the analysis is still far away from coming up with an adequate theory of accumulation. All of this suggests, as Robinson earlier indicated, that there is "much work to be done". Furthermore, in tackling the manifold and complex features of the accumulation process, Robinson's "bit by bit" approach is still relevant. She also added the important proviso: "that we give up the search for grand general laws and are content to try to enquire how things happen." (Robinson, 1980, p. 95).

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³⁶ Some of the behavioral issues that arise in seeking to adapt the Sraffian model to "historical time" are explored at length in Harris (1988; 1991).

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