Fundamentals of Wireless Communication[1][2]

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[1]Draft. Comments will be much appreciated; please send them to dtse@eecs.berkeley.edu or pramodv@uiuc.edu. Please do not distribute the notes without the authors' consent.

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Chapter 1

Introduction and Book Overview

1.1 Book Objective

Wireless communication is one of the most vibrant research areas in the communication field today. While it has been a topic of study since the 60's, the past decade has seen a surge of research activities in the area. This is due to a confluence of several factors. First is the explosive increase in demand for tetherless connectivity, driven so far mainly by cellular telephony but is expected to be soon eclipsed by wireless data applications. Second, the dramatic progress in VLSI technology has enabled small-area and low-power implementation of sophisticated signal processing algorithms and coding techniques. Third, the success of second-generation (2G) digital wireless standards, in particular the IS-95 Code Division Multiple Access (CDMA) standard, provides a concrete demonstration that good ideas from communication theory can have a significant impact in practice. The research thrust in the past decade has led to a much richer set of perspectives and tools on how to communicate over wireless channels, and the picture is still very much evolving.

There are two fundamental aspects of wireless communication that makes the problem challenging and interesting. These aspects are by and large not as significant in wireline communication. First is the phenomenon of *fading*: the time-variation of the channel strengths due to the small-scale effect of multipath fading, as well as larger scale effects such as path loss via distance attenuation and shadowing by obstacles. Second, unlike in the wired world where each transmitter-receiver pair can often be thought of as an isolated point-to-point link, wireless users communicate over the air and there is significant *interference* between them in wireless communication. The interference can be between transmitters communicating with a common receiver (e.g. uplink of a cellular system), between signals from a single transmitter to multiple receivers (e.g. downlink of a cellular system), or between different transmitter-receiver pairs (e.g. interference between users in different cells). How to deal with fading and with interference is central to the design of wireless communication systems, and will be the central themes of this book. Although this book takes a physical-layer perspective, it will be seen that in fact the management of fading and interference has ramifications across multiple layers.

The book has two objectives and can be roughly divided into two corresponding parts. The first part focuses on the basic and more traditional concepts of the field: modeling of multipath fading channels, diversity techniques to mitigate fading, coherent and noncoherent receivers, as well as multiple access and interference management issues in existing wireless systems. Current digital wireless standards will be used as examples. The second part deals with the more recent developments of the field. Two particular topics are discussed in depth: opportunistic communication and space-time multiple antenna communication. It will be seen that these recent developments lead to very different points of view on how to deal with fading and interference in wireless systems. A particular theme is the multifaceted nature of channel fading. While fading has traditionally be viewed as a nuisance to be counteracted, recent results suggest that fading can in fact be viewed as beneficial and exploited to increase the system spectral efficiency.

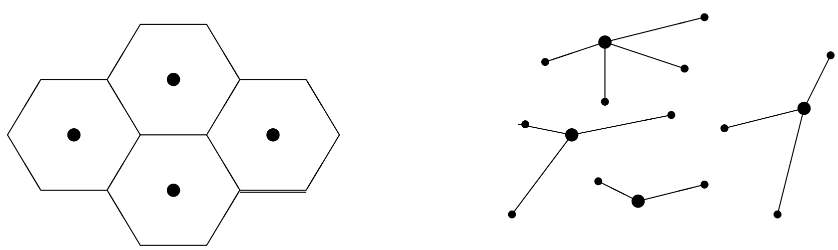
The expected background are solid undergraduate courses in signal and systems, probability and digital communication. It is expected that the readers of this book may have a wide range of backgrounds, and some of the appendices will be catered to providing supplementary background material. We will also try to introduce concepts from first principles as much as possible. Information theory has played a significant role in many of the recent developments in wireless communication, and we will use it as a coherent framework throughout the book. The level of sophistication at which we use information theory is however not high; we will cover all the required background in this book.

1.2 Wireless Systems

Wireless communication, despite the hype of the popular press, is a field that has been around for over a hundred years, starting around 1897 with Marconi's successful demonstrations of wireless telegraphy. By 1901, radio reception across the Atlantic Ocean had been established; thus rapid progress in technology has also been around for quite a while. In the intervening hundred years, many types of wireless systems have flourished, and often later disappeared. For example, television transmission, in its early days, was broadcast by wireless radio transmitters, which is increasingly being replaced by cable transmission. Similarly, the point to point microwave circuits that formed the backbone of the telephone network are being replaced by optical fiber. In the first example, wireless technology became outdated when a wired distribution network was installed; in the second, a new wired technology (optical fiber) replaced the older technology. The opposite type of example is occurring today in telephony, where wireless (cellular) technology is partially replacing the use of the wired telephone network (particularly in parts of the world where the wired network is not well developed). The point of these examples is that there are many situations in which there is a choice between wireless and wire technologies, and the choice often changes when new technologies become available.

In this book, we will concentrate on cellular networks, both because they are of great current interest and also because the features of many other wireless systems can be easily understood as special cases or simple generalizations of the features of cellular networks. A cellular network consists of a large number of wireless subscribers who have cellular telephones (mobile users), that can be used in cars, in buildings, on the street, or almost anywhere. There are also a number of fixed base stations, arranged to provide coverage (via wireless electromagnetic transmission) of the subscribers.

The area covered by a base station, i.e., the area from which incoming calls reach that base station, is called a cell. One often pictures a cell as a hexagonal region with the base station in the middle. One then pictures a city or region as being broken up into a hexagonal lattice of cells (see Figure 1.2a). In reality, the base stations are placed somewhat irregularly, depending on the location of places such as building tops or hill tops that have good communication coverage and that can be leased or bought (see Figure 1.2b). Similarly, the mobile users connected to a base station are chosen by good communication paths rather than geographic distance.



(a) Part (a): an oversimplified view in which each cell is hexagonal.

(b) Part (b): a more realistic case where base stations are irregularly placed and cell phones choose the best base station

Figure 1.1: Cells and Base stations for a cellular network

When a mobile user makes a call, it is connected to the base station to which it appears to have the best path (often the closest base station). The base stations in a given area are then connected to a *mobile telephone switching office* (MTSO, also called a *mobile switching center* MSC) by high speed wire connections or microwave links. The MTSO is connected to the public wired telephone network. Thus an incoming call from a mobile user is first connected to a base station and from there to the MTSO and then to the wired network. From there the call goes to its destination, which might be an ordinary wire line telephone, or might be another mobile subscriber. Thus, we see that a cellular network is not an independent network, but rather an appendage to the wired network. The MTSO also plays a major role in coordinating which base station will handle a call to or from a user and when to handoff a user from one base station to another.

When another telephone (either wired or wireless) places a call to a given user, the reverse process takes place. First the MTSO for the called subscriber is found, then the closest base station is found, and finally the call is set up through the MTSO and the base station. The wireless link from a base station to a mobile user is interchangeably called the *downlink* or the *forward channel*, and the link from a user to a base station is called the *uplink* or a *reverse channel*. There are usually many users connected to a single base station, and thus, for the forward channels, the base station must multiplex together the signals to the various connected users and then broadcast one waveform from which each user can extract its own signal. The combined channel from the one base station to the multiple users is called a *broadcast channel*. For the reverse channels, each user connected to a given base station transmits its own waveform, and the base station receives the sum of the waveforms from the various users plus noise. The base station must then separate out the signals from each user and forward these signals to the MTSO. The combined channel from each user to the base station is called a *multiaccess channel*.

Older cellular systems, such as the AMPS system developed in the U.S. in the 80's, are analog. That is, a voice waveform is modulated on a carrier and transmitted without being transformed into a digital stream. Different users in the same cell are assigned different modulation frequencies, and adjacent cells use different sets of frequencies. Cells sufficiently far away from each other can reuse the same set of frequencies with little danger of interference.

All of the newer cellular systems are digital (i.e., they have a binary interface). Since these cellular systems, and their standards, were originally developed for telephony, the current data rates and delays in cellular systems are essentially determined by voice requirements. At present, these systems are mostly used for telephony, but both the capability to send data and the applications for data are rapidly increasing. Later on we will discuss wireless data applications at higher rates than those compatible with voice channels.

As mentioned above, there are many kinds of wireless systems other than cellular. First there are the broadcast systems such as AM radio, FM radio, TV, and paging systems. All of these are similar to the broadcast part of cellular networks, although the data rates, the size of the areas covered by each broadcasting node, and the frequency ranges are very different. Next, there are wireless LANs (local area networks) These are designed for much higher data rates than cellular systems, but otherwise are similar to a single cell of a cellular system. These are designed to connect PC's, shared peripheral devices, large computers, etc. within an office building or similar local environment. There is little mobility expected in such systems and their major function is to avoid the mazes of cable that are strung around office buildings. There is a similar (even smaller scale) standard called Bluetooth whose purpose is to reduce cabling in an office and simplify transfers between office and hand held devices. Finally, there is another type of LAN called an *ad hoc network*. Here, instead of a central node (base station) through which all traffic flows, the nodes are all alike. The network organizes itself into links between various pairs of nodes and develops routing tables using these links. Here the network layer issues of routing, dissemination of control information, etc. are of primary concern rather than the physical layer issues of major interest here.

One of the most important questions for all of these wireless systems is that of standardization. For cellular systems in particular, there is a need for standardization as people want to use their cell phones in more than just a single city. There are already three mutually incompatible major types of digital cellular systems. One is the GSM system which was standardized in Europe but now used worldwide, another is the TDMA (time-division multiple access) standard developed in the U.S. (IS-136), and a third is CDMA (code division multiple access) (IS-95). We discuss and contrast these briefly later. There are standards for other systems as well, such as the IEEE 802.11 standards for wireless LANs.

In thinking about wireless LANs and wide-area cellular telephony, an obvious question is whether they will some day be combined into one network. The use of data rates compatible with voice rates already exists in the cellular network, and the possibility of much higher data rates already exists in wireless LANs, so the question is whether very high data rates are commercially desirable for the standardized wide-area cellular network. The wireless medium is a much more difficult medium for communication than the wired network. The spectrum available for cellular systems is limited, the interference level is significant, and rapid growth is increasing the level of interference. Adding higher data rates will exacerbate this interference problem. In addition, the screen on hand held devices is small, limiting the amount of data that can be presented and suggesting that many existing applications of such devices do not need very high data rates. Thus whether very high speed data for cellular networks is necessary or desirable in the near future may depend very much on new applications. On the other hand, cellular providers are anxious to provide increasing data rates so as to be viewed as providing more complete service than their competitors.

1.3 Book Outline

The central object of interest is the wireless fading channel. Chapter 2 introduces the multipath fading channel model that we use for the rest of the book. Starting from a continuous-time passband channel, we derive a discrete-time complex baseband model more suitable for analysis and design. We explain the key physical parameters such as coherence time, coherence bandwidth, Doppler spread and delay spread and survey several statistical models for multipath fading (due to constructive and destructive interference of multipaths). There have been many statistical models proposed in the literature; we will be far from exhaustive here. The goal is to have a small set of example models in our repertoire to illustrate the basic communication phenomena we will study.

Chapter 3 introduces many of the issues of communicating over fading channels in the simplest point-to-point context. We start by looking at the problem of detection of uncoded transmission over a narrowband fading channel. We consider both *coherent* and *noncoherent* reception, i.e. with and without channel knowledge at the receiver respectively. We find that in both cases the performance is very poor, much worse than an AWGN channel with the same signal-to-noise ratio (SNR). This is due to a significant probability that the channel is in *deep fade*. We study various *diversity techniques* to mitigate this adverse effect of fading. Diversity techniques increase reliability by sending the same information through multiple independently faded paths so that the probability of successful transmission is higher. Some of these techniques we will study include:

- interleaving of coded symbols over time;
- multipath combining or frequency hopping in spread-spectrum systems to obtain frequency diversity
- use of multiple transmit or receive antennas, via *space-time* coding.
- macrodiversity via combining of signals received from or transmitted to multiple base stations (soft handoff)

In some scenarios, there is an interesting interplay between channel uncertainty and the diversity gain: as the number of diversity branches increase, the performance of the system first improves due to the diversity gain but then subsequently deteriorates as channel uncertainty makes it more difficult to combine signals from the different branches.

In Chapter 4 we shift our focus from point-to-point communication to studying cellular systems as a whole. Multiple access and inter-cell interference management are the key issues that come to the forefront. We explain how existing digital wireless systems deal with these issues. We discuss the concepts of frequency reuse and cell sectorization, and contrast between narrowband systems such as GSM and IS-136, where users within the same cell are kept orthogonal and frequency is reused only in cells far away, and CDMA systems, where the signals of users both within the same cell and across different cells are spread across the same spectrum, i.e. frequency reuse factor of 1. We focus particularly on the design principles of spread-spectrum CDMA systems.

In addition to the diversity techniques of time-interleaving, multipath combining and soft handoff, *power control* and *interference averaging* are the key mechanisms to manage intra-cell and inter-cell interference respectively. All five techniques strive toward the same system goal: to maintain the channel quality of each user, as measured by the signal-to-interference-and-noise ratio (SINR), as constant as possible. We conclude this chapter with the discussion of a wideband orthogonal frequency division multiplexing system (OFDM) which combines the advantages of CDMA and narrowband systems.

In Chapter 5 we study the basic information theory of wireless channels. This gives us a higher level view of the tradeoffs involved in the earlier chapters as well as lay the foundation for understanding the more modern developments in the subsequent chapters. We use as a baseline for comparison the performance over the (non-faded) additive white Gaussian noise (AWGN) channel. We introduce the information theoretic concept of *channel capacity* as the basic performance measure. The capacity of a channel provides the fundamental limit of communication achievable by any scheme. For the fading channel, there are several capacity measures, relevant for different scenarios. Using these capacity measures, we define several resources associated with a fading channel: 1) diversity; 2) number of degrees of freedom; 3) received power. These three resources form a basis for assessing the nature of performance gain by the various communication schemes studied in the rest of the book.

Chapters 6 to 9 cover the more recent developments in the field. In Chapter 6 we revisit the problem of multiple access over fading channels from a more fundamental point of view. Information theory suggests that if both the transmitters and the receiver can track the fading channel, the optimal strategy to maximize the total system throughput is to allow only the user with the best channel to transmit at any time. A similar strategy is also optimal for the downlink (one-to-many). Opportunistic strategies of this type yield a system wide *multiuser diversity* gain: the more users in the system, the larger the gain, as there is more likely to have a user with a very strong channel. To implement the concept in a real system, three important considerations are: 1) *fairness* of the resource allocation across users, 2) *delay* experienced by the individual user waiting for its channel to become good, and 3) measurement inaccuracy and delay in feeding back the channel state to the transmitters. We discuss how these issues are addressed in the context of IS-865 (also called HDR or CDMA 2000 1x EV-DO), a third-generation wireless data system.

A wireless system consists of multiple dimensions: time, frequency, space and users. Opportunistic communication maximizes the spectral efficiency by measuring when and where the channel is good and only transmit in those degrees of freedom. In this context, channel fading is *beneficial* in the sense that the fluctuation of the channel across the degrees of freedom ensures that there will be some degrees of freedom in which the channel is very good. This is in sharp contrast to the diversity-based approach we will discuss in Chapter 3, where channel fluctuation is always detrimental and the design goal is to average out the fading to make the overall channel as constant as possible. Taking this philosophy one step further, we discuss a technique, called *opportunistic beamforming*, in which channel fluctuation can be *induced* in situations when the natural fading has small dynamic range and/or is slow. From the cellular system point of view, this technique also increases the fluctuations of the *interference* imparted on adjacent cells, and presents an opposing philosophy to the notion of interference averaging in CDMA systems.

Chapters 7, 8 and 9 discuss multi-input multi-output (MIMO) systems. It has been known for a while that a multiaccess system with multiple receive antennas allows several users to simultaneous communicate to the receiver. The multiple antennas in effect increase the number of degrees of freedom in the system and allows spatial separation of the signals from the different users. It has recently been shown that a similar effect occurs for point-to-point channel with multiple transmit *and* receive antennas, i.e. even when the antennas of the multiple users are co-located. This holds provided that the scattering environment is rich enough to allow the receive antennas separate out the signal from the different transmit antennas. This allows the *spatial multiplexing* of information. We see yet another example where channel fading is in fact beneficial to communication.

Chapter 7 starts with a discussion of MIMO channel models. Capacity results in the point-to-point case are presented. We then describe several signal processing and coding schemes which achieve or approach the channel capacity. These schemes are based on techniques including singular-value decomposition, linear and decision-feedback equalization (also known as successive cancellation). As shown in Chapter 3, multiple antennas can also be used to obtain diversity gain, and so a natural question arises as how diversity and spatial multiplexing can be put in the same picture. In Chapter 8, the problem is formulated as a *tradeoff* between the diversity and multiplexing gain achievable, and it is shown that for a given fading channel model, there is an optimal tradeoff between the two types of gains achievable by any space-time coding scheme. This is then used as a unified framework to assess both the diversity and multiplexing performance of several schemes. Finally, in Chapter 9, we extend our discussion to multiuser and multi-cellular systems. Here, in addition to providing spatial multiplexing and diversity, multiple antennas can also be used to suppress interference.

Chapter 2

The Wireless Channel

A good understanding of the wireless channel, its key physical parameters and the modeling issues, lays the foundation for the rest of the course. This is the goal of the chapter. We start with the physical modeling of the wireless channel in terms of electromagnetic waves. We then derive an input-output linear time-varying model for the channel, and define some important physical parameters. Finally we introduce a few statistical models of the channel variation over time and over frequency.

2.1 Physical Modeling for Wireless Channels

Wireless channels operate through electromagnetic radiation from the transmitter to the receiver. In principle, one could solve the electromagnetic field equations, in conjunction with the transmitted signal, to find the electromagnetic field impinging on the receiver antenna. This would have to be done taking into account the obstructions[1] caused by ground, buildings, vehicles, etc. in the vicinity of this electromagnetic wave.

Cellular communication is limited by the Federal Communication Commission (FCC), and by similar authorities in other countries, to one of three frequency bands, one around 0.9 Ghz, one around 1.9 GHz, and one around 5.8 GHz. The wavelength $\Lambda(f)$ of electromagnetic radiation at any given frequency f is given by $\Lambda = c/f$, where $c = 3\times 10^8$ m/s is the velocity of light. The wavelength in these cellular bands is thus a fraction of a meter, so to calculate the electromagnetic field at a receiver, the locations of the receiver and the obstructions would have to be known within sub-meter accuracies. The electromagnetic field equations are therefore too complex to solve, especially on the fly for mobile users. Thus we have to ask what we really need to know about these channels, and what approximations might be reasonable.

[1]By obstructions, we mean not only objects in the line of sight between transmitter and receiver, but also objects in locations that cause non-negligible changes in the electromagnetic field at the receiver; we shall see examples of such obstructions later.

One of the important questions is where to place the base stations, and what range of power levels are then necessary on the downlink and uplink channels. To some extent this question must be answered experimentally, but it certainly helps to have a sense of what types of phenomena to expect. Another major question is what types of modulation and detection techniques look promising. Here again, we need a sense of what types of phenomena to expect. To address this, we will construct stochastic models of the channel, assuming that different channel behaviors appear with different probabilities, and change over time (with specific stochastic properties). We will return to the question of why such stochastic models are appropriate, but for now we simply want to explore the gross characteristics of these channels. Let us start by looking at several over-idealized examples.

2.1.1 Free space, fixed transmitting and receive antennas

First consider a fixed antenna radiating into free space. In the far field,[2] the electric field and magnetic field at any given location are perpendicular both to each other and to the direction of propagation from the antenna. They are also proportional to each other, so it is sufficient to know only one of them (just as in wired communication, where we view a signal as simply a voltage waveform or a current waveform). In response to a transmitted sinusoid $\cos 2\pi ft$, we can express the electric far field at time t as

$$E(f,t,(r,\theta,\psi)) = \frac{\alpha_s(\theta,\psi,f)\cos 2\pi f(t-r/c)}{r} \tag{2.1}$$

Here (r,θ,ψ) represents the point $\mathbf{u}$ in space at which the electric field is being measured, where r is the distance from the transmitting antenna to $\mathbf{u}$ and where (θ,ψ) represent the vertical and horizontal angles from the antenna to $\mathbf{u}$. The constant $c = 3\times 10^8$ m/s is the velocity of light, and $\alpha_s(\theta,\psi,f)$ is the radiation pattern of the sending antenna at frequency f in the direction (θ,ψ); it also contains a scaling factor to account for antenna losses. Note that the phase of the field varies with fr/c, corresponding to the delay caused by the radiation traveling at the speed of light.

We are not concerned here with actually finding the radiation pattern for any given antenna, but only with recognizing that antennas have radiation patterns, and that the free space far field behaves as above.

It is important to observe that as the distance r increases, the electric field decreases as r^{-1} and thus the power per square meter in the free space wave decreases as r^{-2}. This is expected, since if we look at concentric spheres of increasing radius r around the antenna, the total power radiated through the sphere remains constant, but the surface area increases as r^2. Thus the power per unit area must decrease as r^{-2}. We

[2] The far field is the field far enough away from the antenna that eqn (2.1) is valid. For cellular systems, it is a safe assumption that the receiver is in the far field.

will see shortly that this r^{-2} reduction of power with distance is often not valid when there are obstructions to free space propagation.

Next, suppose there is a fixed receive antenna at location $\mathbf{u} = (r, \theta, \psi)$. The received waveform (in the absence of noise) in response to the above transmitted sinusoid is then

$$E_r(f, t, u) = \frac{\alpha(\theta, \psi, f) \cos 2\pi f(t - r/c)}{r} \tag{2.2}$$

where $\alpha(\theta, \psi, f)$ is the product of the antenna patterns of transmitting and receive antennas in the given direction. We have done something a little strange here in starting with the free space field at $\mathbf{u}$ in the absence of an antenna. Placing a receive antenna there changes the electric field in the vicinity of $\mathbf{u}$, but this is taken into account by the antenna pattern of the receive antenna.

Now suppose, for the given $\mathbf{u}$, that we define

$$H(f) := \frac{\alpha(\theta, \psi, f) e^{-j2\pi f r/c}}{r}. \tag{2.3}$$

We then have $E_r(f, t, u) = \Re\left[H(f) e^{j2\pi f t}\right]$. We have not mentioned it yet, but (2.1) and (2.2) are both linear in the input. That is, the received field (waveform) at $\mathbf{u}$ in response to a weighted sum of transmitted waveforms is simply the weighted sum of responses to those individual waveforms. Thus, $H(f)$ is the system function for an LTI (linear time invariant) channel, and its inverse Fourier transform is the impulse response. The need for understanding electromagnetism is to determine what this system function is. We will find in what follows that linearity is a good assumption for all the wireless channels we consider, but that the time invariance does not hold when either the antennas or obstructions are in relative motion.

2.1.2 Free space, moving antenna

Next consider the fixed antenna and free space model above with a receive antenna that is moving with velocity v in the direction of increasing distance from the transmitting antenna. That is, we assume that the receive antenna is at a moving location described as $\mathbf{u}(t) = (r(t), \theta, \psi)$ with $r(t) = r_0 + vt$. Using (2.1) to describe the free space electric field at the moving point $u(t)$ (for the moment with no receive antenna), we have

$$E(f, t, (r_0 + vt, \theta, \psi)) = \frac{\alpha_s(\theta, \psi, f) \cos 2\pi f(t - r_0/c - vt/c)}{r_0 + vt}. \tag{2.4}$$

Note that we can rewrite $f(t - r_0/c - vt/c)$ as $f(1 - v/c)t - fr_0/c$. Thus the sinusoid at frequency f has been converted to a sinusoid of frequency $f(1 - v/c)$; there has been a *Doppler shift* of $-fv/c$ due to the motion of the observation point[3]. Intuitively, each

[3]The reader should be familiar with the Doppler shift associated with moving cars. When an ambulance is rapidly moving toward us we hear a higher frequency siren. When it passes us we hear a rapid shift toward lower frequencies

successive crest in the transmitted sinusoid has to travel a little further before it gets observed at this moving observation point. If the antenna is now placed at $u(t)$, and the change of field due to the antenna presence is again represented by the receiver antenna pattern, the received waveform, in analogy to (2.2), is

$$E_r(f,t,(r_0+vt,\theta,\psi)) = \frac{\alpha(\theta,\psi,f)\cos 2\pi\left[f(1-v/c)t - fr_0/c\right]}{r_0+vt}. \tag{2.5}$$

This channel cannot be represented as an LTI channel. If we ignore the time varying attenuation in the denominator of (2.5), however, we can represent the channel in terms of a system function followed by translating the frequency f by the Doppler shift $-fv/c$. It is important to observe that the amount of shift is dependent on the frequency f. We will come back to discussing the importance of this Doppler shift and of the time varying attenuation after considering the next example.

The above analysis does not depend on whether it is the transmitter or the receiver (or both) that are moving. So long as $r(t)$ is interpreted as the distance between the antennas (and the relative orientations of the antennas are constant), (2.4) and (2.5) are valid.

2.1.3 Reflecting wall, fixed antenna

Consider Figure 2.1 below in which there is a fixed antenna transmitting the sinusoid $\cos 2\pi ft$, a fixed receive antenna, and a single perfectly reflecting large fixed wall. We assume that in the absence of the receive antenna, the electromagnetic field at the point where the receive antenna will be placed is the sum of the free space field coming from the transmit antenna plus a reflected wave coming from the wall. As before, in the presence of the receive antenna, the perturbation of the field due to the antenna is represented by the antenna pattern. An additional assumption here is that the presence of the receive antenna does not appreciably affect the plane wave impinging on the wall. In essence, what we have done here is to approximate the solution of

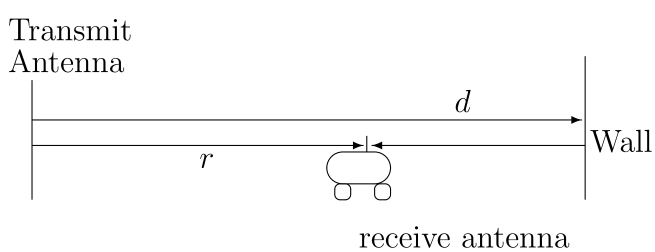


Figure 2.1: Illustration of a direct path and a reflected path

Maxwell's equations by a method called *ray tracing*. The assumption here is that the received waveform can be approximated by the sum of the free space wave from the sending transmitter plus reflected free space waves from each of the reflecting obstacles.

In the present situation, if we assume that the wall is very large, the reflected wave at a given point is the same (except for a sign change) as the free space wave that would exist on the opposite side of the wall if the wall were not present (see Figure 2.2). This means that the reflected wave from the wall has the intensity of a free space wave at a distance equal to the distance to the wall and then back to the receive antenna, i.e., $2d - r$. Using (2.1) for both the direct and the reflected wave, and assuming the same antenna gain α for both waves,

$$E_r(f,t) = \frac{\alpha \cos 2\pi \left(ft - \frac{fr}{c}\right)}{r} - \frac{\alpha \cos 2\pi \left(ft + \frac{fr - 2fd}{c}\right)}{2d - r}. \tag{2.6}$$

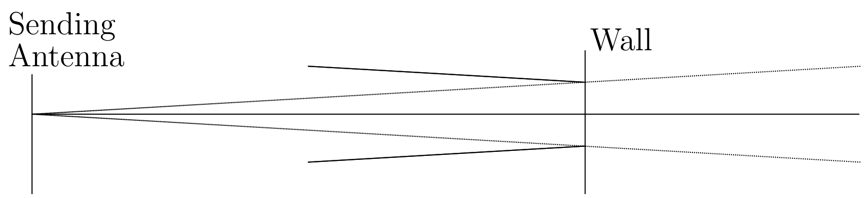


Figure 2.2: Relation of reflected wave to wave without wall.

The received signal is a superposition of two waves both of frequency f. The phase difference between the two waves is:

$$\Delta\theta = \left(\frac{(2\pi f)\, 2d - r}{c} + \pi\right) - \left(\frac{2\pi f r}{c}\right) = \frac{4\pi f}{c}(d - r) + \pi \tag{2.7}$$

When the phase difference is 0, the two waves add *constructively*, and the received signal is strong. When the phase difference is π, the two waves add *destructively*, and the received signal is weak. As a function of r, this translates into a spatial pattern of constructive and destructive interference of the waves. The distance from a peak to a valley is $\lambda/4$, where $\lambda := c/f$ is the wavelength of the transmitted sinusoid.

The constructive and destructive interference pattern also depends on the frequency f: for a fixed r, if f changes by

$$\frac{1}{2}\left(\frac{2d - r}{c} - \frac{r}{c}\right)^{-1}$$

then we move from a peak to a valley. The quantity

$$T_d := \left(\frac{2d - r}{c} - \frac{r}{c}\right) \tag{2.8}$$

is called the *delay spread* of the channel: it is the difference between the propagation delays along the two signal paths. Thus, the constructive and destructive interference pattern changes significantly if the frequency changes by an amount of the order of $1/T_d$.

2.1.4 Reflecting wall, moving antenna

Suppose the receive antenna is now moving at velocity v (Figure 2.3). As it moves through the pattern of constructive and destructive interference created by the two waves, the strength of the received signal increases and decreases. This is the phenomenon of *multipath fading*. The time taken to travel from a peak to a valley is $c/(4fv)$: this is the time-scale at which the fading occurs.

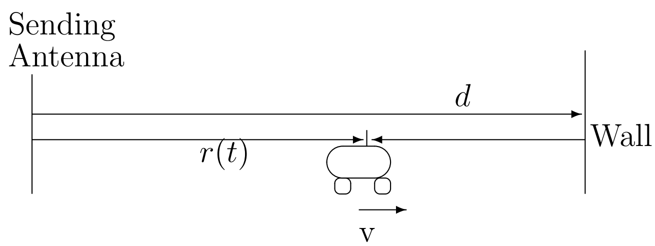


Figure 2.3: Illustration of a direct path and a reflected path

To see this more explicitly, suppose the receive antenna is at location r_0 at time 0. Taking $r = r_0 + vt$ in (2.6), we get:

$$E_r(f,t) = \frac{\alpha \cos 2\pi[f(1-\frac{v}{c})t - \frac{fr_0}{c}]}{r_0 + vt} - \frac{\alpha \cos 2\pi\left[f(1+\frac{v}{c})t + \frac{fr_0 - 2fd}{c}\right]}{2d - r_0 - vt}. \tag{2.9}$$

The first term, the direct wave, is a sinusoid of slowly decreasing magnitude at frequency $f(1-v/c)$. The second is a sinusoid of smaller but increasing magnitude at frequency $f(1+v/c)$. The combination of the two creates a beat frequency at fv/c, which is the rate of traversal across the interference pattern. As an example, if the mobile is moving at 60 km/hr and $f = 900$ MHz, this beat frequency is 50 Hz. The waveform can be visualized most easily when the mobile is much closer to the wall than to the transmit antenna. In this case we can approximate the denominator of the second term by $r_0 + vt$. Then, combining the exponentials, we get

$$E_r(f,t) \approx -\frac{2\alpha \sin 2\pi\left[\frac{fv}{c}t + \frac{2\pi f(r_0 - d)}{c}\right] \cos 2\pi[ft - \frac{fd}{c}]}{r_0 + vt}. \tag{2.10}$$

This is the product of two sinusoids, one at the input frequency f, which is typically on the order of GHz, and the other at the Doppler shift fv/c, which might be on the order of 50Hz. Thus the response to a sinusoid at f is another sinusoid at f whose amplitude is varying with peaks going to zeros every 5 ms or so. In this case, since the direct and reflected signals are of similar strength, there is complete cancellation at the valleys of the interference pattern. Note that in (2.9) we are viewing the response as the sum of two sinusoids, each of different frequency, while in (2.10), we are viewing the response as a single sinusoid of the original frequency with a time varying amplitude. These are just two different ways to view the same phenomenon.

We now see why we have partially ignored the denominator terms in (2.9) and (2.10). When the difference between two paths changes by a quarter wavelength, the phase difference between the responses on the two paths change by $\pi/2$, which causes a very significant change in the overall received amplitude. Since the carrier wavelength is very small relative to the path lengths, the time over which this phase effect causes a significant change is far smaller than the time over which the denominator terms cause a significant change. The effect of the phase changes is on the order of milliseconds, whereas the effect of changes in the denominator are relevant over periods of seconds or minutes. In terms of modulation and detection, the time scales of interest are in the range of milliseconds and less, and the denominators are effectively constant over these periods.

The reader might notice that we are constantly making approximations in trying to understand wireless communications, much more so than for wired communications. This is partly because wired channels are typically time-invariant over a very long time-scale, while wireless channels are typically time varying, and appropriate models depend very much on the time scales of interest. For wireless systems, the most important issue is what approximations to make. Solving and manipulating equations is far less important. Thus it is important to understand these modeling issues thoroughly.

2.1.5 Reflection from a Ground Plane

Consider a transmitting and receive antenna, both above a plane surface such as a road (see Figure 2.4). When the horizontal distance r between the antennas becomes very large relative to their vertical displacements from the ground plane, a very surprising thing happens. In particular, the difference between the direct path length and the reflected path length goes to zero as r^{-1} with increasing r (see Exercise 2.5)).

When r is large enough, this difference between the path lengths becomes small relative to a wavelength c/f. Since the sign of the electric field is reversed on the reflected path, these two waves start to cancel each other out. The electric wave at the receiver is then attenuated as r^{-2}, and the received power decreases as r^{-4}. What this example shows is that the received power can decrease with distance considerably faster than r^{-2} in the presence of disturbances to free space. This situation is particularly

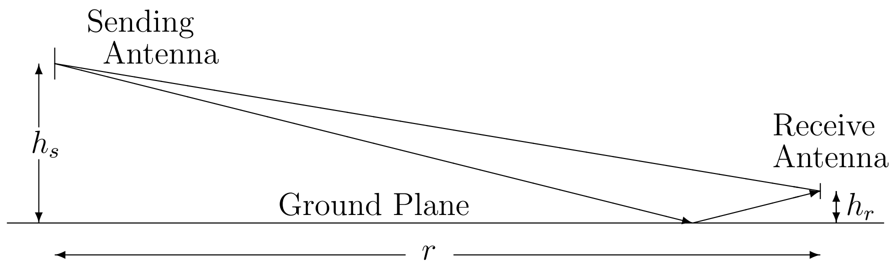


Figure 2.4: Illustration of a direct path and a reflected path off of a ground plane

important in rural areas where base stations tend to be placed on roads. Note, however, that the way the power decreases with distance is both helpful and harmful. It is helpful in reducing the interference between adjacent cells, but it is harmful in reducing the coverage of cells. As cellular systems become more popular, however, the major determinant of cell size is the number of mobiles in the cell. The size of cells has been steadily decreasing, and one talks of micro cells and pico cells as a response to this effect.

2.1.6 Shadowing

Shadowing is a phenomenon that occurs when partially absorbing materials lie between the sending and receive antenna. This is called shadowing because it is similar to the effect of clouds partly blocking sunlight. Shadowing occurs when mobiles are inside buildings and the electromagnetic wave must pass through building walls. It also occurs when an outside mobile is temporarily shielded from the base station by a building or some other structure.

The effect of shadow fading differs from multipath fading in two important ways. First, the duration of a shadow fade lasts for multiple seconds or minutes. For this reason, shadow fading is often called slow fading and multipath fading is called fast fading. Second, the attenuation due to shadowing is exponential in the width of the barrier that must be passed through. Thus the overall attenuation contains not only the r^{-2} effect of free space transmission, but also the exponential attenuation over the depth of the obstructing material.

2.1.7 Moving Antenna, Multiple Reflectors

Dealing with multiple reflectors, under the assumption of ray tracing, is in principle simply a matter of modeling the received waveform as the sum of many responses from different paths rather than just two paths. We have seen enough examples, however, to understand that finding the magnitude and phase of these responses is no simple task. Even for the very simple large wall assumed in Figure 2.1, the reflected field calculated in (2.6) is valid only at small distances from the wall relative to the dimensions of the wall. At very large distances, the total power reflected from the wall is proportional to both d^{-2} and to the cross section of the wall. The part of this reaching the receiver is proportional to $(d - r(t))^{-2}$. Thus the power attenuation from transmitter to receiver (for the large distance case) is proportional to $[d(d-r(t)]^{-2}$ rather than to $[2d-r(t)]^{-2}$. This shows that ray tracing must be used with some caution. Fortunately, however, linearity still holds in these more complex cases.

Another type of reflection is known as scattering and can occur in the atmosphere or in reflections from very rough objects. Here there are a very large number of individual paths, and the received waveform is better modeled as an integral over infinitesimally small paths rather than as a sum.

Knowing how to find the amplitude of the reflected field from each type of reflector above is an important topic if our objective is trying to determine where to place base stations, since this type of analysis is helpful in determining the coverage of a base station (although ultimately experimentation is necessary). Studying this in more depth, however, would take us too far afield and too far into electromagnetic theory. In addition, we are primarily interested in questions of modulation, detection, multiple access, and network protocols rather than location of base stations. Thus, we turn our attention to understanding the nature of the aggregate received waveform, given a representation for each reflected wave. Thus we turn to modeling the input/output behavior of a channel rather than the detailed response on each path.

2.2 Input/Output Model of the Wireless Channel

We derive an input/output model in this section. We first show that the multipath effects can be modeled as a linear time-varying system. We then obtain a baseband representation of this model. The continuous-time channel is then sampled to obtain a discrete-time model. Finally we incorporate additive noise.

2.2.1 The Wireless Channel as a Linear Time-Varying System

In the previous section we focussed on the response to the sinusoidal input $\phi(t) = \cos 2\pi f t$. The received signal can be written as $\sum_i a_i(f,t)\phi(t - \tau_i(f,t))$, where $a_i(f,t)$ and $\tau_i(f,t)$ are respectively the overall attenuation and propagation delay at time t from the transmitter to the receiver on path i. The overall attenuation is simply the product of the attenuation factors due to the antenna pattern of the transmitter, the antenna pattern of the receiver, the reflector, as well as a factor that is a function of the distance from transmitting antenna to reflector and from reflector to receive antenna. This attenuation describes the channel effect at a particular frequency f. If we further assume that the $a_i(f,t)$'s and the $\tau_i(f,t)$'s do not depend on the frequency f, then we can use the principle of superposition to generalize the above input-output relation to an arbitrary input $x(t)$ with nonzero bandwidth.

$$y(t) = \sum_i a_i(t)x(t - \tau_i(t)) \tag{2.11}$$

In practice the attenuations and the propagation delays are usually slowly varying functions of frequency. These variations follow from the time varying path lengths and also from frequency dependent antenna gains. However, we are primarily interested in transmitting over bands that are narrow relative to the carrier frequency, and over such ranges we can omit this frequency dependence. It should however be noted that although the *individual* attenuations and delays are assumed to be independent of the frequency, the *overall* channel response can still vary with frequency due to the fact that different paths have different delays.

For the example of a perfectly reflecting wall in Figure 2.3, then,

$$a_1(t) = \frac{|\alpha|}{r_0 + vt} \qquad\qquad a_2(t) = \frac{|\alpha|}{2d - r_0 - vt} \tag{2.12}$$

where the first expression is for the direct path and the second for the reflected path, and

$$\tau_1(t) = \frac{r_0 + vt}{c} - \frac{\angle\phi_1}{2\pi f} \qquad\qquad \tau_2(t) = \frac{2d - r_0 - vt}{c} - \frac{\angle\phi_2}{2\pi f}. \tag{2.13}$$

The term $\angle\phi_j$ here is to account for possible phase changes at the transmitter, reflector, and receiver. For the example here, there is a phase reversal at the reflector so we can take $\phi_1 = 0$ and $\phi_2 = \pi$.

Since the channel (2.11) is linear, it can be described by the response $h(\tau, t)$ at time t to an impulse τ seconds earlier at the input. In terms of $h(\tau, t)$, the input-output relationship is given by:

$$y(t) = \int_{-\infty}^{\infty} h(\tau, t)x(t - \tau)d\tau \tag{2.14}$$

Comparing (2.14) and (2.11), we see that the impulse response for the fading multipath channel is:

$$h(\tau, t) = \sum_i a_i(t)\delta(\tau - \tau_i(t)). \tag{2.15}$$

This expression is really quite nice. It says that the effect of mobile users, arbitrarily moving reflectors and absorbers, and all of the complexities of solving Maxwell's equations, finally reduce to an input/output relation between transmit and receive antennas which is simply represented as the impulse response of a linear time varying channel filter.

The effect of the Doppler shift is not immediately evident in this representation. From (2.13) for the single reflecting wall example, $d\tau_i(t)/dt = v_i/c$ where v_i is the velocity with which the i^{th} path length is increasing. Thus the Doppler shift on the i^{th} path is $-f d\tau_i(t)/dt$.

In the special case when the transmitter, receiver and the environment are all stationary, the attenuations $a_i(t)$'s and propagation delays $\tau_i(t)$'s do not depend on time t, and we have the usual linear time-invariant channel with an impulse response

$$h(\tau) = \sum_i a_i \delta(\tau - \tau_i).$$

For the time-varying impulse response $h(\tau, t)$, we can define a time-varying frequency response

$$H(f;t) := \int_{-\infty}^{\infty} h(\tau, t) e^{-j2\pi f \tau} d\tau = \sum_i a_i(t) e^{-j2\pi f \tau_i(t)}. \tag{2.16}$$

In the special case when the channel is time-invariant, this reduces to the usual frequency response. One way of interpreting $H(f;t)$ is to think of the system as a slowly varying function of t with a frequency response $H(f;t)$ at each fixed time t. Corresponding, $h(\tau, t)$ can be thought of as the impulse response of the system at a fixed time t. This is a legitimate and useful way of thinking about multipath fading channels, as the time-scale at which the channel varies is typically much longer than the delay spread of the impulse response at a fixed time. In the reflecting wall example in Section 2.1.4, the time taken for the channel to change significantly is of the order of milliseconds while the delay spread is of the order of microseconds. Fading channels which have this charracteristic are sometimes called *underspread* channels.

2.2.2 Baseband Equivalent Model

In typical wireless applications, communication occurs in a passband $[f_c - \frac{W}{2}, f_c + \frac{W}{2}]$ of bandwidth W around a centered frequency f_c, the spectrum having been specified by regulatory authorities. However, most of the processing, such as coding/decoding, modulation/demodulation, synchronization, etc., is actually done at the baseband. At the transmitter, the last stage of the operation is to "up-convert" the signal to the carrier frequency and transmit it via the antenna. Similarly, the first step at the receiver is to "down-convert" the RF (radio-frequency) signal to the baseband before further processing. Therefore from a communication system design point of view, it is most useful to have a baseband equivalent representation of the system. We first start with defining the baseband equivalent representation of signals.

Consider a real signal $s(t)$ with Fourier transform $S(f)$, bandlimited in $[f_c - W/2, f_c + W/2]$ with $W < 2f_c$. Define its *complex baseband equivalent* $s_b(t)$ as the signal having Fourier transform:

$$S_b(f) = \begin{cases} \sqrt{2} S(f + f_c) & f > -f_c \\ 0 & f \leq -f_c \end{cases} \tag{2.17}$$

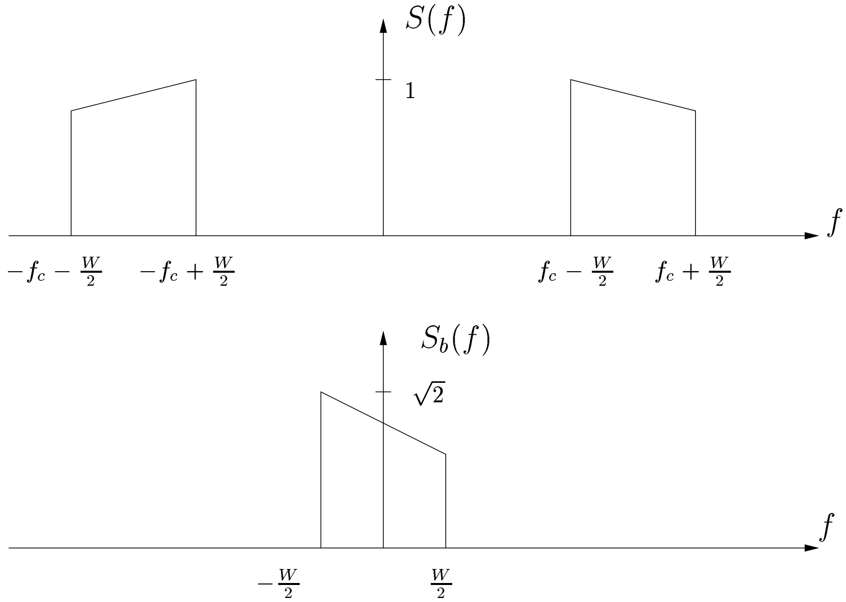


Figure 2.5: Illustration of the relationship between a passband spectrum $S(f)$ to its baseband equivalent $S_b(f)$.

Since $s(t)$ is real, its Fourier transform is symmetrical around $f = 0$, which means that $s_b(t)$ contains exactly the same information as $s(t)$. The factor of $\sqrt{2}$ is quite arbitrary but chosen to normalize the energies of $s_b(t)$ and $s(t)$ to be the same. Note that $s_b(t)$ is bandlimited in $[-W/2, W/2]$. See Figure 2.5.

To reconstruct $s(t)$ from $s_b(t)$, we observe that:

$$\sqrt{2}S(f) = S_b(f - f_c) + S_b^*(-f - f_c). \tag{2.18}$$

Taking inverse Fourier transforms, we get

$$s(t) = \frac{1}{\sqrt{2}} \left\{ s_b(t)e^{j2\pi f_c t} + s_b^*(t)e^{-j2\pi f_c t} \right\} = \sqrt{2}\Re \left[s_b(t)e^{j2\pi f_c t} \right]. \tag{2.19}$$

In terms of real signals, the relationship between $s(t)$ and $s_b(t)$ is shown in Figure 2.6. The passband signal $s(t)$ is obtained by modulating $\Re[s_b(t)]$ by $\sqrt{2}\cos 2\pi f_c t$ and $\Im[s_b(t)]$ by $-\sqrt{2}\sin 2\pi f_c t$ and summing (up-conversion). The baseband signal $\Re[s_b(t)]$ (resp. $\Im[s_b(t)]$) is obtained by modulating $s(t)$ by $\sqrt{2}\cos 2\pi f_c t$ (resp. $\sqrt{2}\sin 2\pi f_c t$) followed by ideal low-pass filtering at the baseband $[-W/2, W/2]$ (down-conversion).

Let us now go back to the multipath fading channel (2.11) with impulse response given by (2.15). Figure 2.7 shows the system diagram from the baseband transmitted signal $x_b(t)$ to the baseband received signal $y_b(t)$. This implementation of a passband

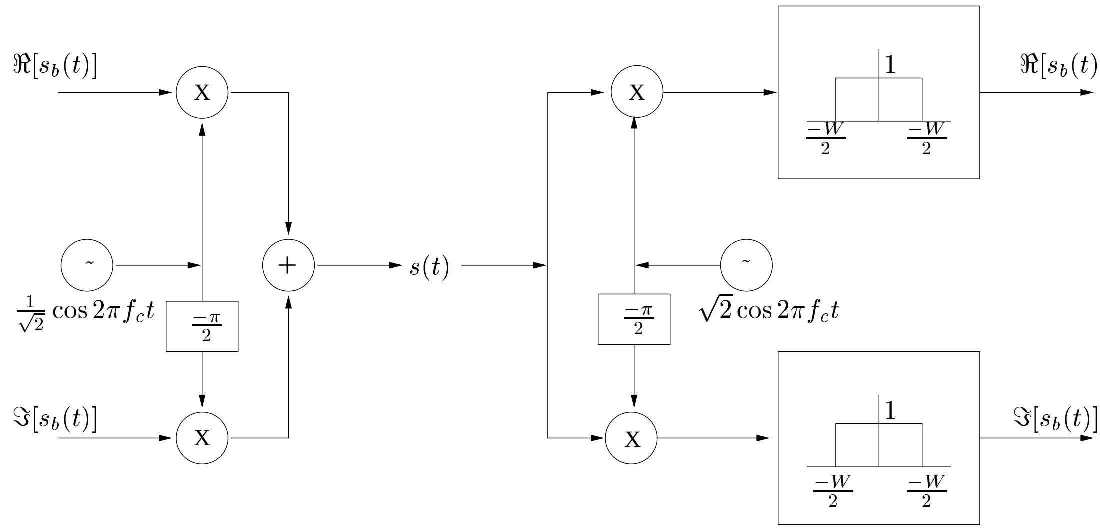


Figure 2.6: Illustration of upconversion from $s_b(t)$ to $s(t)$ and followed by downconversion from $s(t)$ back to $s_b(t)$.

communication system is known as *quadrature amplitude modulation* (QAM). The signal $\Re[x_b(t)]$ is sometimes called the in-phase component (I) and $\Im[x_b(t)]$ the quadrature component (Q) (rotated by $\pi/2$.) We now calculate the baseband equivalent channel. Substituting $x(t) = \sqrt{2}\Re[x_b(t)e^{j2\pi f_c t}]$ and $y(t) = \sqrt{2}\Re[y_b(t)e^{j2\pi f_c t}]$ into (2.11) we get:

$$\begin{aligned}\Re[y_b(t)e^{j2\pi f_c t}] &= \sum_i a_i(t)\Re[x_b(t-\tau_i(t))e^{j2\pi f_c(t-\tau_i(t))}] \\ &= \Re\left[\left\{\sum_i a_i(t)x_b(t-\tau_i(t))e^{-j2\pi f_c\tau_i(t)}\right\}e^{j2\pi f_c t}\right]\end{aligned}$$

Similarly, one can see that

$$\Im[y_b(t)e^{j2\pi f_c t}] = \Im\left[\left\{\sum_i a_i(t)x_b(t-\tau_i(t))e^{-j2\pi f_c\tau_i(t)}\right\}e^{j2\pi f_c t}\right]$$

(Verify!) Hence, the baseband equivalent channel is:

$$y_b(t) = \sum_i a_i^b(t)x_b(t-\tau_i(t)), \tag{2.20}$$

where

$$a_i^b(t) := a_i(t)e^{-j2\pi f_c\tau_i(t)}.$$

This is also a linear time-varying system, and the baseband equivalent impulse response is:

$$h_b(\tau,t) = \sum_i a_i^b(t)\delta(t-\tau_i(t)).$$

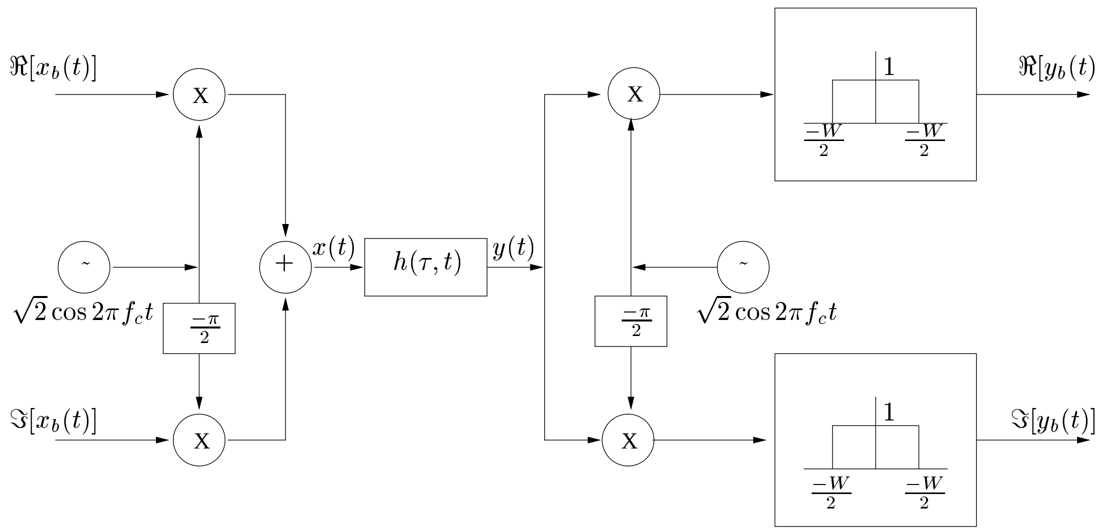


Figure 2.7: System diagram from the baseband transmitted signal $x_b(t)$ to the baseband received signal $y_b(t)$.

This is easy to interpret in the time domain, and the effect of the carrier frequency can be seen explicitly. The baseband output is the sum, over each path, of the delayed replicas of the baseband input. The magnitude of the i^{th} such term is the magnitude of the response on the given path; this changes slowly, with significant changes occuring on the order of seconds or more. The phase is changed by $\pi/2$ (i.e. is changed significantly) when the delay on the path changes by $1/(4f_c)$, or equivalently, when the path length changes by a quarter wavelength, i.e. by $c/(4f_c)$. If the path length is changing at velocity v, the time required for such a phase change is $c/(4f_c v)$. Recalling that the Doppler shift D at frequency f is fv/c, and noting that $f \approx f_c$ for narrow band communication, the time required for a $\pi/2$ phase change is $1/(4D)$. For the single reflecting wall example, this is about 5 ms (assuming $f_c = 900$ MHz and $v = 60$ km/hr). The phases of both paths are rotating at this rate but in opposite directions.

Note that the Fourier transform $H_b(f;t)$ of $h(\tau, t)$ for a fixed t is simply $H(f - f_c; t)$, i.e. the frequency response of the original system (at a fixed t) shifted by the carrier frequency. This provides another way of thinking about the baseband equivalent channel.

2.2.3 A Discrete Time Baseband Model

The next step in creating a useful channel model is to convert the continuous time channel to a discrete time channel. We take the usual approach of the sampling theorem. Assume that the input waveform $x(t)$ is bandlimited to W Hz. The baseband equivalent is then limited to $W/2$ and can be represented as

$$x_b(t) = \sum_n x[n]\text{sinc}(Wt - n) \tag{2.21}$$

where $x[n]$ is given by $x_b(n/W)$ and sinc(t) is defined as:

$$\text{sinc}(t) := \frac{\sin(\pi t)}{\pi t}. \tag{2.22}$$

This representation follows from the sampling theorem, which says that any waveform bandlimited to $W/2$ can be expanded in terms of the orthogonal basis $\{\text{sinc}(Wt - n)\}_n$, with coefficients given by the samples.

Using (2.20), the baseband output is given by

$$y_b(t) = \sum_n x[n] \sum_i a_i^b(t)\text{sinc}[tW - \tau_i(t)W - n]. \tag{2.23}$$

The sampled outputs at multiples of $1/W$, $y[m] := y_b(m/W)$, are then given by

$$y[m] = \sum_n x[n] \sum_i a_i^b(m/W)\text{sinc}[m - n - \tau_i(m/W)W]. \tag{2.24}$$

The sampled output $y[m]$ can equivalently be thought as the projection of the waveform $y_b(t)$ onto the waveform $W\text{sinc}(Wt - m)$. Let $\ell := m - n$. Then

$$y[m] = \sum_\ell x[m - \ell] \sum_i a_i^b(m/W)\text{sinc}[\ell - \tau_i(m/W)W]. \tag{2.25}$$

By defining

$$h_\ell[m] := \sum_i a_i^b(m/W)\text{sinc}[\ell - \tau_i(m/W)W], \tag{2.26}$$

(2.25) can be written in the simple form

$$y[m] = \sum_\ell h_\ell[m]\, x[m - \ell]. \tag{2.27}$$

We denote $h_\ell[m]$ as the ℓ^{th} (complex) channel filter tap at time m. Its value is a function of mainly the gains $a_i^b(t)$ of the paths whose delays $\tau_i(t)$ are close to ℓ/W. As we discuss later, the number of channel filter taps (i.e. different values of ℓ) for which $h_\ell[m]$ is significantly non-zero is usually quite small. If for each ℓ, the ℓ^{th} tap is unchanging with m, then the channel is linear time invariant. If each tap changes slowly with m, then we call the channel slowly time varying. As seen in the next subsection, cellular systems and most of the other wireless systems of current interest are slowly time varying.

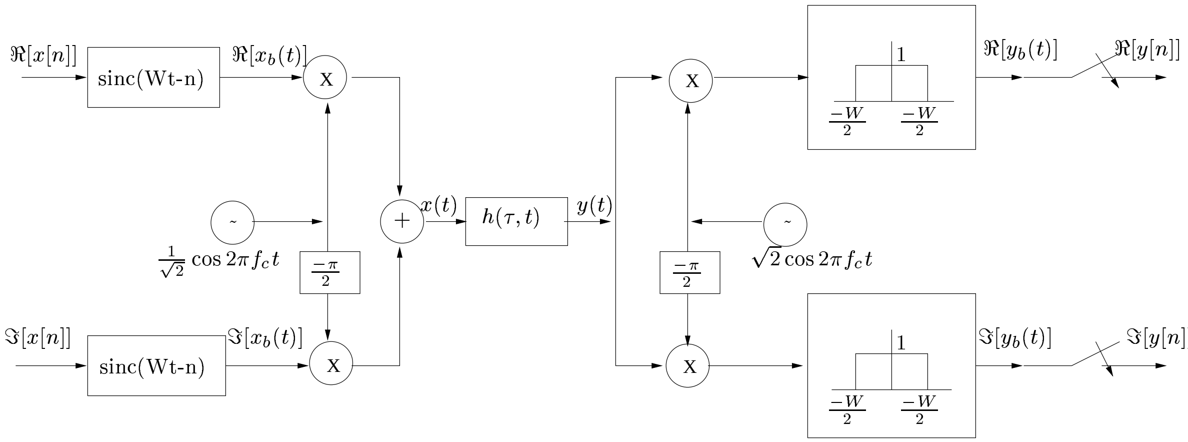


Figure 2.8: System diagram from the baseband transmitted symbol $x[n]$ to the baseband sampled received signal $y[n]$.

We can interpret the sampling operations as modulation and demodulation in a communication system. At time n, we are modulating the complex symbol $x[n]$ (in-phase plus quadrature components) by the sinc pulse before the up-conversion. At the receiver, the received signal is sampled at times m/W at the output of the low-pass filter. Figure 2.8 shows the complete system. In practice, other transmit pulses, such as the raised cosine pulse, are often used in place of the sinc pulse, which has rather poor time-decay property and tends to be more susceptible to timing errors. This necessitates sampling at a rate above the Nyquist sampling rate, but does not affect too much what we are doing in this book. Hence we will confine to Nyquist sampling.

Due to the Doppler shift, the bandwidth of the output $y_b(t)$ is generally slightly larger than the bandwidth $W/2$ of the input $x_b(t)$, and thus the output samples $\{y[m]\}$ do not fully represent the output waveform. This problem is usually ignored in practice, since the Doppler shifts are small (of the order of 10's-100's of Hz) compared to the bandwidth W. Since channels are typically underspread, the taps are slowly varying, they behave like an LTI filter over the periods of interest and essentially model the channel. Also, it is very convenient for the sampling rate of the input and output to be the same. Alternatively, it would be possible to sample the output at twice the rate of the input. This would recapture all the information in the received waveform. The number of taps would be almost doubled because of the reduced sample interval, but it would typically be somewhat less than double since the representation would not spread the path delays so much.

2.2.4 Additive White Noise

As a last step we include additive noise in our input/output model. We make the standard assumption that the noise $w(t)$ is zero-mean additive white Gaussian (AWGN) with power spectral density $N_0/2$ (i.e. $E[w(0)w(t)] = \frac{N_0}{2}\delta(t)$.). The model (2.11) is now modified to be:

$$y(t) = \sum_i a_i(t)x(t - \tau_i(t)) + w(t).$$

See Figure 2.9. The discrete-time baseband-equivalent model (2.27) now becomes

$$y[m] = \sum_\ell h_\ell[m]x[m - \ell] + w[m] \tag{2.28}$$

where $w[m]$ is the low-pass filtered noise at the sampling instant m/W. Just like the signal, the white noise $w(t)$ is down-converted, filtered at baseband and ideally sampled. Thus, it can be verified (see Exercise 2.8) that

$$\Re(w[m]) = \int_{-\infty}^{\infty} w(t)\psi_{m,1}(t)dt \tag{2.29}$$

$$\Im(w[m]) = \int_{-\infty}^{\infty} w(t)\psi_{m,2}(t)dt \tag{2.30}$$

where

$$\psi_{m,1}(t) := \sqrt{2}W \cos 2\pi f_c t \text{sinc}(Wt - m), \qquad \phi_{m,2}(t) := -\sqrt{2}W \sin 2\pi f_c t \text{sinc}(Wt - m).$$

It can further be shown that $\{\psi_{m,1}(t), \psi_{m,2}(t)\}_m$ forms an *orthogonal set* of waveforms, i.e. the waveforms are orthogonal to each other. (See Exercise 2.9.) A basic property of Gaussian white noise says that projections onto orthogonal waveforms are uncorrelated and hence independent. (This is verified in Exercise 2.10.) Hence the discrete-time noise process $\{w[m]\}$ is white, i.e. independent over time, and moreover the real and imaginary components are independent and identically distributed (i.i.d.) Gaussians with variances $N_0/2$. A complex Gaussian random variable X whose real and imaginary components are i.i.d. satisfies a *circular symmetry* property: $e^{j\phi}X$ has the same distribution as X for any ϕ. We shall call such a random variable *circular symmetric complex Gaussian*, denoted by $\mathcal{CN}(0, \sigma^2)$, where $\sigma^2 = E[|X|^2]$.

The assumption of AWGN essentially means that we are assuming that the primary source of the noise is at the receiver or is radiation impinging on the receiver that is independent of the paths over which the signal is being received. This is normally a very good assumption for most communication situations.

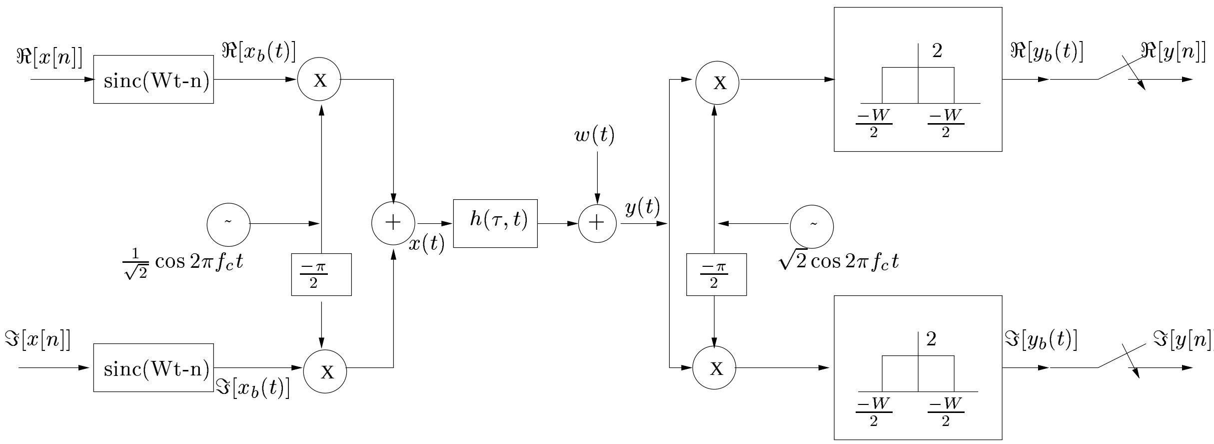


Figure 2.9: A complete system diagram.

2.3 Time and Frequency Coherence; Multipath Spread

An important channel parameter is the time-scale of the variation of the channel. How fast do the taps $h_\ell[m]$ vary as a function of time m? Recall that

$$\begin{aligned} h_\ell[m] &= \sum_i a_i^b(m/W)\text{sinc}[\ell - \tau_i(m/W)W] \\ &= \sum_i a_i(m/W)e^{-j2\pi f_c \tau_i(m/W)}\text{sinc}[\ell - \tau_i(m/W)W]. \end{aligned} \tag{2.31}$$

Let us look at this expression term by term. From Section 2.2.2 we gather that significant changes in a_i occur over periods of seconds or more. Significant changes in the phase of each path occur at intervals of $1/(4D)$, where D is the Doppler shift for that path. When the different paths contributing to the tap have different Doppler shifts, the magnitude of $h_\ell[m]$ changes significantly. This is happening at the time-scale inversely proportional to the largest difference between the Doppler shifts, called the *Doppler spread* D_s. Typical intervals for such changes are on the order of 10 ms. Finally, changes in the sinc term of (2.31) due to the time variation of each $\tau_i(t)$ are proportional to the bandwidth, whereas those in the phase are proportional to the carrier frequency, which is much larger. Essentially, it takes much longer for a path to move from one tap to the next than for its phase to change significantly. Thus, the fastest changes in the filter taps occur because of the phase changes, and these are significant over delay changes of $1/(4D_s)$.

The time coherence, T_c, of a wireless channel is defined (in an order of magnitude sense) as the interval over which $h_\ell[m]$ changes significantly as a function of m. What we have found, then, is the important relation:

$$T_c = \frac{1}{4D_s}. \tag{2.32}$$

This is a somewhat imprecise relation, since the largest Doppler shifts may belong to paths that are too weak to make a difference. We could also view a phase change of $\pi/4$ to be significant, and thus replace the factor of 4 above by 8. Many people instead replace the factor of 4 by 1. The important thing is to recognize that the major effect in determining time coherence is the Doppler shift, and that the relationship is reciprocal; the larger the Doppler shift, the smaller the time coherence.

Another important general parameter of a wireless system is the multipath delay spread, T_d, defined as the difference in propagation time between the longest and shortest path, where we assume, in all of the above sums over different paths, that only the significant paths are included. Thus,

$$T_d := \max_i \tau_i(t) - \min_i \tau_i(t). \tag{2.33}$$

This is defined as a function of t, but we regard it as an order of magnitude quantity, like the time coherence and Doppler shift. If a cell or LAN has a linear extent of a few kilometers or less, it is very unlikely to have path lengths that differ by more than 300 to 600 meters. This corresponds to path delays of one or two μs. As cells become smaller due to increased cellular usage, T_d also shrinks. Typical wireless channels are underspread, which means that the delay spread T_d is much smaller than the coherence time T_c.

The bandwidths of cellular systems range between several hundred kHz and several MHz, and thus, for the above multipath delay spread values, all the path delays in (2.26) lie within the peaks of 2 or 3 sinc functions; more often, they lie within a single peak. Adding a few extra taps to each channel filter because of the slow decay of the sinc function, we see that cellular channels can be represented with at most 4 or 5 channel filter taps.

When we study modulation and detection for these channels, we shall see that the receiver must estimate the values of these channel filter taps. These taps are estimated via transmitted and received waveforms, and thus the receiver makes no explicit use of (and usually does not have) any information about individual path delays and path strengths. This is why we have not studied the details of propagation over multiple paths with complicated types of reflection mechanisms. All we really need is the aggregate values of gross physical mechanisms such as Doppler spread, time coherence, and multipath spread.

There is one additional gross mechanism called *frequency coherence*. Wireless channels change both in time and frequency. The time coherence shows us how quickly the channel changes in time, and similarly, the frequency coherence shows how quickly it changes in frequency. We first understood about channels changing in time, and correspondingly about the duration of fades, by studying the simple example of a direct path and a single reflected path. That same example also showed us how channels change with frequency. We can see this in terms of the frequency response as well.

Key Channel Parameters and Time Scales	Symbol	Representative Values
carrier frequency	f_c	1 GHz
communication bandwidth	W	1 MHz
distance between transmitter and receiver	d	1 km
velocity of mobile	v	40 mph
Doppler shift for a path	$D = \frac{f_c v}{c}$	50 Hz
Doppler spread of paths corresponding to a tap	D_s	100 Hz
time scale for change of path amplitude	$\frac{d}{v}$	1 minute
time scale for change of path phase	$\frac{1}{4D}$	5 ms
time scale for a path to move over a tap	$\frac{c}{vW}$	20 s
coherence time interval	$T_c = \frac{1}{4D_s}$	2.5 ms
delay spread	T_d	1 μ s
coherence bandwidth	$W_c = \frac{1}{2T_d}$	500 kHz

Table 2.1: A summary of the physical parameters of the channel and the time scale of change of the key parameters in its discrete-time baseband model.

Recall that the frequency response at time t is

$$H(f;t) = \sum_i a_i(t) e^{-j2\pi f \tau_i(t)}.$$

The contribution due to a particular path has linear phase in f. For multiple paths, there is a differential phase, $2\pi f(\tau_i(t) - \tau_k(t))$. This differential phase causes selective fading in frequency. This says that not only does $E_r(f,t)$ change significantly when t changes by $1/(4D)$, but also when f changes by $1/2T_d$. This argument extends to an arbitrary number of paths, so the coherence bandwidth, W_c is given by

$$W_c = \frac{1}{2T_d}. \tag{2.34}$$

This relationship, like (2.32) is intended as an order of magnitude relation, essentially pointing out that coherence bandwidth is reciprocal to multipath spread. When the bandwidth of the input is considerably less than W_c, the channel is usually referred to as *flat fading*, and, in essence, a single channel filter tap is sufficient to represent the channel. Note that flat fading is not a property of the channel alone, but of the relationship between the bandwidth W and the delay spread T_d.

The physical parameters and the time scale of change of key parameters of the discrete-time baseband channel model are summarized in Table 2.1.

2.4 Statistical Channel Models

We defined Doppler spread and multipath spread in the previous section as quantities associated with a given receiver at a given location, velocity, and time. However, we are interested in a characterization that is valid over some range of conditions. That is, we recognize that the channel filter taps, $\{h_\ell[m]\}$ must be measured, but we want a longer term characterization of how many taps are necessary and how quickly they change.

Such a characterization requires a probabilistic model of the channel tap values, perhaps gathered by statistical measurements of the channel. We are familiar with describing additive noise by such a probabilistic model, and familiar with evaluating error probability for coding and detection using such models. Those error probability evaluations, however, depend critically on the independence and assumed Gaussian distribution of the noise variables.

It should be clear from the description of the physical mechanisms generating Doppler spread and multipath spread that probabilistic models for the channel filter taps are going to be far less believable than the models for noise. On the other hand, we need such models, even if they are quite inaccurate. Without models, systems are designed using experience and experimentation, and creativity becomes somewhat stifled. Even with highly over-simplified models, we can compare different system approaches and get a sense of what types of approaches are worth pursuing.

To a certain extent, all analytical work is done with simplified models. For example, white Gaussian noise (WGN) is often assumed in communication models, although we know the model is valid only over sufficiently small frequency bands. With WGN, however, we expect the model to be quite good when used properly. For wireless channel models, however, probabilistic models are quite poor and only provide order-of-magnitude guides to system design and performance. We will see that we can define Doppler spread, multipath spread, etc. much more cleanly with probabilistic models, but the underlying problem remains that these channels are very different from each other and cannot really be characterized by probabilistic models. At the same time, there is a large literature based on probabilistic models for wireless channels, and it has been highly useful for providing insight into wireless systems.

There is another question in deciding what to model. Recall the continuous-time multipath fading channel

$$y(t) = \sum_i a_i(t) x[t - \tau_i(t)] + w(t). \tag{2.35}$$

This contains an exact specification of the delay and magnitude of each path. From this, we derived a discrete time baseband model in terms of channel filter taps as

$$y[m] = \sum_\ell h_\ell[m] x[m - \ell] + w[m] \tag{2.36}$$

where

$$h_\ell[m] = \sum_i a_i(m/W) e^{-j2\pi f_c \tau_i(m/W)} \text{sinc}[\ell - \tau_i(m/W)W], \tag{2.37}$$

We used the sampling theorem expansion in which $x[m] = x_b(m/W)$ and $y[m] = y_b(m/W)$. Each channel tap $h_\ell[m]$ contains an aggregate of paths, with the delays smoothed out by the baseband signal bandwidth.

Fortunately, it is the filter taps that must be modeled for input/output descriptions, and also fortunately, the filter taps often contain a sufficient path aggregation so that a statistical model might have a chance of success.

The simplest probabilistic model for the channel filter taps is based on the assumption that there are a large number of statistically independent reflectors with random amplitudes in the delay window corresponding to a single tap. Since the reflectors are far away relative to the carrier wavelength, it is also reasonable to assume that the phase for each path is uniformly distributed between 0 and 2π and that these phases are independent. Thus each tap $h_\ell[m]$ can be modeled as a random variable which is the sum of a large number of small complex random variables each of uniformly distributed phase. It follows that $\Re(h_\ell[m])$ is the sum of many small independent real random variables, so it can reasonably be modeled as a zero-mean Gaussian random variable. Similarly, because of the uniform phase, $\Re(h_\ell[m]e^{j\phi})$ is similarly Gaussian with the same variance. With a little thought, it can be seen that this assures us that $h_\ell[m]$ is in fact circular symmetric $\mathcal{CN}(0, \sigma_\ell^2)$. It is assumed here that the variance of $h_\ell[m]$ is a function of the tap ℓ, but independent of time m (there is little point in creating a probabilistic model that depends on time). With this assumed Gaussian probability density, we show in Exercise 2.11 that the magnitude $|h_\ell[m]|$ of the ℓ^{th} tap is a *Rayleigh* random variable with density

$$f(x) = \frac{x}{\sigma_\ell^2} \exp\left\{\frac{-x^2}{2\sigma_\ell^2}\right\}. \tag{2.38}$$

We later address how these random variables are related between different m and ℓ. This model, which is called *Rayleigh fading*, is quite reasonable for scattering mechanisms where there are many small reflectors, but is adopted primarily for its simplicity in typical cellular situations with a relatively small number of reflectors. The word *Rayleigh* is almost universally used for this model, but the assumption is that the tap gains are circularly symmetric complex Gaussian random variables.

There is a frequently used alternative model in which the line of sight path (often called a *specular* path) is large and has a known magnitude, and that there are also a large number of independent paths. In this case, $h_\ell[m]$, at least for one value of ℓ, can be modeled as a complex Gaussian random variable with a mean (corresponding to the specular path) plus real and imaginary i.i.d. fluctuations around the mean. The magnitude of such a random variable is said to have a *Rician* distribution. Its density has quite a complicated form; it is often a better model of fading than the Rayleigh model.

Modeling each $h_\ell[m]$ as a complex random variable provides part of the statistical description that we need, but this is not the most important part. The more important issue is how these quantities vary with time. As we will see in the rest of the course, the rate of channel variation has significant impact on several aspects of the communication problem. A statistical quantity that models this relationship is known as the *tap gain correlation function*, $R_\ell^W[n]$. It is defined as

$$R_\ell^W[n] := W\mathbb{E}\left\{h_\ell^*[m]h_\ell[m+n]\right\}. \tag{2.39}$$

The reason for using the bandwidth W as a multiplicative factor in this equation will be discussed later. For each tap ℓ, this gives the autocorrelation function of the sequence of random variables modeling that tap as it evolves in time. We are tacitly assuming that this is not a function of time m. Since the sequence of random variables $\{h_\ell[m]\}$ for any given ℓ has both a mean and covariance function that does not depend on m, this sequence is wide sense stationary. We also assume that, as a random variable, $h_\ell[m]$ is independent of $h_{\ell'}[m']$ for all $\ell \neq \ell'$ and all m, m'. This final assumption is intuitively plausible[4] since paths in different ranges of delay contribute to $h_\ell[m]$ for different values of ℓ.

In terms of $R_\ell^W[n]$, the multipath spread T_d can be defined as the product of W times the range of ℓ over which $R_\ell^W[0]$ is significantly non-zero. This is somewhat preferable to our previous "definition" in that the statistical nature of T_d becomes explicit and the reliance on some sort of stationarity becomes explicit. We can also now define the time coherence T_c more explicitly as the smallest value of $n > 0$ for which $R_\ell^W[n]$ is significantly different from $R_\ell^W[0]$. With both of these definitions, we still have the ambiguity of what 'significant' means, but we are now facing the reality that these quantities must be viewed as statistics rather than as instantaneous values.

The tap gain correlation function is useful as a way of expressing the statistics for how tap gains change given a particular bandwidth W, but gives little insight into questions related to choice of a bandwidth for communication. If we visualize increasing the bandwidth, we can see several things happening. First, the ranges of delay that are separated into different taps ℓ become narrower, so there are fewer paths corresponding to each tap, and thus the Rayleigh approximation becomes poorer. Second, the sinc functions of (2.37) become narrower, so the path delays spill over less in time, and consequently, T_d as described above becomes smaller. For this same reason, $R_\ell^W[0]$ gives a finer grained picture of the amount of power being received in the delay window of width ℓ/W. In summary, as we try to apply this model to larger W, we get more detailed information about delay and correlation at that delay, but the information becomes more questionable.

[4] One could argue that a moving reflector would gradually travel from the range of one tap to another, but as we have seen this is typically happening over a very slow time-scale.

The discussion above focuses on statistical modeling of *small-scale* multipath fading. The dynamics of this take place at the spatial scale of the order of wavelengths of the carrier. At a larger spatial scale, channel variations occur due to shadowing and varying distances between the transmitter and the receiver. Small scale variations are more relevant to issues such as channel measurement and dynamic resource allocation. Larger spatial scale variation is more relevant for issues such as cell-site planning and coverage and capacity analysis. We will discuss more about large-scale fading models when we touch on these issues in Chapter 4.

Exercises

EXERCISE 2.1. 1. Equation (2.4) is derived under the assumption that the motion is in the direction of the line of sight from sending antenna to receive antenna. Find this field under the assumption that ϕ is the angle between the line of sight and the motion of the receiver. Assume that the range of time of interest is small enough that changes in (θ, ψ) can be ignored.

2. Explain why, and under what conditions, it is a reasonable approximation to ignore the change in (θ, ψ) over small intervals of time.

EXERCISE 2.2. Eq. (2.10) was derived under the assumption that $r(t) \approx d$. Derive an expression for the received waveform for general $r(t)$. Break the first term in (6) into two terms, one with the same numerator but the denominator $2d - r_0 - vt$ and the other with the remainder. Interpret your result.

EXERCISE 2.3. In the two-path example in Section 2.1.4, the wall is on the left side of the receiver so that the reflected wave and the direct wave travels in opposite direction. Suppose now the reflecting wall is on the left side of transmitter. Redo the analysis. What is the nature of the multipath fading, both over time and over frequency? Explain any similarity or difference with the case considered in Section 2.1.4.

EXERCISE 2.4. In the lecture, we show for the two-path example that the coherence bandwidth should be inversely proportional to the delay spread. Explain this physically in terms of the constructive and destructive interference pattern of the waves.

EXERCISE 2.5. 1. Let r_1 be the length of the direct path in Figure 2.1. Let r_2 be the length of the reflected path (summing the path length from the transmitter to ground plane and the path length from ground plane to receiver). Show that $r_2 - r_1$ is asymptotically equal to b/r and find the value of the constant b. Hint: Recall that for x small, $\sqrt{1+x} \approx (1+x/2)$ in the sense that $[\sqrt{1+x}-1]/x \to 1/2$ as $x \to 0$.

2. Assume that the received waveform at the receive antenna is given by

$$E_r(f,t) = \frac{\alpha \cos 2\pi[ft - fr_1/c]}{r_1} - \frac{\alpha \cos 2\pi[ft - fr_2/c]}{r_2} \qquad (a)$$

Approximate the denominator r_2 by r_1 in (a) and show that $E_r \approx \beta/r^2$ for r^{-1} much smaller than c/f. Find the value of β.

3. Explain why this asymptotic expression remains valid without first approximating the denominator r_2 in (a) by r_1.

Exercise 2.6. Assume that a communication channel first filters the transmitted passband signal before adding WGN. Suppose the channel is known and the channel filter has an impulse response $h(t)$ and a system function $\hat{h}(f) = \int_{\infty}^{\infty} h(t) \exp(-2j\pi f t)\, dt$. Suppose that a QAM system with symbol duration T is developed without knowledge of the channel filtering. A baseband filter $\theta(t)$ is developed satisfying the Nyquist property that $\{\theta(t - kT)\}$ is an orthonormal set. The matched filter $\theta(-t)$ is used at the receiver before sampling and detection.

Knowing about the channel filtering, you now want to redesign either the baseband filter at the transmitter or the baseband filter at the receiver so that there is no intersymbol interference between receiver samples and so that the noise on the samples is iid.

1. Which filter do you want to redesign?

2. Give an expression for the impulse response of the redesigned filter (assume a carrier frequency f_c).

3. Draw a figure of the various filters at passband to show why your solution is correct. (We suggest you do this before answering the first two parts.)

Exercise 2.7. For the two-path example in 2.1.4 with $d = 2$ km and the receiver at 1.5 km from the transmitter, plot in MATLAB the time variation of the taps of the discrete-time baseband channel. You can assume a carrier frequency $f_c = 900$ MHz. Give a few plots for several bandwidths W so as to exhibit both flat and frequency-selective fading cases.

Exercise 2.8. Verify (2.29) and (2.30).

Exercise 2.9. 1. Show that if the waveforms $\{\theta(t - nT)\}_n$ form an orthogonal set, then the waveforms $\{\psi_{n,1}, \psi_{n,2}\}_n$ also form an orthogonal set, provided that $\theta(t)$ is bandlimited to $[-f_c, f_c]$. Here,

$$\begin{aligned} \psi_{n,1}(t) &= \theta(t - nT)\cos 2\pi f_c t \\ \psi_{n,2}(t) &= \theta(t - nT)\sin 2\pi f_c t. \end{aligned}$$

How should we normalize the energy of $\theta(t)$ to make the $\psi(t)$'s to be *orthonormal*?

2. For a given f_c, find an example where the result in (a) is false when the condition that $\theta(t)$ is bandlimited to $[-f_c, f_c]$ is violated.

Exercise 2.10. 1. Let $\mathbf{w} \sim N(0, \sigma^2 I)$ (n-dimensional i.i.d. jointly Gaussian real random vector.) Let $\mathbf{Q}$ be an n by n orthogonal matrix. Show that $\mathbf{Qw}$ has the same distribution as $\mathbf{w}$. Give a geometric interpretation to this result.

2. Let $\{w(t)\}$ be white Gaussian noise with power spectral density $\frac{N_0}{2}$. Let $\mathbf{s_1}, \ldots, \mathbf{s_M}$ be a set of finite orthonormal waveforms (i.e. orthogonal and unit energy), and define $z_i = \int_{-\infty}^{\infty} w(t) s_i(t) dt$. Find the joint distribution of $\mathbf{z}$.

3. Explore the analogy between parts (a) and (b).

4. Let $\mathbf{w}$ now be an n-dimensional i.i.d. *complex* Gaussian random vector (i.e. the real and imaginary parts are jointly Gaussian.) Let $\mathbf{U}$ be an n by n unitary matrix (i.e. $\mathbf{U}^\dagger \mathbf{U} = \mathbf{U}\mathbf{U}^\dagger = I$). Is it true that $\mathbf{Uw}$ has the same distribution as $\mathbf{w}$? If so, explain. If not, give further conditions on $\mathbf{w}$ for this to be true.

EXERCISE 2.11. Compute the probability density function of the magnitude $|X|$ of a complex circular symmetric Gaussian random variable X with variance σ^2. What is the probability density function of $|X|^2$?

EXERCISE 2.12. In class we have discussed the various factors why the channel tap gains $h_\ell[m]$ vary in time (as a function of m) and how the various dynamics operate at different time-scales. The analysis is based on the assumption that communication takes place in a bandwidth W around a carrier frequency f_c, with $f_c \gg W$. This assumption is not valid for *ultra-wideband* communication systems, where the transmission bandwidth is from 3 GHz to 10 GHz, as regulated by the FCC. Redo the analysis for this system. What is the main mechanism that causes the tap gains to vary at the fastest time-scale, and what is this fastest time-scale determined by?

EXERCISE 2.13. 1. Consider the detection problem:

$$\mathbf{r} = \mathbf{h}x + \mathbf{w},$$

where $x = -1$ or $+1$ with equal probability, $\mathbf{h}$ is a given n-dimensional real vector, and $\mathbf{w}$ is i.i.d. real Gaussian noise. Extract a scalar sufficient statistic and derive the MAP detection rule and compute its probability of error.

2. Suppose $\mathbf{h}$ is now complex and $\mathbf{w}$ is i.i.d. complex Gaussian. Can you still find a scalar sufficient statistic for the problem? If so, explain. If not, give further conditions on $\mathbf{w}$ for you to do this. Compute the probability of error of the MAP rule.

3. Suppose x is now equally probable to be $\pm 1 \pm j$. Redo the previous part.

EXERCISE 2.14. Give a convincing argument why it is reasonable to assume that the complex random vector

$$\mathbf{h} := \begin{pmatrix} h_\ell[m] \\ h_\ell[m+1] \\ \cdot \\ \cdot \\ h_\ell[m+n] \end{pmatrix}$$

is circular symmetric. Here, $h_\ell[n]$ is the complex ℓ^{th} tap at time m of the baseband model for the multipath fading channel.

EXERCISE 2.15. Correlation between mobile and base station antennas

Often in modeling multiple input multiple output (MIMO) fading channels the fading coefficients between different transmit and receive antennas are assumed to be independent random variables. This problem explores whether this is a reasonable assumption based on the scattering model and the antenna separation.

1. (Antenna separation at the mobile) Assume a mobile with velocity v moving away from the base station, with uniform scattering ($p(\psi) = 1/(2\pi)$).
   (a) Compute the Doppler spread D for a carrier frequency f_c, and the corresponding coherence time T_c.
   (b) Assuming that fading states separated by T_c are approximately uncorrelated, at what distance should we place a second antenna at the mobile to get an independently faded signal ? (Hint: how much distance does the mobile travel in T_c ?)
2. (Antenna separation at the base station) Assume that the scattering ring has radius R and that the distance between the base station and the mobile is d. Further assume for a moment that the base station is moving away from the mobile with velocity v'. Repeat the previous part to find the minimum antenna spacing at the base station for uncorrelated fading. (Hint: is the scattering still uniform around the base station ?) Typically the scatterers are locally around the mobile and far away from the mobile. What is the implication of your result for this scenario?

EXERCISE 2.16. Verify the equation above eqn. (2.13) in the notes:

$$\Im[y_b(t)e^{j2\pi f_c t}] = \Im\left[\left\{\sum_i a_i(t)x_b(t-\tau_i(t))e^{-j2\pi f_c \tau_i(t)}\right\}e^{j2\pi f_c t}\right]$$

Does this equation contain any more information about the communication system in Figure 2.4 beyond what is in the equation:

$$\Re[y_b(t)e^{j2\pi f_c t}] = \Re\left[\left\{\sum_i a_i(t)x_b(t-\tau_i(t))e^{-j2\pi f_c \tau_i(t)}\right\}e^{j2\pi f_c t}\right]?$$

Explain.

EXERCISE 2.17. Consider the two-path example in Section 2.3.1 with $d = 2$ km and the receiver at 1.5 km from the transmitter moving at velocity 60 km/hr away from the transmitter. The carrier frequency is 900 MHz.

1. Plot in MATLAB the magnitudes of the taps of the discrete-time baseband channel at a fixed time t. Give a few plots for several bandwidths W so as to exhibit both flat and frequency-selective fading.

2. Plot the time variation of the phase and magnitude of a typical tap of the discrete-time baseband channel for a bandwidth where the channel is (approximately) flat and for a bandwidth where the channel is frequency selective. How does the time-variations depend on the bandwidth? Explain.