Johnson Noise:

Determinations of k and Absolute Zero

Edwin Ng | 12 December 2011

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- Two resistors R connected by a wire: $I = V^2/2R$
- By equipartition, each mode at frequency f has energy kT

$$\mathrm{d}\langle P\rangle = \frac{\mathrm{d}\langle V^2\rangle}{4R} = kT\,\mathrm{d}f$$

Theory of Johnson Noise (cont.)

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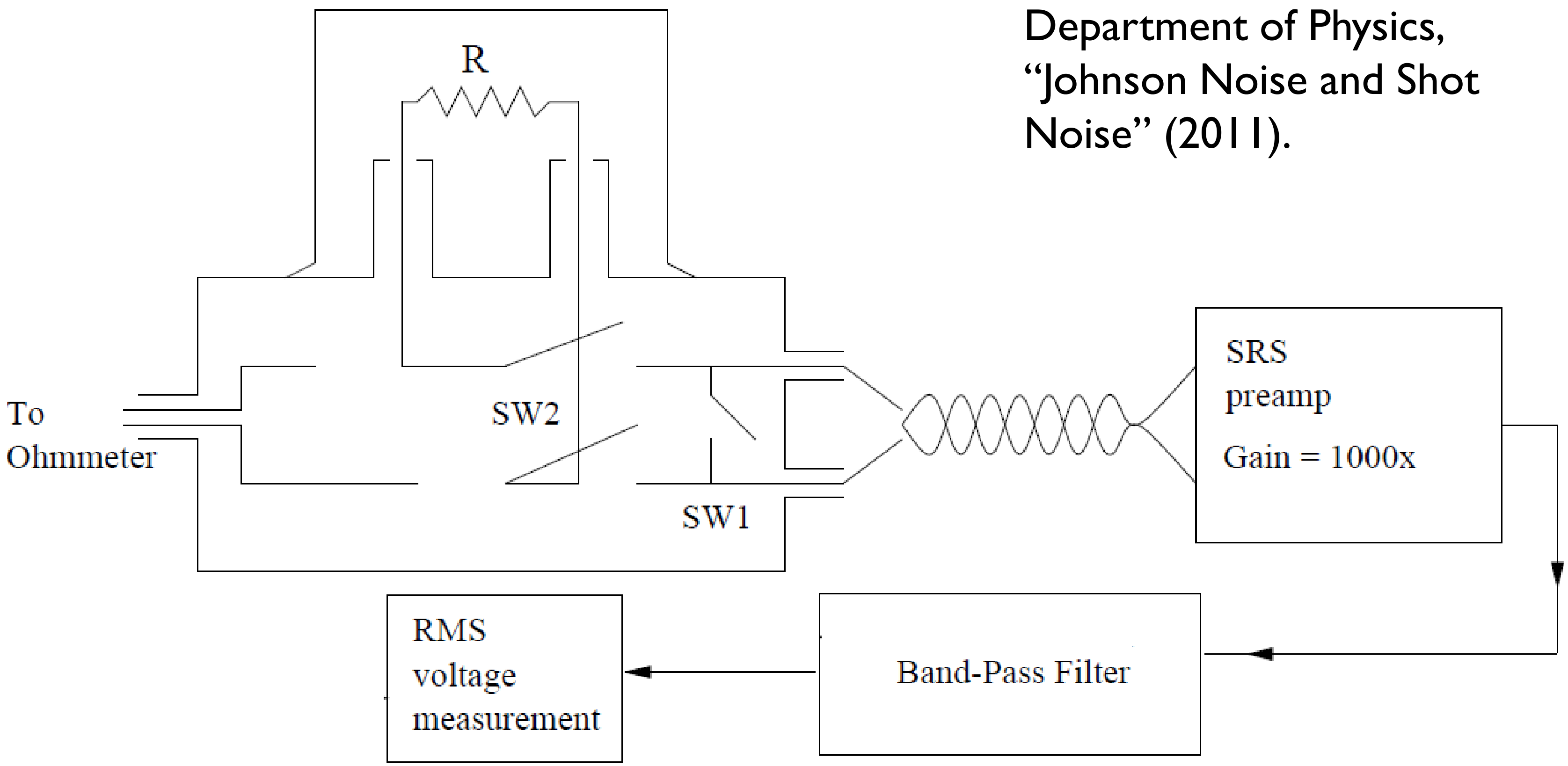
- Governing formula for Johnson-Nyquist noise:

$$\mathrm{d}\langle V^2 \rangle = 4kTR \frac{\mathrm{d}f}{1 + (2\pi f RC)^2}$$

Johnson Noise Setup

Adapted from MIT Department of Physics, "Johnson Noise and Shot Noise" (2011).



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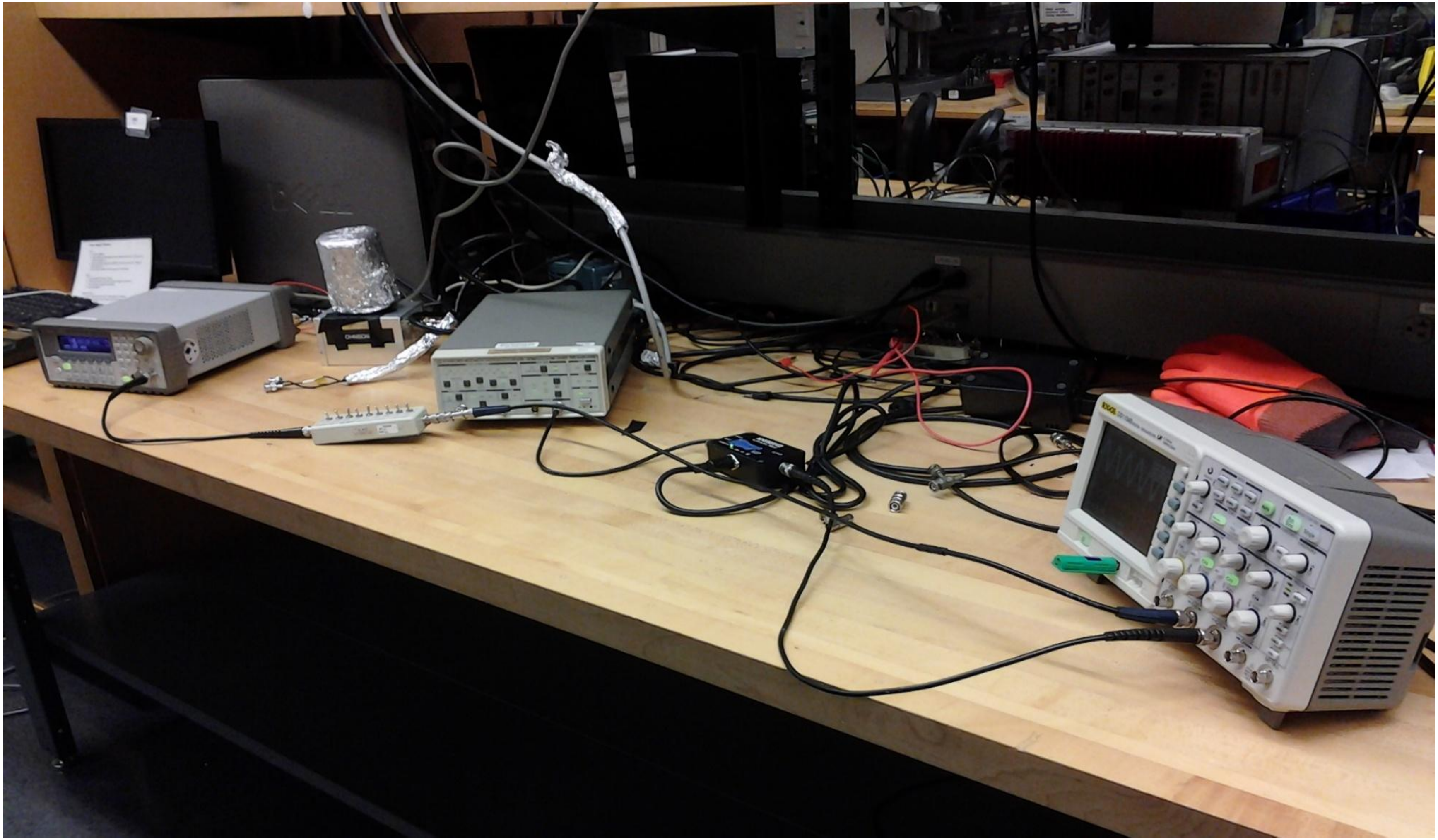

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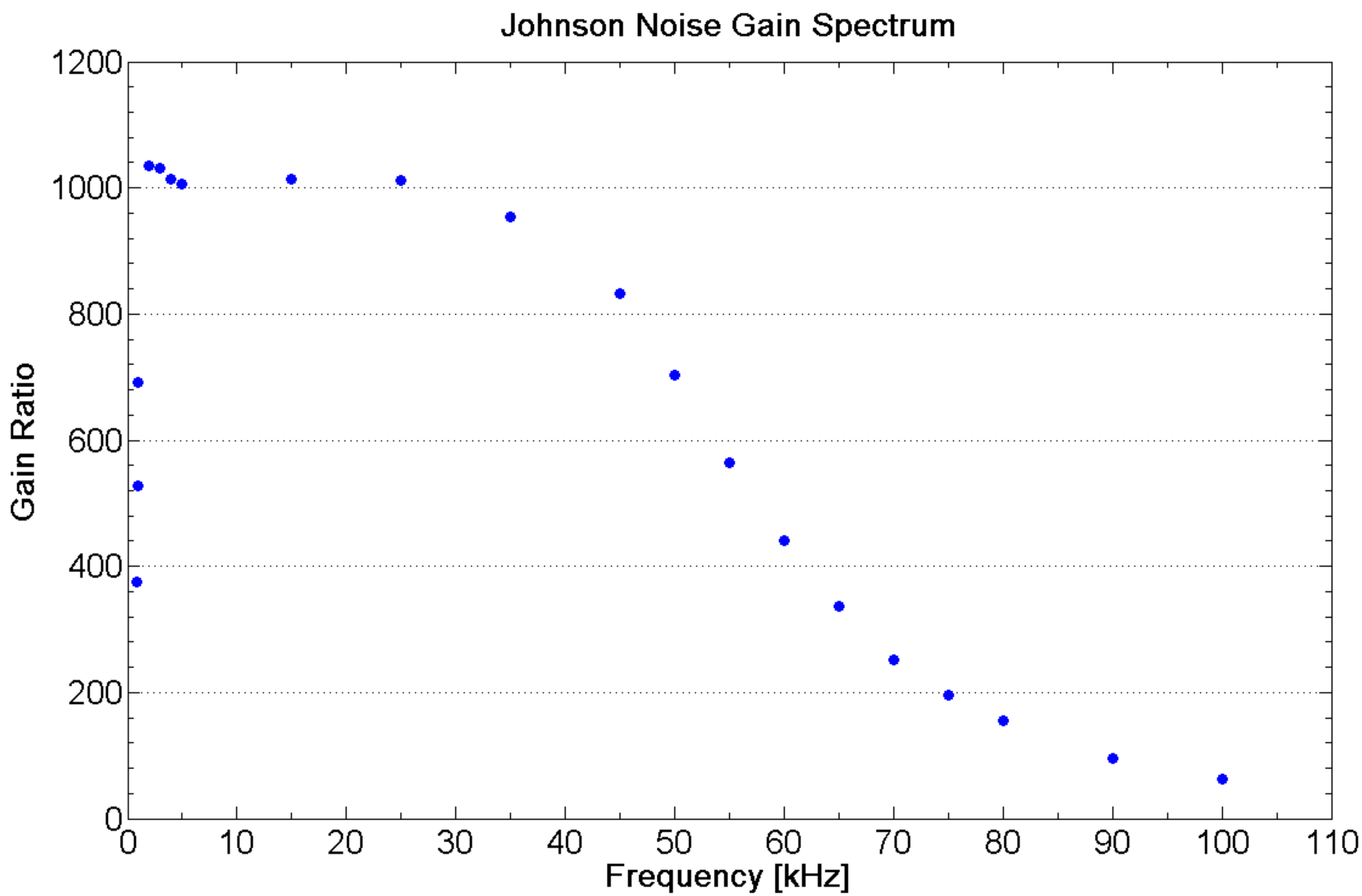
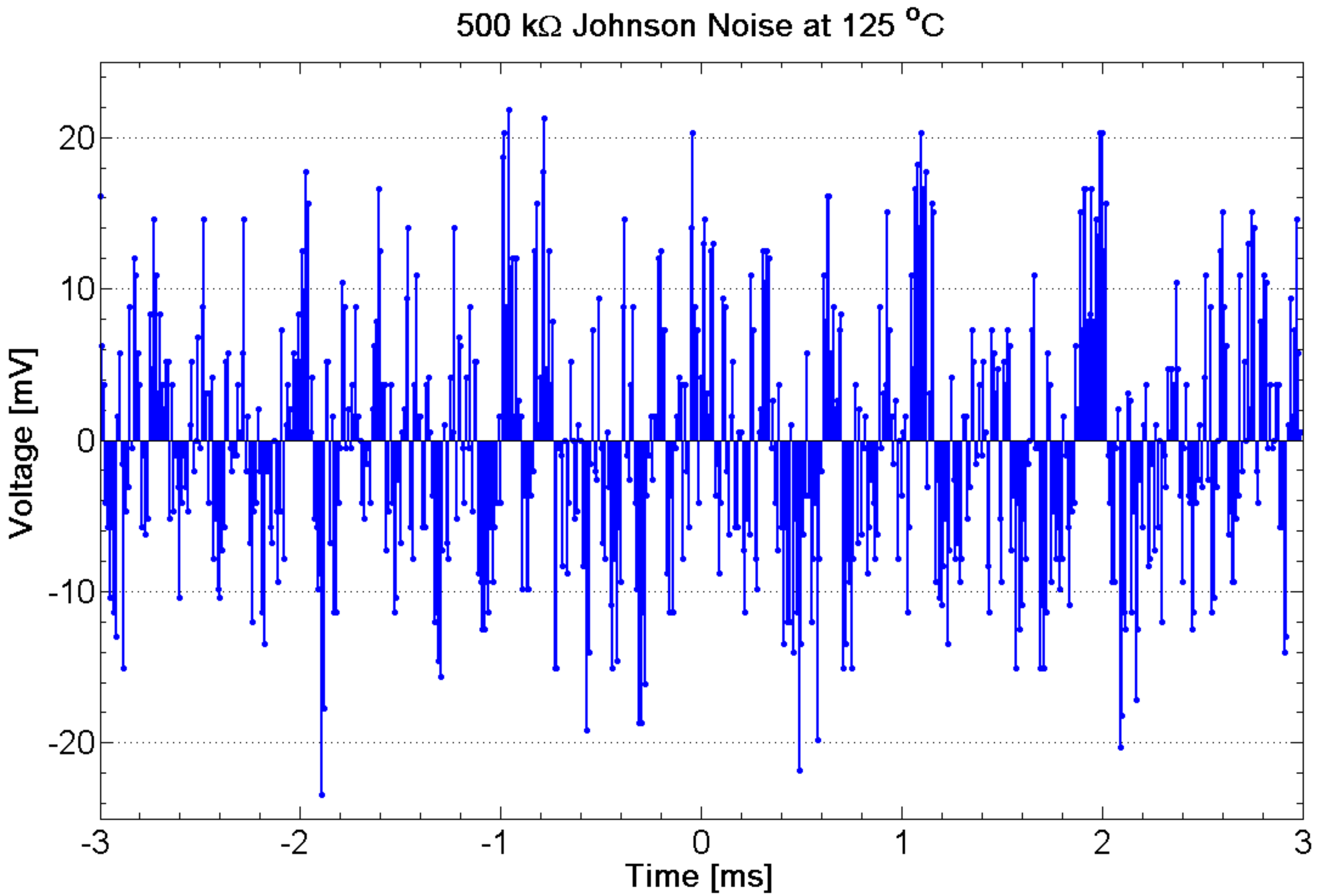
- Will measure RMS voltage through band pass filter (from ~1kHz to ~50kHz)
- Define gain ratio $g(f) = V_{out}(f)/V_{in}(f)$
- Integrate the Johnson noise with $G(R, C)$ integral

$$\langle V^2 \rangle = 4kTR \int_0^\infty \frac{g^2(f)}{1 + (2\pi f C R)^2} \mathrm{d}f$$

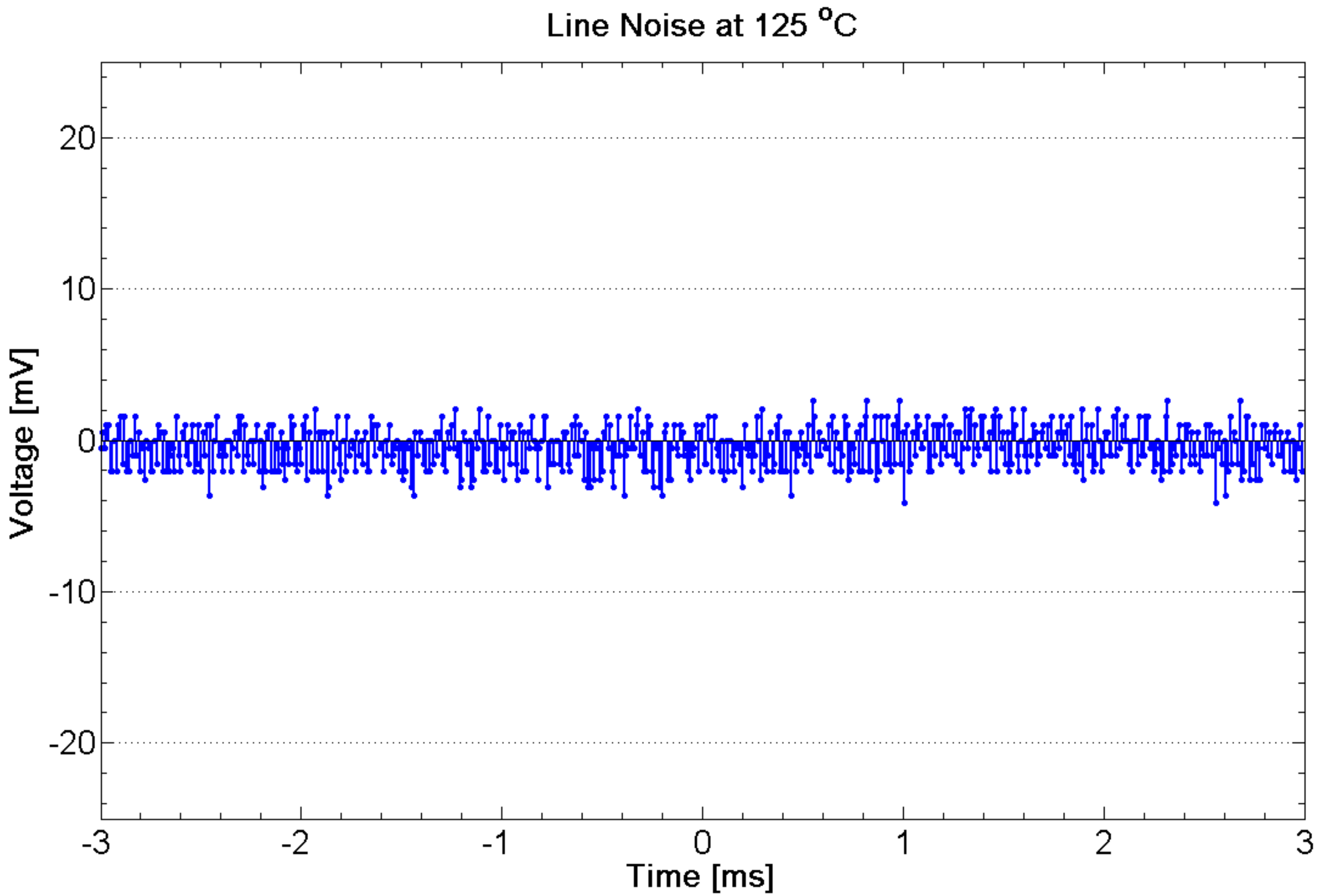
Gain and Band Calibration (cont.)



Measuring RMS Voltages



Measuring RMS Voltages (cont.)



Resistance Measurements

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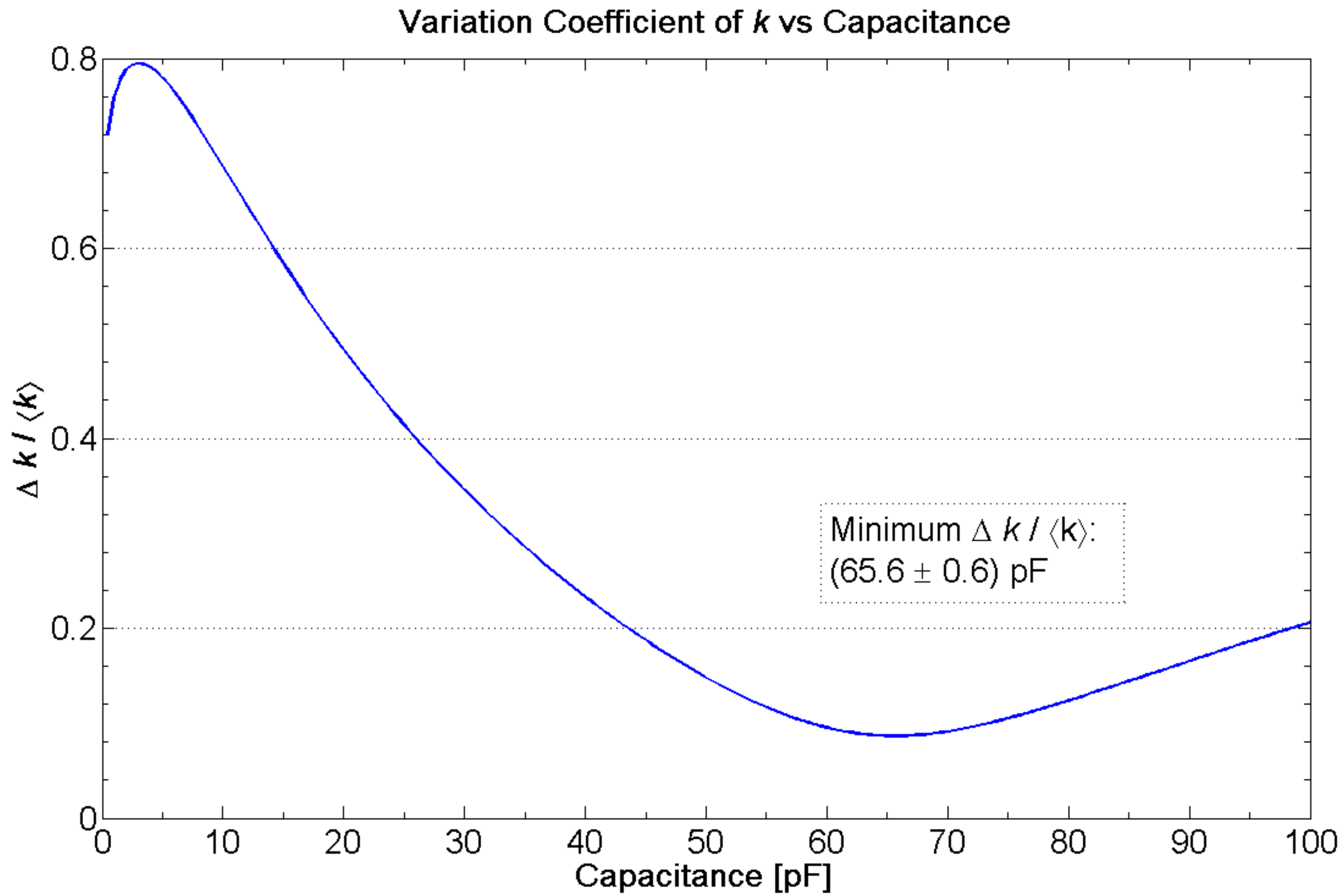
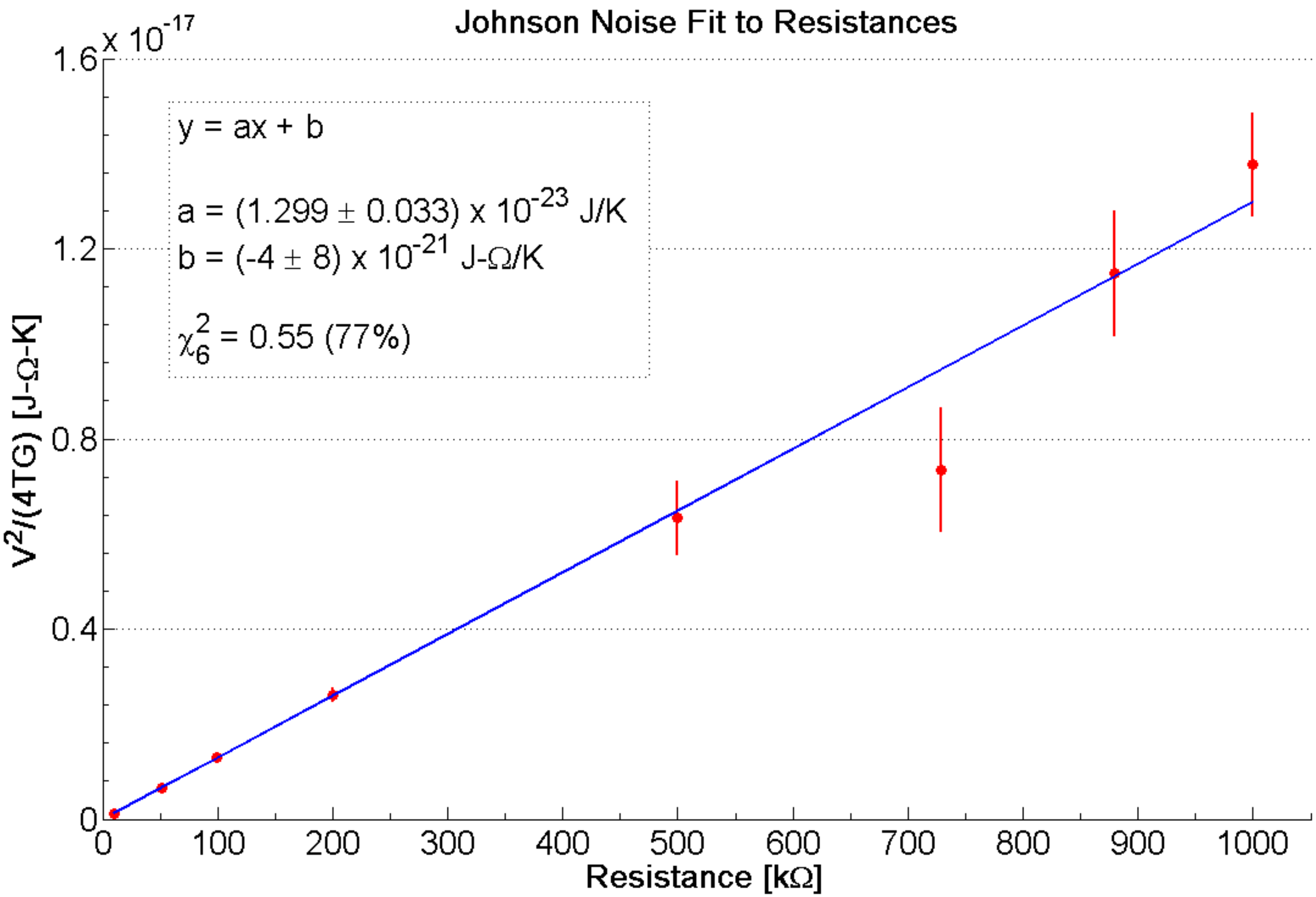
- Need to determine *C*

Determination of Capacitance



Determination of *k* with Resistances



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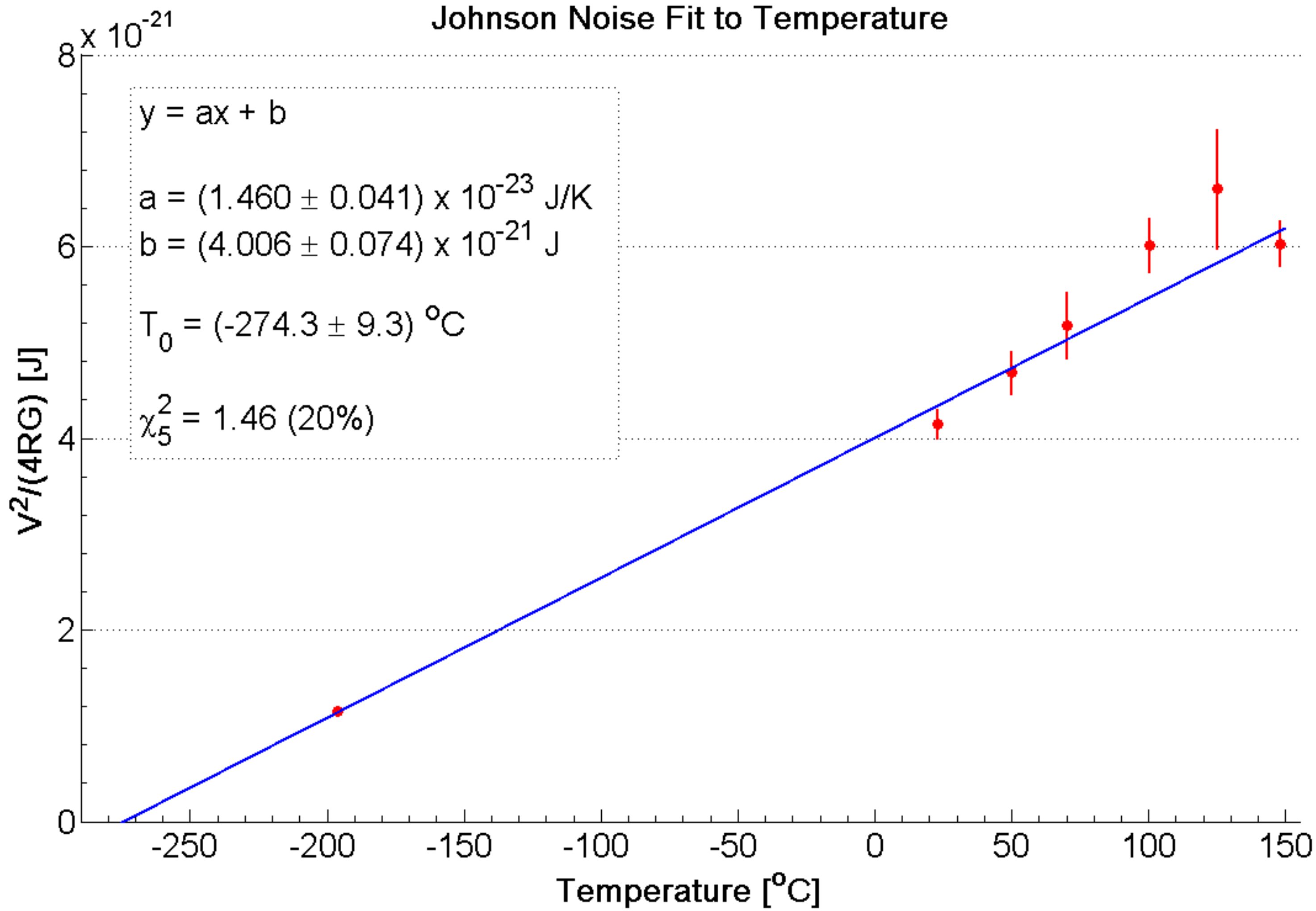
- T_c measured in Celsius: T_0 is absolute zero

k and T_0 with Temperature



Conclusions

- Best estimate on *k*
  - $(1.361 \pm 0.026_{rand.} \pm 0.081_{syst.}) \times 10^{-23}$ J/K
  - Correct value: 1.381×10^{-23} J/K (≈ 1.5% error)
- Determination of absolute zero
  - $T_0 = (-274.3 \pm 9.3)$ °C
  - Correct value: -273.15 °C (≈ 2.0% error)
- Verified existence and behavior of Johnson-Nyquist noise

Question and Answer