of $p(x)$ actually increases variance.

But in method 2, the covariance be 1st and 2nd stage so that asympt. var. is actually reduced.

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**Topic:** Distributional Effects  
(Ferpo (2007))

**Quantile Treatment Effect (Quantile ATT and quantile ITE).**

Motivation: We want to compare other aspects of the distribution of $Y_1(1), Y_0(1)$. We observe $Y_1, D=1$, $Y_0, D=0$. (eq. income if jointed, living in vs. not jointed union.

We don't observe $Y_0, D=0$. (This is the counterfactual)

We assume $Y_0, Y_1 \perp D \perp X$.

So, the motivation here is to find something like...

\[ \text{med}(Y_1 | D=1) - \text{med}(Y_0 | D=1) \]

+ How?

Let $\beta = \text{med}(Y_0 | D=1)$

\[ P(Y_0 \leq \beta | D=1) = 2 \]

\[ \Rightarrow E[2(Y_0 \leq \beta | D=1)] = 2 \]

\[ \downarrow \]

Now, $E[g(Y_0, \beta) | D, X] = \int E[g(Y_0 \leq \beta) | D=1, x] f(x | D=1) \, dx$.

By cond. indep.

\[ \Rightarrow \int E[g(Y_0 \leq \beta) | D=0, x] \frac{f(x | D=1)}{f(x | D=0)} \, dx \]
\[
\int \frac{\hat{p}(x)}{\hat{p}(x) + \frac{1}{1 - \hat{p}(x)}} \frac{d}{d} f(x; \theta = 0) dx.
\]

So, we want to find \( \beta \) s.t.

\[
\mathbb{E} \left[ \mathbb{I}(Y_0 < \beta) \frac{\hat{p}(x)}{\hat{p}(x) + \frac{1}{1 - \hat{p}(x)}} \mid D = 0 \right] = z.
\]

How? The above is just a \textit{GMM} moment condition. We can solve for...

\[
\min_{\beta} \left( \frac{1}{\eta_0 \sum_{j=1}^{n_0} \mathbb{I}(Y_j < \beta) \frac{\hat{p}(x_j)(1 - \hat{p}(x_j))}{\hat{p}(x_j) + \frac{1}{1 - \hat{p}(x_j)}} - z} \right)^2
\]

This is precisely \( \text{FE} \) or CATE,

\( w/ z = \frac{z}{2} \).