The Tragedy of the Commons, Livestock Cycles, and Sustainability [†]

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This paper revisits the tragedy of the commons and examines the conditions under which externalities contribute to livestock cycles. Using a stylized intertemporal model capturing the main characteristics of African livestock producers, we show that externalities magnify livestock cycles triggered by occasional droughts. This is true even when producers are fully rational. Two forces fuel such cycles: producers' concerns with consumption smoothing; and expected capital gains when demand for livestock products is inelastic. Implications regarding African livestock production are discussed.

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Since the works of Hardin (see Hardin and Baden, 1977) and others (e.g., Clark (1976), Berck (1979)), economists know that excessive exploitation of a resource occurs when individual users fail to internalize the negative externality they generate for other users. This situation can in principle be remedied by inducing users to internalize the externality they create, either directly by reallocating property rights, or indirectly through regulation and cooperation. But free riding cannot always be fully prevented. This paper revisits the effect of externalities a dynamic setting and examines their relationship with production cycles.

The common observation that livestock population goes through large swings in areas with extensive grazing on common pasture form the starting point for this work (e.g., Livingstone (1986), Sandford (1983)). These swings are wasteful and can be extremely disruptive, as they trigger famines among herders and farmers impoverished by the loss of animals (e.g, Sen (1981)). Swings in livestock population are customarily attributed to droughts. At the same time, many suspect that overgrazing makes high levels of livestock production unsustainable, thereby magnifying the effect of droughts on livestock. This paper formalizes these ideas and demonstrates that externalities increase the variance of livestock cycles otherwise triggered by droughts.

Although the paper is organized around the externalities that potentially arise from open or common access to pasture, we do not dispute that many herder societies have developed institutions to internalize externalities by combining informal cooperation and reciprocal arrangements, political control over grazing resources, and property rights over water points (Sanford (1983)). The point of this paper is that if these institutions and

regulations are unable to eliminate free-riding entirely, the resulting externalities exacerbate livestock cycles. Numerical simulation indeed indicate that even a little bit of externalities can be very damaging for efficiency and welfare.

We also investigate the effect that livestock markets may have on overgrazing and livestock cycles. In the West African semi-arid tropics, Fafchamps and Gavian (1996, 1997) have indeed shown that livestock markets, though responsive to large demand shocks, are not fully integrated spatially and that local prices vary much more that would be consistent with spatial arbitrage. In their study of livestock in Burkina Faso, Fafchamps, Udry, and Czukas (1996) show that sales of livestock in the wake of a drought are much smaller than anticipated. They speculate that the poor integration of livestock markets could account for the relatively small role that livestock play against income shocks. This hypothesis is investigated further in this paper.

The modeling approach used here departs from other works in several respects. Cycles have been shown to occur when producers are myopic or motivated by non-economic considerations (e.g., Dahl and Hjort (1976), Tacher (1983), Krummel, O'Neil and Mankin (1986)). We assume here that producers are individually rational and correctly anticipate the consequences of their actions (e.g., Berck and Perloff (1984)). Second, it is well known that stochastic livestock cycles can arise even in the absence of externalities (Jarvis (1974), Rosen, Murphy, and Scheinkman (1994), Mundlak and Huang (1996)). Such cycles are ultimately due to the existence of gestation lags in animal reproduction. Lags are not the emphasis here. We focus instead on externalities as an alternative mechanism by which cycles arise or are magnified. Third, to facilitates the analysis of cycles, we use a discrete time modeling framework, a formal departure from other works in continuous time (e.g., Berck (1979), Dasgupta and Heal (1979)).

The paper is organized as follows. In Section 1 we present background material on livestock raising in semi-arid Africa to illustrate how the tragedy of the commons plays out in practice. A stylized model of livestock raising with externalities and endogenous prices is introduced in Section 2. As it turns out, much can be learned about the relationship between externalities and livestock cycles from a careful study of the deterministic case. The properties of the steady state in the absence of production shocks are examined in detail in Section 3 where we demonstrate that externalities can lead to deterministic cycles. Close examination of these cycles reveals that two forces can induce producers to hold onto livestock even when they anticipate losing part of their herd: concerns with consumption smoothing; and expected capital gains when demand for livestock products is inelastic. Production shocks are then introduced in Section 4. We show that conditions that lead to deterministic cycles also raise the variance of production in the presence of stochastic production shocks, hence justifying *ex post* our detailed examination of the deterministic case. Numerical simulations illustrate how externalities and concerns with consumption smoothing magnify livestock cycles in the presence of production shocks. Conclusions and implications regarding African livestock producers are presented at the end.

Section 1. Livestock Raising in Semi-Arid Africa

Livestock raising is a major production activity in most semi-arid areas of Africa.[1] While milk is typically consumed by producers directly (e.g., McCabe (1987), p.289; Smith (1975), p.81; Loutan (1985), Bernus (1980)) or exchanged with villagers (e.g.,

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[1] For a description of livestock production in Africa, see for instance Sandford (1983), Monod (1975), Shapiro (1979), Livingstone (1986, 1991), Cossins and Upton (1988), Swift (1986), de Leeuw and de Haan (1983), White (1984), Frantz (1975).

Sandford (1983), Sutter (1987), pp.203-206), livestock for meat consumption goes prici-pally to urban centers. *Trypanosomiasis,* a livestock disease transmitted by the tse-tse fly, precludes livestock production in most humid areas of Sub-Saharan Africa. As a result, livestock raising is concentrated in semi-arid areas such as the Sahel, the Eastern African lowlands, and the like (e.g., Shapiro (1979), Staatz (1979), Sandford (1977), Jarvis and Erikson (1986)). Due to the difficulties inherent in transporting animals over extremely long distances, livestock markets remain poorly integrated (Fafchamps and Gavian (1996)). Livestock producers seldom limit their activities to animals; most of them cul-tivate food crops as well. This enables them to take advantage of the positive interac-tions between animals and crops -- i.e. manure as source of soil fertility and crop residues as animal feed (e.g., Sakurai (1995), Saha, Stroud, and Goube (undated)).

Common Access to Pasture

In semi-arid areas, common access to pasture is the rule (e.g., Sandford (1983), Upton (1986)). Access to pasture itself is often completely open. Control over water holes, however, may be in the hands of members of a certain ethnic group or lineage. Water holes *de facto* command access to neighboring pastures since there is a limit to the distance animals can walk without drinking.[2] Attempts at establishing privately or pub-licly run ranches in dry areas of West Africa and the Horn have generally failed. Priva-tizing access to pasture, when attempted, has only been successful in areas with more

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[2] There is disaggreement among experts regarding the extent to which *access* to water holes is restricted to certain users, even when the *control* over those holes is in the hands of a well defined group (e.g, Sandford (1983)). New boreholes put in place in recent years by African governments are usually open to all (e.g, Sandford (1983), Jarvis (1993), Monod (1975), de Leeuw and Tothill (1990), Scoones (1989), Oba and Lusigi (1987)). Preexisting water holes are more likely to be restricted to a group of users, but in certain areas access to them is completely open -- e.g., north of Gao and Timbuctu in Mali (conversation with Pierre Hiernaux, ILCA Range Ecologist). Even when access is restricted, enforcing property rights is problematic since water holes are often left unattended.

secure rainfall, like the Kenyan highlands or semi-humid areas of Zimbabwe. The uneven density of humid air currents in semi-arid tropics results in rainfall that is extremely variable non only over time but also across space. Pooling pasture resources is an effective insurance mechanism against localized shortages of rain and pasture (e.g., Sandford (1983), Nugent and Sanchez (1990, 1995)). In the foreseable future, common access to pasture is thus likely to remain the dominant form of livestock production in much of semi-arid Africa.

As is well known, in the absence of any control mechanism, common or open access to a productive resource leads to its overexploitation (e.g., Gordon (1954), Hardin and Baden (1977)). The reason is that individual users realize that, if they restrain their own use of the resource, it will benefit others, not them. If individual users were to interact over an extended period of time, however, they may be able to coordinate their actions to reduce overexploitation and increase their joint welfare (e.g., Runge (1981)). This argument has best been formalized as the folk theorem of repeated games (e.g, Fudenberg and Maskin (1986)). The question then is: have institutional mechanisms evolved that limits overexploitation of pasture resources in semi-arid Africa?

The available evidence on this issue is mixed. Early anthropological and historical accounts suggest that in previous centuries herder societies in the Sahel and elsewhere had evolved strong political institutions (e.g, Hopkins (1973)). These institutions seem to have played a role in keeping livestock pressure under control and ensuring cooperation among herders. Herders also used their political strength to restrict access to pasture by sedentary agricultural producers and, in some cases, to enslave them. The political influence of herders' groups weakened considerably during the colonial era, in part because they clashed with colonial authorities and were actively fought. With few

exceptions, independent African states have pursued similar policies. By loosing their political supremacy, herders have also largely lost the ability to limit access to pasture. This may have undermined their ability to enforce cooperative institutions and agreements aimed at mitigating the natural tendency toward the overexploitation of open pasture.[3]

Overgrazing and Cycles

Livestock raising in semi-arid areas of Africa has long been dominated by cycles of drought, range degradation, destocking of animals, range recovery, and restocking of animals, followed by a new cycle of drought and recovery (e.g., Livingstone (1991), Franke (1982), Toulmin (1994), Scoones (1989), p.14; Staatz (1979), pp.31-34; Shapiro (1979), pp.160 & 368).[4] The processes at work in these cycles are well known and have been discussed extensively elsewhere (e.g, Sandford (1983), Jarvis (1993), Livingstone (1986, 1991), Swift (1986), Cossins and Upton (1988), Oba and Lusigi (1987)). It is generally agreed that the overexploitation of the available pasture is at the root of livestock cycles, and that this overexploitation -- or overgrazing -- is the result of common or open

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[3] Today many African states, weakened economically, face difficulties in containing herders' desire to dispose of political institutions at their service. Clashes involving the Twaregs in Niger and Mali, the Moors in Mali and Mauritania, the Tubus in Chad, the Afars in Ethiopia, Erytrea, and Djibouti, the Nubas and others in Sudan, and various factions in Somalia may all be symptoms of the fact that herders feel a deep need for political institutions that restrict access to pasture. Of course, the temptation is always there to transfer the burden of restricted access onto a weaker group, whether sedentary farmers or other herder groups -- as in-fighting among herders in Chad, Somalia and Sudan has shown. The fact that herders have become increasingly vocal over the last decades, however, suggests that the present institutional setup is not able to achieve the coordination required to reduce livestock pressure. One interpretation of herder activism is that herders believe that coordination and enforcement cannot be achieved without some measure of political control.

[4] Following the dramatic Sahelian drought of 1974, it was suggested by some (e.g., Wade (1974)) that overgrazing resulted in permanent damage to the range and that the desertification of Africa was to be feared. Rapid herd recovery in the second half of the 1970's, followed by a new drought in 1984 and a new recovery in the late 1980's have reduced the fears that the successive collapses of the West African livestock economy are irreversible (e.g., Sandford (1983), Jarvis (1993), Warren and Khogali (1992)).

access (e.g., Sandford (1983), Jarvis (1993), Oba and Lusigi (1987), Turton (1977), Lyne and Nieuwoudt (1990)). Although the timing of cycles depends on exogenous rainfall shocks -- range degradation is normally triggered by two consecutive years of low rainfall following a period of steady livestock recovery (e.g., Solod (1990)) -- it fundamentally results from the accumulation of animals beyond the carrying capacity of the range (e.g., Sandford (1983), Turton (1977), de Leeuw and Tothill (1990)).[5]

Whatever the reason, these cycles are massive: some authors cite figures as high as 50 to 80% of cattle being 'lost' during drought years;[6] the reported numbers are somewhat smaller for sheep and goats, but still large -- e.g., 30% (e.g., Livingstone (1991), Franke (1982), Toulmin (1994), Scoones (1989), p.14; Staatz (1979), pp.31-34; Shapiro (1979), pp.160 & 368). These losses lead to entitlement failures and foster famines. Sen (1981), for instance, documents how herders who lost their livestock were among the worst hit during the 1973 Ethiopian famine. Farmers who rely on livestock as a form of precautionary saving are also adversely affected. Livestock cycles in Sub-Saharan Africa thus play a determinant role in the effectiveness of the risk coping strategies of the rural poor. To make these strategies more effective, a proper understanding of the reasons for the amplitude of livestock cycles is essential. There is general agreement that livestock cycles are due to the overexploitation of pasture resources, but the effect that various exogenous factors have on these cycles is not well understood. The objectives of this paper is to devise a framework in which the effect of these factors can be analyzed.

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[5] The effect of epidemics on animals may also be compounded by drought (e.g., de Leeuw and de Haan (1983)).

[6] These numbers must nevertheless be interpreted with caution because they may include physical loss and as well as distress sales.

Section 2. A Model of Livestock Raising with Externalities

We begin by constructing a model that represents, in a stylized fashion, producers' decisions about investment and production when faced with common pasture. Market conditions, alternative sources of income, and the possibility of negative returns to capital are all captured in the model. Prices are endogenous. We first define the production technology, individual preferences, and market conditions. We then describe how common access is parameterized so as to separate the effect of population pressure from those of institutions governing access to a common resource. Proofs of all the propositions are presented in Appendix A.

We consider a large but fixed population H of infinitely lived producers who share a given pasture area. Producers all have identical preferences, technology and initial capital stock.[7] Total pasture is assumed equally divided among groups of M herders who share pasture among themselves but not with outsiders. Private pasture ownership arises when $M = 1$; setting $M = H = \infty$ equals open access. Intermediate values of M correspond to common access with partial internalization of externalities through imperfect monitoring and enforcement institutions. To capture the long gestation lags involved in livestock production (e.g., Rosen (1987), Rosen, Murphy and Scheinkman (1994)), producers' decisions are modeled in discrete time. This choice also facilitates the study of steady state stability and random cycles that constitute the focus of this paper.

Each herder $i \in H$ at time t keeps livestock, denoted $L_{i,t}$, which can be bought or sold at market price p_t. Variable $L_{i,t}$ is regarded as an index of livestock body mass, corrected for variations in animal weight and meat quality.[8] Variable p_t is thus the price

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[7] We abstract here from the issue of heterogeneity among producers (see Baland and Platteau (1996), Dayton-Johnson and Bardhan (1996), Sakurai (1995), Dercon (forthcoming)).

of livestock body mass, i.e., meat. Producers collectively face a constant downward slop-ing demand schedule, meant to represent exogenous demand for livestock products (e.g., Staatz (1979), Fafchamps and Gavian (1997)). Producers behave as price takers.

The physical rate of return to one unit of livestock, net of depreciation and variable costs, is denoted α_t. Non-marketed forms of output are discussed at the end of the sec-tion. The combined value of output and capital stock at the beginning of period t is $p_t(1+\alpha_t)L_{i,t}$. Physical returns to livestock depend on the total number of animals sharing the available pasture. Let $L_{a,t}$ denote the average number of animals per unit of pasture area, i.e.:

$$L_{a,t} \equiv \frac{1}{M} \sum_{i \in M} L_{i,t}$$

Competition for pasture means that $\alpha_t = \alpha(L_{a,t})$, with $\alpha'(L_{a,t}) < 0$ for all $L_{a,t}$: the more animals, the less pasture per animal, and the less weight gain and offspring livestock pro-duce. Function $\alpha(.)$ is taken to be bounded, continuous, and continuously differentiable.[9] For the moment, $\alpha(.)$ is taken to be deterministic; production shocks are introduced in Section 4. We also postulate that $\alpha''(.) \leq 0$ or, at least, that it is small.[10]

Since $\alpha(.)$ is a monotonically decreasing function, returns to livestock are single-peaked, i.e. there exists a unique $\hat{L}$ such that $1+\alpha(\hat{L})+\alpha'(\hat{L})\hat{L} = 0$. Value $\hat{L}$ is the max-imum sustainable livestock population achievable with production technology $\alpha(.)$. We

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[8] For tractability reasons, we abstract from issues of herd composition, such as the relative proportions of reproductive animals in the herd.

[9] Function $\alpha(.)$ can be understood as a reduced, discrete time version of a prey-predator model involving pasture -- the prey -- and livestock -- the predator. The detailed dynamics of pasture herbage growth are ignored here.

[10] A large positive α'' would mean that returns to livestock decrease at a rapidly decreasing rate. In the case of exploitation of common pasture, returns are more likely to decrease at a constant or increasing rate due to overcrowding. Consequently, α'' can safely be assumed non-positive or, at most, positive but negligible. When the production function is quadratic (see infra), $\alpha'' = 0$.

define the absolute upper bound on production as the point $\bar{L}$ at which $\alpha(\bar{L}) = -1$: if all producers hold $\bar{L}$ units of livestock, the overuse of pasture is so great that all livestock starve. With $\alpha''(.) \leq 0$, $\bar{L}$ is finite. A simple functional form that satisfies all the above requirements is $\alpha(L) = \omega(\bar{L} - L) - 1$, with ω between $1/\bar{L}$ and $4/\bar{L}$.[11] In this case, the maximum sustainable livestock population $\hat{L}$ is equal to $\bar{L}/2$.

The marginal return to livestock perceived by an individual producer i is:

$$1 + \alpha(L_{a,t}) + \alpha'(L_{a,t})\frac{dL_{a,t}}{dL_{i,t}} \tag{1}$$

In a symmetrical Cournot/Nash equilibrium where producers treat the production level of others as independent of their own, $\dfrac{dL_{a,t}}{dL_{i,t}}$ boils down to the effect that producer i has on the average number of livestock heads per unit of pasture $L_{a,t}$, i.e., to $\dfrac{L_{i,t}}{M}$. In this case, externalities, and thus incentives to free ride increase with the size of the group M; efficiency requires private ownership. Repeated interaction between herders may nevertheless enable them to set up common property institutions and encourage produc-ers to internalize the effect that their use of pasture resources has on others. External institutions such as laws and courts may also help mitigate free riding. We capture these institutional arrangements in a stylized manner by defining a parameter N such that per-ceived marginal returns to livestock are equal to:

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[11] In the absence of human intervention, the average livestock population would in this case evolve according to the well known quadratic map $L_{t+1} = \omega(\bar{L} - L_t)L_t$. When the value of ω is progressively increased from $3/\bar{L}$ to $4/\bar{L}$, this map has been shown to bifurcate between cycles of different lengths, including cycles of infinite periodicity (chaos) and cycles of periodicity three. See for instance Grandmont (1988), Baumol and Benhabib (1989), and the references cited therein. This map can thus reproduce, in a discrete time framework, the cyclic behavior common in predator-prey models, pasture being the prey and livestock the predator in this case (e.g., Hirsch and Smale (1974), chapter 12).

$$1 + \alpha(L_{a,t}) + \alpha'(L_{a,t})\frac{L_{i,t}}{N} \tag{2}$$

Parameter N, which by construction must satisfy $1 \leq N \leq M$, measures the success with which institutions induce producers to internalize externalities: $N = 1$ defines complete internalization, while $N = M$ characterizes no internalization. Values of N between 1 and M represent intermediate levels of institutional failure. Varying parameter N thus makes it possible to investigate, in a stylized manner, the effect that various degrees of common access and institutional efficiency have on livestock dynamics.

Producers maximize a discounted, additively separable intertemporal utility function $U(c)$ defined over consumption c. The instantaneous utility function $U(.)$ is assumed to have the usual smoothness properties: it is bounded, continuous, strictly increasing in consumption, concave, continuously differentiable, and $\lim_{c \to \infty} U(c) = -\infty$.[12] Producers have an alternative source of income y that is constant over time in real terms; y is assumed sufficient to allow accumulation from zero wealth -- i.e. $U(y-e) > -\infty$ for some $e \geq 0$. The discount factor β, common to all herders, lies between 0 and 1. Livestock are the only asset at producers' disposal. The existence of financial markets can, however, be approximated by letting the intertemporal elasticity of substitution go to infinity. The producer's optimization problem then reduces to the maximization of discounted profits and the discount factor β can be interpreted as $\frac{1}{1+r}$ where r is the prevailing interest rate r. In this case, arbitrage among assets requires that perceived returns to livestock equal r.

We are interested in understanding how droughts and externalities affect the cyclic behavior of the livestock population $L_{a,t}$. It turns out, quite surprisingly, that much can

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[12] This condition ensures that it is never optimal for livestock producers to consume nothing, i.e., to starve.

be learned about the relationship between externalities and livestock cycles from a careful study of the deterministic case. This is because the presence of externalities potentially leads to deterministic equilibrium cycles. Although we do not believe that deterministic cycles are relevant to explain movements in African livestock populations, a formal examination of these cycles provides useful insights as to why, in equilibrium, rational livestock producers would hang onto livestock even when they know for certain that they will lose some of their animals. Similar issues arise just before droughts when the reconstituted livestock population exceeds the carrying capacity of the range and it is clear to everyone that low rainfall will unmistakably trigger massive loss of animals (e.g., Livingstone (1986, 1991), Sandford (1983)). Production risk is introduced in Section 4 where we show that all the factors that favor the emergence of cycles in the deterministic case also magnify drought triggered cycles in the stochastic case.

Section 3. Cycles in the Absence of Production Shocks

In this section we examine the long-term equilibrium behavior of the livestock population in the absence of shocks. Individual optimization and competitive equilibrium are presented first and the general properties of the solution path are derived. The stability of the steady state is studied next. Non-marketed outputs like prestige and lifestyle are discussed at the end of the section.

Individual Optimization

The optimization problem of producer *i* can be written:

$$\underset{\{c_{i,t},\,L_{i,t}\}}{Max} \sum_{t=0}^{\infty} \beta^t U(c_{i,t})$$

$$s.t. \ \ 0 \le c_{i,t} \le y + p_t((1 + \alpha(L_{a,t-1}))L_{i,t-1}) \tag{3}$$

where marginal returns to livestock are computed according to equation (2). Inequality

(3) is the usual budget constraint. A solution to the above optimization problem is a plan of consumption and livestock holdings $\{(c_{i,t}, L_{i,t})\}$ that maximizes utility subject to equation (2), the budget constraint (3), a path of prices $\{p_t\}$, and a path of livestock holdings of other producers in their group, $\{\sum_{j \neq i} L_{j,t}\}$. Let a solution be denoted $\{(c_{i,t}^o, L_{i,t}^o)\}$; it implicitly depends on $\{p_t\}$ and $\{\sum_{j \neq i} L_{j,t}\}$. The following proposition establishes that the producer's optimization problem is well behaved.

Proposition 1:

(1) The sequence $\{(c_{i,t}^o, L_{i,t}^o)\}_{t=1}^{\infty}$ with

$$0 < c_{i,t}^o = y + p_t((1 + \alpha(L_{a,t-1}^o))L_{t-1}^o - L_{i,t}^o)$$
$$\text{and } 0 < L_{i,t}^o < M\bar{L} - L_t^i$$

is optimal if it satisfies the Euler equations

$$-p_t U'[y + p_t((1 + \alpha(L_{a,t-1}^o)L_{i,t-1}^o) - L_{i,t}^o)]$$
$$+ \beta p_{t+1} U'[y + p_t((1 + \alpha(L_{a,t}^o))L_{i,t}^o) - L_{i,t+1}^o)]$$
$$(1 + \alpha(L_{a,t}^o) + \alpha'(L_{a,t}^o)\frac{L_{i,t}^o}{N}) = 0 \qquad (4)$$

at all t, and the transversality condition

$$\lim_{t \to \infty} \beta^t U(c_{i,t}^o)c_{i,t}^o = 0. \qquad (5)$$

(2) If $L_{a,t} = 0$ whenever $L_{i,t} = 0$, the above Euler equations, together with the transversality condition, fully characterize the optimal plan.

Competitive Equilibrium

Let $L_t \equiv \frac{1}{H}\sum_{i \in H} L_{i,t}$ be the average livestock holding in the economy at the end of period t. The livestock price p_t depends on aggregate or, equivalently, average net sales of livestock during the period:

$$p_t = p((1 + \alpha(L_{t-1}))L_{t-1} - L_t) \tag{6}$$

with $\infty > p(x) > 0$ for all finite x. Demand is downward sloping, implying that p_t decreases monotonically with net sales -- i.e. $p'(x) < 0$.

A competitive equilibrium can be defined as a path of consumption and capital for each producer $i \in H$ such that equation (6) is satisfied in all periods and all producers are individually maximizing their utility. Since all producers and all groups are identical, we restrict our attention to symmetric competitive equilibria. Along a symmetric equilibrium, $c_{i,t}^o = c_t$ and $L_{i,t}^o = L_t$, for all $i \neq j$, and all t. Returns to livestock can be written $\alpha(L_t)$. Replacing for $L_{a,t}$ and p_t in equation (4), any symmetric equilibrium must satisfy the following Euler equation:

$$-U'[y + p(L_{t-1}(1 + \alpha(L_{t-1})) - L_t)((1 + \alpha(L_{t-1}))L_{t-1}) - L_t)]p(L_{t-1}(1 + \alpha(L_{t-1})) - L_t)$$
$$+ \beta U'[y + p(L_t(1 + \alpha(L_t)) - L_{t+1})((1 + \alpha(L_t))L_t) - L_{t+1})]$$
$$p(L_t(1 + \alpha(L_t)) - L_{t+1})(1 + \alpha(L_t) + \alpha'(L_t)\frac{L_t}{N}) = 0 \tag{7}$$

Together with the initial endowment L_0 and the transversality condition (5), equation (7) fully describes the dynamic behavior of the economy.

The Steady State

The steady state of the economy is the constant level of livestock that satisfies equation (7). Straightforward manipulation yields:

$$\frac{1}{\beta} = 1 + \alpha(L^*) + \alpha'(L^*)\frac{L^*}{N} \tag{8}$$

Equation (8) shows that, along the steady state, perceived returns to livestock must equal producers' rate of impatience. Steady state livestock holdings do not depend on demand conditions, consumption preferences, or other sources of income. They are only a function of the rate of time preference β, the parameter N which measures externalities, and

function $\alpha(.)$ which determines physical returns to livestock. When $1 + \alpha(L)$ takes the form $\omega(\bar{L} - L)$, then $L^* = \dfrac{N}{N+1}(\bar{L} - \dfrac{1}{\omega\beta})$, which shows that L^* also depends on the level of population pressure $\bar{L}$. This leads to the following proposition:

Proposition 2:

(1) The steady state L^* exists and is unique. A sufficient condition for L^* to be in the interior of the domain of L is that $1 + \alpha(0) > 1/\beta$. If this condition is not satisfied, the steady state is zero.

(2) L^* increases with β and N.

(3) When $1 + \alpha(L)$ takes the form $\omega(\bar{L} - L)$, then L^* increases with ω.

(4) The social marginal return on livestock $1 + \alpha + \alpha'L^*$ is smaller than the privately perceived marginal return $1 + \alpha + \alpha'L^*/N$ whenever $N > 1$. The higher the value of N, the lower the social marginal return.

(5) There exist values of β and N above which the social marginal return is negative at the steady state.

Proposition 2 demonstrates that, as expected, steady state livestock holdings increase monotonically with the extent to which producers fail to internalize externalities. The model thus reproduces the well known tragedy of the commons (Gordon (1954), Hardin and Baden (1977)). Proposition 2 further suggests that overexploitation can be so bad as to induce a *negative* social marginal return to livestock. The perceived private marginal return, however, remains positive.

The steady state livestock population is shown to be higher when producers are patient. For β sufficiently low, the only steady state is shutdown: producers consume all

their livestock in finite time and leave the range undisturbed from then on. These results run somewhat contrary to the commonly held view that producers with a short time horizon exhaust natural resources more rapidly (e.g., Hardin and Baden (1977), Low (1980), Hubbard (1982)). The reason is that producers must invest in livestock to exploit common pasture. Impatient producers cause less damage to pasture because they are unwilling to delay consumption and do not accumulate as much livestock.[13] A corrolary of the above is that one should expect less overexploitation of the commons when producers have a short time horizon, as would probably be the case when they are poor and unable to delay consumption. Overgrazing may increase when producers get more prosperous and presumably more capable of accumulating capital in the form of livestock. By the same token, granting cheap credit to producers may encourage the purchase of livestock and result in a worsening of the commons.

When α, the return to livestock, is high -- i.e. when ω is large -- the steady state livestock population is large too. This implies that increasing the rate at which livestock can grow, say, by drilling boreholes or by extending veterinary services, without improving the availability of pasture[14] -- i.e., without changing $\bar{L}$ and $\hat{L}$ -- is likely to exacerbate overexploitation of the common resource. This observation has been made by numerous authors (e.g., Sandford (1983), Monod (1975), pp.167-168).

Stability of the Steady State

Knowing that an economy has a steady state does not describe its equilibrium

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[13] Transposed to fisheries, this result means that sufficiently impatient fisherman will run down their production capital -- boats, nets -- before they can exhaust the fish stock.

[14] Boreholes can improve the availability of pasture if they are drilled in areas with good pasture but no source of livestock water.

behavior if this steady state is not stable. We now show that conditions exist under which the steady state of the deterministic model is not stable. Livestock assets then follow a limit cycle. To analyze the stability of the steady state, we form the characteristic polynomial of equation (7).[15] Let $g(L_{t+1}, L_t, L_{t-1}) = 0$ be shorthand for equation (7). Form the characteristic matrix by totally differentiating g:

$$\begin{bmatrix} -\dfrac{\partial g/\partial L_t}{\partial g/\partial L_{t+1}} & -\dfrac{\partial g/\partial L_{t-1}}{\partial g/\partial L_{t+1}} \\ 1 & 0 \end{bmatrix} \tag{9}$$

Provided that matrix (9) is well defined and non-singular, it can be used to form the characteristic polynomial $P(\lambda)$. The stability of the steady state can then be studied by examining the two roots of $P(\lambda)$. If the steady state is stable, they correspond to the stable and unstable manifolds going through L^*. The speed of convergence depends on the absolute magnitude of the stable root: a larger stable in absolute value means that convergence is slower. In the stable root is negative, convergence is oscillatory. If both roots are larger than one in absolute value, the steady state is unstable. To summarize:

Proposition 3:

(1) Except for a set of parameter configurations of measure zero, the characteristic matrix exists and is non-singular.

(2) A steady state is locally stable if one of the roots of $P(\lambda)$, called the stable root, is smaller than one in absolute value. It is unstable if both roots are greater than one in absolute value.

(3) Convergence to a stable steady state is faster if the stable root is small in absolute value; it is oscillatory if the stable root is negative.

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[15] Judd (1990)'s approach based on the policy function leads to similar formulas and identical results.

Although $P(\lambda)$ is a complex algebraic expression, it is quadratic in λ and it tends to $+\infty$ when λ tends to $+$ or $-\infty$. Its behavior can thus be analyzed by studying the value it takes at $\lambda=1$ and $\lambda=-1$. If $P(\lambda)$ is negative at both points, both roots are larger than one in absolute value, no stable manifold exists leading to L^*, and the steady state is unstable. If $P(\lambda)$ is positive at either $\lambda=1$ or $\lambda=-1$ but negative at the other, one of its roots is smaller than one in absolute value, a stable manifold exists, and the steady state is stable. Parameter configurations for which $P(\lambda)$ is zero at either $\lambda=1$ or -1 are bifurcation points. This leads to the following proposition:

Proposition 4:

(1) $P(1) < 0$ and $P(-1) < 0$ are both necessary conditions for the instability of an interior steady state.

(2) There exist a set of parameter configurations for which both $P(-1)$ and $P(1) < 0$. For parameter configurations within that set, the steady state is unstable. The set has full Lebesgue measure.

Having established that there are parameter values at which the steady state is unstable and, consequently, that livestock assets may cycle in the long run, we now examine how model parameters affect the stability of the steady state. This is done by investigating how parameters influence the characteristic polynomial $P(\lambda)$ at $\lambda=1$ and $\lambda=-1$. Let Ψ denote the coefficient of relative risk aversion $-\dfrac{U''c^*}{U'}$ evaluated at the steady state; Ψ rises whith producers' desire to smooth consumption over time. Parameter s is defined as the share of livestock income $p\alpha L^*$ in total steady state consumption c^*, and $\varepsilon > 0$ is the elasticity of consumption demand for livestock at the steady state $p\alpha L^*$. With these definitions, the following proposition can be derived:

Proposition 5:

(1) When production externalities are absent (i.e., $N=1$), the steady state is stable.

(2) For the steady state to be unstable, L^* must be greater than the maximum sustainable herd, $\hat{L}$; β and N must be large (in a sense made precise in Appendix A); and $\frac{\Psi s - 1}{\Psi s}$ must be smaller than ε.

(3) Instability of the steady state arises either if (a) $\varepsilon > 1$ and Ψ and s are sufficiently large; or if (b) $\varepsilon < 1$ and Ψ and s are sufficiently small. (Precise conditions are given in Appendix A.)

The first part of proposition 5 indicates that deterministic production cycles can only occur in the presence of externalities. The second part of the proposition suggests that deterministic cycles can only arise when externalities are large and producers are patient. The instability of an interior steady state also requires that the demand elasticity be sufficiently high (high ε), relative to intertemporal substitutability Ψ and the share s of livestock income in total steady state consumption. The stated condition holds for many reasonable parameter values; when $\Psi < 1$, it is satisfied for all demand elasticities since, by definition, $s \leq 1$ always.

The role of demand elasticity ε in the third part brings out the complex relationship between livestock markets and deterministic cycles. The intuition behind this relationship can be illustrated as follows. High externalities induce producers to accumulate too many animals. Comes a point when physical returns to livestock become negative. Producers then realize that, if they do not sell livestock right away, they will lose some of their animals. Normally, this realization should induce producers to liquidate livestock, thereby bringing the economy back to the steady state. Two distinct forces may

nevertheless be sufficiently strong to induce producers to hold onto their animals in spite of the anticipated loss of physical capital, thereby keeping the economy away from the steady state. The first of these forces is producers' desire to smooth consumption; the second is price effects. The interaction between these forces is what gives rise to cycles.

Consider first the consumption smoothing motive. Excess sales today would indeed imply the need to rebuild one's herd -- and thus to buy livestock -- tomorrow. Producers who are highly dependent on livestock income to finance consumption may choose to hold onto their animals, knowing that they will lose some, rather than face the prospect of having insufficient livestock income to cover their future consumption needs. Producers then overaccumulate to ensure that they have enough animals for minimal consumption in lean years (e.g., Sandford (1983), Livingstone (1986)). It is the absence of alternative assets and income opportunities that leads to cycles; if producers could sell their livestock and save the proceeds instead of incurring a capital loss, they would do so and a cycle would not arise.[16]

The second of the two forces is driven by the recognition that, if demand is inelastic, total producer revenue is higher when supply is low and prices high. If producers accumulate livestock to the point where physical returns become negative, some producers will initially seek to liquidate their animals. This will not only drive the current price down but also raise the future price in anticipation of reduced supply. If demand is sufficiently inelastic, and hence the price difference large enough, the anticipated price gain more than compensates the physical loss. Producers then want to hold onto their

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[16] Note that, in this case, livestock play a buffer role against income fluctuations, but this buffer role manifests itself not by sales of livestock in bad times, but rather by a physical loss of animals, in line with much of the anthropology literature on herders' "love affair" with cattle (e.g., Monod (1975)), and in agreement with Fafchamps, Udry and Czukas (1996)'s results.

animals even though they realize their will lose some of them. This mechanism is what makes the steady state unstable. Cycles driven by the price motive may arise even in the presence of perfect capital markets: producers hold onto livestock because they hope to reap a capital gain, not because they wish to smooth consumption. In fact, keeping livestock when demand is inelastic tends to make consumption more variable, not less. This is why price effects and consumption smoothing effects can, in some cases, cancel each other out.

The interaction between the two forces is what explains the last part of Proposition 9. In case (a), demand is elastic and the price effect is not present. Cycles then arise if the consumption smoothing desire is sufficiently strong, i.e., if Ψ and s are large. In constrast, in case (b), the price effect dominates; it leads to in cycles if the consumption smoothing motive is not too strong. As part 2 of Proposition 9 indicates, when externalities are very strong, producers are very patient, and returns to livestock are very non-linear (i.e., ω large), the range of values of Ψ, s and ε for which the steady state is unstable can be quite large. It is easy to verify numerically that cycles can arise for any arbitrary value of Ψ and ε, provided N, β and ω are large enough.

An immediate corollary of Propositions 4 and 5 is that when the above effects are not sufficiently strong to destabilize the steady state, they nevertheless slow down convergence because they pull the roots of $P(\lambda)$ away from 0. We shall see in Section 4 that, when production shocks are introduced, slower convergence translates into greater variance of livestock assets. The reason is that the motives discussed above induce producers to hold onto livestock even when they anticipate losing animals to production shocks, thereby magnifying the effect of these shocks.

The Role of Non-Marketed Output

Herders often derive from livestock other sources of satisfaction than that obtained from the sale of animals. They may enjoy their way of life and the direct contact with nature that it entails; they may derive utility from various non-marketed 'by-products', like the prestige associated with the possession of livestock in general and of cattle in particular (Monod (1975), pp.130-134), and the insurance that livestock provide against shortfalls in other sources of income (Binswanger and McIntire (1987), Rosenzweig and Wolpin (1993)).[17] Herders also derive a large portion of their calories from milk (Loutan (1985), Bernus (1980)), a significant proportion of which is not marketed.

The model can be expanded to show that the existence of non-marketed output encourages overgrazing. Add a new argument m_t to the producers' utility function, and let the production of m_t be proportional to total output:

$$m_t = \kappa(1 + \alpha(L_{t-1}))L_{t-1} \tag{10}$$

where κ is a constant of proportionality. Producers' desire to consume the non-marketed output induces them to a maintain a larger capital stock and thus to exploit the common access resource more intensively. The satisfaction they derive from the consumption of non-marketed output may even induce them to accept a negative rate of return on the portion of their output that is marketed. As shown in the earlier part of this section, these factors are precisely those that make the occurrence of cycles more likely. These results are summarized in the following proposition:

Proposition 6:

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[17] See, however, Fafchamps, Udry, and Czukas (1996).

(1) L^* decrease as κ or $U_m \to 0$: the existence of non-marketed output encourages over-grazing.

(2) In the presence of non-marketed output, private returns to capital from marketed output may be negative at the steady state.

(3) A high κ broadens the range of other parameter values for which the steady state is unstable.

Section 4. Livestock Cycles and Droughts

After a long detour examining deterministic cycles, we now return to our main interest: livestock cycles and droughts. We proceed as follows. Letting returns to livestock depend on exogenous production shocks, we first demonstrate that the producers' decision problem has a solution and that a policy function exists. The transition function of the equilibrium livestock path may, however, have several invariant measures -- the stochastic equivalent of deterministic cycles. We then cash in on our examination of the deterministic case and demonstrate that, even if the invariant measure is unique, the exact same conditions that lead to deterministic cycles increase the variance of equilibrium livestock holdings in the stochastic case. In other words, all the factors that favor equilibrium cycles in the deterministic case also magnify the effect of production shocks on the livestock population. At the end of the section, numerical simulations are used to graphically illustrate the pernicious effect that externalities have on livestock production cycles.

A Model of Livestock Raising with Production Shocks

Let z_t stand for an external shock affecting production at time t.[18] To focus the

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[18] For the sake of brevity, shocks that affect the outside demand for meat are not covered here (e.g., Fafchamps and Gavian (1997)). The effect of external shocks on the local demand for livestock as a

discussion on collective shocks such as droughts or livestock epidemics, assume that z_t affects all producers equally. Returns to livestock become $\alpha(L_t , z_{t+1})$. Rank states of nature in such a way that low values of z_t mean low returns to capital and vice versa: $\alpha_z(L , z) > 0$ for all z, L. Let the domain of z_t be $Z = [0 , \bar{z}]$ with $0 < \bar{z} < +\infty$, and let $\mathbf{Z}$ be the associated Borel set. Let Q be a transition function defined on $(Z, \mathbf{Z})$, with the Feller property. The sequence of random shocks $\{z_t\}$ is a Markov process generated by Q. All other assumptions of the model remain unchanged. The optimization problem facing producers is now:

$$Max\ E\{\sum_{t=0}^{\infty} \beta^t U(c_t)\}$$

$$s.t.\ \ 0 \le c_t \le y + p_t((1 + \alpha(L_{t-1} , z_t))L_{t-1} - L_t) \qquad (11)$$

Perceived marginal returns to livestock are still given by equation (2). Let the feasible set $\Gamma(L_{t-1} , z_t)$ be defined as the set of consumption and livestock holding plans (c_t , L_t) that satisfies equation (11) above. The Belman functional equation can then be written:

$$v(L_{t-1} , z_t) = \underset{(c_t , L_t) \in \Gamma(L_{t-1} , z_t)}{Max} [U(c_t) + \beta \int_Z v(L_t , z_{t+1})Q(z_t , dz_{t+1})] \qquad (12)$$

As the following proposition shows, the individual optimization problem remains well behaved:

Proposition 7:

(1) There exists a unique bounded continuous function v satisfying the Belman equation.

(2) v is strictly concave in L_{t-1} and the policy correspondence is a continuous single-valued function.

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productive asset and as a form of precautionary saving is included in the analysis. Income effects on the local demand for meat are ignored, but they are likely to be small in most rural areas of Africa. The possible correlation between livestock production shocks and non-livestock income is ignored here to keep the model tractable. A combined treatment is left for future research.

The presence of uncertainty introduces a precautionary motive for holding livestock as a self-insurance mechanism against the vagaries of external shocks and their effect on income (e.g., Kimball (1990), Fafchamps, Udry, and Czukas (1996), Fafchamps and Pender (1997), Dercon (forthcoming)). This may constitute an additional reason for accumulating capital and thus contributes to the overexploitation of the resource. Moving on to the competitive equilibrium, let $L(L_t , z_{t+1})$ denote the choice of livestock holdings at which $L_{i,t} = L_{j,t}$ for all $i \neq j$, i.e.:

$$L_{t+1} = L((1 + \alpha(L_t , z_{t+1}))L_t , z_{t+1}) \tag{13}$$

The function α and the decision rule L, together with the probability measure for z, define a Markov transition function P on the domain of L. The properties of this transition function describe the behavior of equilibrium livestock holdings over time.

Proposition 8:

P has at least one invariant measure; it may have several.

The above proposition indicates that the economy has at least one invariant measure, that is, one Markov matrix that describes how the long term livestock holdings evolve over time, conditional upon the state they are in at any particular time. The proposition also states that it may have multiple invariant measures or cycles. The reason is that one of the key assumptions for uniqueness[19] is not satisfied: *P* is not monotone in *L* since the policy function itself in quadratic in *L*. That *P* may have several invariant measures is, of course, not surprising, given that the corresponding deterministic model exhibits cyclic behavior. To see why, suppose for a moment that the distribution of z_t is degenerate. The system then collapses onto the deterministic problem analyzed in Sec-

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[19] See Stokey and Lucas (1989), theorem 12.12.

tion 2. Let parameter values be such that deterministic livestock holdings cycle between L_1 and L_2. Now introduce a little bit of uncertainty into the model. For small enough shocks, the economy will continue to cycle between the neighborhoods of L_1 and L_2. In other words, P will cycle through invariant measures and the ergodic set within which livestock holdings evolve in the long run has cyclically moving subsets. Conditions under which cyclically moving subsets arise are similar to those under which deterministic cycles occur.

Stability of the Deterministic Steady State and Variance of Livestock Holdings

Even if the economy has a single invariant measure, factors that slow down convergence to the deterministic steady state or that work against its stability also increase the variability of livestock holdings in the presence of production risk. This can be shown by differentiating the decision rule with respect to the variance of z around the deterministic steady state:

Proposition 9: For a small variance of z, the variance of L around its deterministic steady state increases as the roots of the characteristic polynomial studied in Section 2 get larger in absolute value.

Proposition 9 demonstrates that the forces that undermine the stability of the deterministic steady state and slow down convergence are the same forces that raise the variance of the stochastic path of livestock holdings in the presence of production shocks. This is hardly surprising: we saw in Section 3 that concerns for consumption smoothing and capital gain considerations may induce producers to hold onto livestock even when they anticipate incurring physical losses. The exact same forces are at work here, and they operate in the same fashion. An immediate and important corollary of Proposition 9

is that externalities raise the variance of livestock.

A Numerical Illustration

To illustrate how externalities result in a more volatile livestock population, consider the following example, constructed so as to mimic the droughts that decimate livestock at irregular interval in the Sahel and elsewhere. Assume that returns to livestock are given by $\omega(\bar{L}(z)-L_t)-1$ where $\bar{L}$ takes on two values, $\bar{L}^u$ and $\bar{L}^d$ with $\bar{L}^u \geq \bar{L}^d$, with probability τ and $1-\tau$, respectively. Given these assumptions, an algorithm is constructed that iterates on a modified Belman equation (see Appendix B), taking other producers' livestock holdings as given. Equilibrium conditions are then imposed. Unpon convergence, this algorithm produces an approximation of the equilibrium policy function. This approximation is then used to simulate the path of livestock holdings over time, given an arbitrary realization of production shocks.

Examples of livestock paths are given in Figures 1 and 2. We had no difficulty reproducing the cyclic livestock paths described in Livingstone (1986). Figure 1 demonstrates vividly that externalities result in stochastic cycles of wider amplitude. Similar qualitative results are obtained with other parameter values. The Figure further indicates that much economic efficiency is lost even with a little bit of externalities, i.e., when *N* goes from 1 -- first best -- to 2 -- what would formally be achieved if all pasture land was divided into pairs of household using it jointly. Other simulations (not shown) indicate that livestock are more variable when producers are patient and ω is large.

The desire to smooth consumption, represented by a larger value of Ψ, is also shown to increase the variance of livestock holdings when demand in elastic (see Figure 2). The reason is that producers seek to smooth consumption by overaccumulating

livestock. This strategy is largely futile since other producers do the same, thereby fueling overgrazing and precipitating a collapse of the livestock population when comes the next drought. This result may explain Fafchamps, Udry and Czukas' (1996) findings that livestock play a minor role in helping households stabilize their consumption in the wake of a major drought. *Given* that others overgraze, however, accumulating livestock still remains in the interest of individual producers.

Conclusion

Livestock play an important role in the livelihood of many rural dwellers in semi-arid areas of the Third World, particularly in Africa. Drought-driven livestock cycles, possibly magnified by externalities, represent a wasteful and often life-threatening process. This paper has investigated the theoretial relationship between these livestock cycles and externalities. We have shown that, when production shocks are present and producers fail to fully internalize the effect of their pasture use on other producers, overgrazing results in wide fluctuations of the livestock population. We also noted that overgrazing does not result from producers' impatience, quite the contrary. Because impatient producers are less willing to invest in livestock, they would use pasture resources less intensively. The existence of non-marketed output and sources of 'on-the-job' satisfaction were shown to contribute to the overaccumulation of livestock and, hence, to livestock cycles.

Two distinct forces are capable of inducing producers to hold onto livestock even when they anticipate losing many of their animals. The first force is the desire to smooth consumption when livestock make an essential contribution to household income and other assets are not available; it is probably strongest among pastoralists, i.e., specialized

herders who derive most of their income from livestock and are precluded from accumu-lating grain due to their nomadic lifestyle. This result thus formalizes the numerous fac-tual accounts that describe pastoralists as trying to accumulate 'target herds' of a size sufficient to ensure survival during droughts (e.g., Sandford (1983), Livingstone (1986), Monod (1975)).

The second force is the realization that, if demand for livestock products is inelastic, producer revenues rise when aggregate supply is low and prices are high. Producers may then be inclined to risk physically losing livestock to reap higher prices when livestock supply dries up. This strategy raises the variance of livestock income and is thus more accessible to households with alternative sources of income or with access to financial markets.[20] How strong each of these forces is depends on the elasticity of livestock demand and hence on the spatial integration of livestock market (e.g., Fafchamps and Gavian (1996, 1997)): the second effect is stronger when markets are segmented and local demand inelastic; the first effect is more powerful when livestock prices are fairly stable. If externalities are sufficiently strong, the two effects reinforce each other.

The theoretical analysis presented here opens new avenues of empirical enquiry regarding the relationship between markets, poverty, and the environment. Livestock cycles are a particularly important research topic. Unlike issues pertaining to the preser-vation of wildlife habitat where the interest of the poor and the preservation of the environment are often contradictory,[21] in the case of livestock cycles, they coincide: to the extent that poverty is exacerbated by the magnitude of livestock cycles that are

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[20] Conversations with Maradi residents in 1985 suggested that during the 1984 drought, local wealthy merchants speculated in livestock precisely in the manner predicted by the model.

[21] Unless special programs are put in place that ensure that the poor internalize some of the global benefits of conservation.

themselves caused by overgrazing, reducing overgrazings can only help the poor. What we lack, however, is evidence that overstocking is indeed taking place. The model presented here suggests that a proper assessment of stocking rates must take market conditions into account. Are livestock markets spatially integrated so that local climatic shocks are smoothed out in the aggregate? If not, fluctuations in livestock prices may encourage wealth individuals to take speculative positions during droughts, thereby compounding pasture degradation. Do farmers and herders have access to alternative savings instruments so that they can liquidate livestock if they fear losing animals? If not, they will tend to cling onto their assets as -- highly imperfect -- forms of self-insurance. These issues deserve careful empirical investigation.

Although the analysis presented here did explicitly incorporate policy instruments, it nevertheless suggests that setting up institutions to ensure that externalities are minimized could significantly contribute to the welfare of millions of people. African governments should be encouraged to promote the better management of pasture resources, e.g., by reinforcing the institutions of pastoralists and other rural communities. Our model also provides a useful framework for predicting how the livestock economy of African countries would respond to policy changes without simultaneous improvement in institutions for common pasture management. The construction of boreholes, for instance, is predicted to exacerbate cycles whenever it raises the growth rate of livestock without improving the availability of pasture. Increased livestock prices have the same effect of raising returns to livestock without expanding pasture. They too contribute to livestock cycles in a situation characterized by externalities. Together, these two factors may account for the particularly large swings in cattle population that were observed in the Sahel in the wake of the two Nigerian oil shocks (e.g., Fafchamps and Gavian (1997)).

Results further suggest that the introduction of assets other than livestock (e.g., cereal banks, rural savings and loans cooperatives) would reduce the consumption smoothing motive for holding livestock. But at the same time it would favor speculative hoarding of livestock in markets characterized by inelastic demand and geographical segmentation (e.g., Fafchamps and Gavian (1996)). Consequently, favoring financial intermediation without simultaneously promoting the spatial integration of livestock markets may exacerbate livestock cycles instead of mitigating them, especially if wealthy merchants are able to accumulate speculative herds. These issues deserve further analysis.

Appendix A: Proofs of Propositions

Proof of Proposition 1:

The proof of this proposition is a tedious but straightforward application of the principles of dynamic programming (e.g., Stokey and Lucas (1989)). A detailed proof is available on request from the author.

Proof of Proposition 2:

(1) The right hand side of equation (7) is continuous since $\alpha(.)$ is continuous, and monotonously decreasing in L since $\alpha' \dfrac{N+1}{N} + \alpha'' \dfrac{L}{N} < 0$. Consequently the right hand side of (8) equals $1/\beta$ for at most one value of L. At $L = 0$, the right hand side is strictly greater than the left hand side whenever $1/\beta < 1 + \alpha(0)$. Furthermore, at $L = \overline{L}$, the right hand side is strictly smaller than the left hand side since, by assumption, $\alpha(\overline{L}) = -1$ and $\alpha'' \leq 0$. Consequently, the right hand side of (8) must equal its left hand side for some L^* such that $0 < L^* < \overline{L}.\square$

(2) Totally differentiating (7), one gets:

$$\frac{dL^*}{d\beta} = -\beta^2 [\alpha' \frac{N+1}{N} + \alpha'' \frac{L}{N}] > 0$$

$$\frac{dL^*}{dN} = \frac{[\alpha' \dfrac{N+1}{N} + \alpha'' \dfrac{L}{N}]}{\alpha' \dfrac{L}{N^2}} > 0$$

(3) Differentiating $L^* = \dfrac{N}{N+1}(\overline{L} - \dfrac{1}{\omega\beta})$ with respect to ω yields:

$$\frac{\partial L^*}{\partial \omega} = \frac{N}{N+1} \frac{1}{\omega^2 \beta} > 0 .$$

(4) $1 + \alpha + \alpha' L^* \leq 1 + \alpha + \alpha' \dfrac{L^*}{N}$ since $\alpha' < 0$.

(5) Social returns are positive for $N = 1$. As $N \to \infty$, equation (8) tends to $1/\beta = 1 + \alpha(L^\infty)$. Gross returns to capital $\alpha(L^\infty)$ are still positive, but social returns are negative whenever $-\alpha'(L^\infty)L^\infty > 1/\beta$. For instance, let returns to capital take the quadratic form $\omega(\bar{L} - L)$, and let $\bar{L} = 1$. We have $L^\infty = \dfrac{\beta\omega - 1}{\beta\omega}$. Social returns at L^∞ are negative whenever $\omega > 2/\beta$, a condition that is easy to satisfy since possible parameter values for ω range from $1/\beta$ to 4. Since L^* in monotonically increasing in N and $1 + \alpha + \alpha'L^*$ monotonically decreasing in L^*, social returns will be initially positive, but will decrease and become negative for some finite N whenever $-\alpha'(L^\infty)L^\infty > 1/\beta.\square$

Proof of Proposition 3:

(1) (a) Using equation (7), the partial derivatives of $g\,(.)$ at the steady state are:

$$\frac{\partial g}{\partial L_{t+1}} = -U''p(p + \alpha Lp') - U'p'$$

$$\frac{\partial g}{\partial L_t} = U''p(p + \alpha Lp')(2 + \alpha + \alpha'L)$$

$$+ U'p'(2 + \alpha + \alpha'L) + \beta U'p(\alpha'\frac{N+1}{N} + \alpha''\frac{L}{N})$$

$$\frac{\partial g}{\partial L_{t-1}} = (-U''p(p + \alpha Lp') - U'p')(1 + \alpha + \alpha'L)$$

(b) $-U''p(p + \alpha Lp') - U'p' \neq 0$. Suppose the contrary. Then there is a way to decrease L_{t+1}, increase c_{t+1} and thus utility, and still satisfy the Euler equation. But this contradicts optimality. Thus the above expression must be different from 0 and the matrix (9) is well defined.

(c) Matrix (9) is non-singular if its determinant $\dfrac{\partial g/\partial L_{t-1}}{\partial g/\partial L_{t+1}} = 1 + \alpha + \alpha'L$ is different from 0. This condition holds except for a set of unlikely parameter configurations whereby L^* happens to be exactly such that $1 + \alpha + \alpha'L^* = 0$. That set is of Lebesgue

measure zero.

(2) Given that the steady state is unique, and provided that matrix (9) is well defined and non-singular, the proposition can easily be proved by the same method as that applied in theorems 6.8 and 6.9 of Stokey and Lucas (1989), p. 151-154.

(3) Part (3) follows from the theory of difference equations.□

Proof of Proposition 4:

(1) $P(\lambda)$ is quadratic in λ and therefore continuous. Furthermore, it is positive for large positive and large negative values of λ. Using cumbersome but straightforward algebraic manipulations of the characteristic polynomial, it can be shown that the roots of $P(\lambda)$ cannot simultaneously be above 1 or below -1. $P(-1) < 0$ and $P(1) < 0$ thus means that $P(\lambda)$ intersects the ordinate axis once above 1 and once below -1. Therefore both λ roots are real and larger than one in absolute value and, by proposition 3.2, the steady state is unstable.

(2) We prove part 2 with an example. Suppose that returns to capital are quadratic. Replacing L^* by its steady state value, simple computation shows that $P(-1)$ and $P(1) < 0$ when $\omega \geq 3.2$, $\beta \geq 0.95$, $\Psi \geq .5$, $s \geq 0.4$, $0 < \varepsilon < 0.6$, and $N \geq 100$.□

Proof of Proposition 5:

$P(1)$ and $P(-1)$ can be derived as:

$$P(1) = -\frac{\beta U'p(\alpha'\frac{N+1}{N} + \alpha''\frac{L^*}{N})}{U''p(p + \alpha L^*p') + U'p'} \tag{14}$$

$$P(-1) = 2(2 + \alpha + \alpha'L^*) - P(1) \tag{15}$$

First we focus our attention on $P(1)$, which can be rewritten as:

$$P(1) = \frac{\beta\alpha L^*(\alpha'\dfrac{N+1}{N} + \alpha''\dfrac{L^*}{N})}{\Psi s - \dfrac{1}{\varepsilon}(\Psi s - 1)} \qquad (16)$$

Since by assumption α'' is either negative or positive but small, the numerator of $P(1)$ is always negative: the marginal return to capital is decreasing -- $\alpha' < 0$ -- and all the other terms are positive. The sign of $P(1)$ thus depends on the sign of its denominator, that is, on whether $\dfrac{\Psi s - 1}{\Psi s}$ is greater or smaller than the demand elasticity ε and $P(1) < 0$ when $\dfrac{\Psi s - 1}{\Psi s} < \varepsilon$. Now turn to $P(-1)$. It is composed of two terms. The first, $2(2 + \alpha(L^*) + \alpha'(L^*)L^*)$, is exclusively function of the production technology α and of the steady state level of capital stock L^*. The second is equal to $-P(1)$. When $N=1$, the first term of $P(-1)$ is always positive since $1 + \alpha(L^*) + \alpha'(L^*)L^*$ is the social return to capital which, in the absence of externalities, is equal to $1/\beta$. To summarize, when $N=1$ the first term of $P(-1)$ is always positive. By proposition 4.2, $P(1) < 0$ is a necessary condition for instability of the steady state. But whenever $P(1) < 0$, the second term of $P(-1)$ is positive. Consequently, $P(-1)$ and $P(1)$ can not be simultaneously negative and the steady state is stable. This proves part (1).

We now examine how other model parameters influence the stability of the steady state. Since the steady state can be unstable only when $P(1) < 0$, we must study the factors that are likely to set $P(-1)$ negative when $P(1) < 0$. Denote the first term of $P(-1)$ as A. The second term of $P(-1)$ is positive when $P(1)$ is negative. Therefore $P(1)$ and $P(-1)$ can only be simultaneously negative if A is sufficiently negative to counterbalance the positive $-P(1)$. A is $2(1 +$ the social marginal return to capital). For A to be negative,

the social marginal return to capital must therefore be negative and smaller than -1. The social marginal return to capital is zero at the capital stock $\hat{L}$ that supports the maximum sustainable yield. It becomes negative beyond that level of capital. Consequently, instability of the steady state can only occur when the capital stock corresponding to the maximum sustainable yield is exceeded. This proves part (2).

Differentiating A shows that it decreases as N and β increase, that is, when externalities are important and producers patient. In order for $P(-1)$ to be negative, $P(1)$ must remain small enough in absolute value so that it does not overshadow A's negative effect. Thus, factors that tend to reduce the absolute magnitude of $P(1)$ while preserving its negative sign also contribute to the instability of the steady state. Let the numerator of $P(1)$ be denoted as a, which is negative by construction. Straightforward derivation yields:

$$\frac{\partial P(1)}{\partial \Psi} = \frac{a(1-\varepsilon)\varepsilon s}{(1-\Psi s + \Psi s \varepsilon)^2} \tag{17}$$

$$\frac{\partial P(1)}{\partial s} = \frac{a(1-\varepsilon)\varepsilon \Psi}{(1-\Psi s + \Psi s \varepsilon)^2} \tag{18}$$

$P(1)$ increases with Ψ and s when ε is greater than 1; it decreases with Ψ and s if ε is smaller than 1. We thus have two regimes: one in which demand is elastic, in which case cycles tend to occur when Ψ and s are high as long as $\frac{\Psi s - 1}{\Psi s}$ remains $< \varepsilon$; and one in which demand is inelastic, in which case cycles tend to emerge when Ψ and s are low, again provided that $\frac{\Psi s - 1}{\Psi s} < \varepsilon$. This proves part 3.

Proof of Proposition 6:

The new Euler equation is :

$$U_c(c_{t-1}, m_{t-1})p_{t-1} =$$

$$\beta[U_c(c_t, m_t)p_t + U_m(c_t, m_t)\kappa](1 + \alpha_t + \alpha_t{}'\frac{L_t}{N})$$

and the steady state must now satisfy:

$$\frac{1}{\beta\left[1 + \dfrac{\kappa U_m}{pU_c}\right]} = 1 + \alpha + \alpha{}'\frac{L^*}{N} \tag{19}$$

Differentiating (19) proves Part (1). Reworking the characteristic polynomial with equation (19) proves part (3). Part (2) holds because the left hand side of (19) may be smaller than 1, even though $1/\beta$ is always greater than 1. $\square$

Proof of Proposition 7:

(1) Let the domain of L_t be $[0, \bar{L}]$. Let $\bar{p}$ be the maximum possible price, and define $\hat{\hat{L}}$ the level of capital stock such that $\alpha(\hat{\hat{L}}, \bar{z}) = 0$. Then, without loss of generality, the domain of c_t can be reduced to $[0, \bar{p}(1 + \alpha(\hat{\hat{L}}, \bar{z}))\hat{\hat{L}} + y]$. Thus the domain of (c_t, L_t) is a closed, convex subset of R^l, with its Borel subsets. $\Gamma : (C, L)\times Z \to (C, L)$ is nonempty and compact valued and continuous by the continuity of α. Thus assumptions 9.4-9.7 of Stokey and Lucas are satisfied and part (1) holds by their theorem 9.6.

(2) Since, as in Section 1, U is concave and Γ convex, assumptions 9.10 and 9.11 of Stokey and Lucas are also satisfied and part (2) holds by their theorem 9.8.

Proof of Proposition 8:

α is continuous by assumption, and L is continuous by proposition 7.2. It follows from Stokey and Lucas, p. 237. that P has the Feller property. Since the domain of L_t is compact, Stokey and Lucas's theorem 12.10 applies, which proves the proposition. $\square$

Proof of Proposition 9:

We demonstrate the proposition when prices are constant. The case when prices depend on sales can be analyzed in a similar manner. Let S_t be livestock holdings inherited from the previous period, i.e., $S_t \equiv (1 + \alpha(L_{t-1}, z_t))L_{t-1}$. Let $L(S_t)$ be the policy function solving the stochastic optimization problem with constant prices. Let $V(z)$ denote the variance of z. The variance of L can be approximated as:

$$V(L) \approx V(z) \left[\frac{\partial L}{\partial z} \Big|_{E(z)} \right]^2$$

Around the steady state, we have:

$$\frac{\partial L}{\partial z} = \frac{\partial L}{\partial G} \alpha_z L^*$$

Let $L' = \partial L/\partial G$. Around the steady state, L' can be computed by the method presented in Judd (1990). The Euler equation can be used to derive the following functional equation for $L(G)$:

$$U'(y + p(S_t - L(S_t))) =$$
$$\beta E\{U'[y + p((1 + \alpha(L(S_t), z_{t+1}))L(S_t) - L((1 + \alpha(L(S_t), z_{t+1}))L_{(S_t)}))]$$
$$(1 + \alpha(L(S_t), z_{t+1}) + \alpha'(L(S_t), z_{t+1})\frac{L(S_t)}{N})$$

Let the variance of $z \to 0$. Totally differentiate L's functional equation with respect to G around the steady state. Using equation (7) we get:

$$U''p(1 + L') = \beta[U''pL'(1 - L')(1 + \alpha + \alpha'L)1/\beta + U'L'(\alpha'\frac{N+1}{N} + \alpha''\frac{L}{N})]$$

The above equation is quadratic in L'. Define $X = 1 + \alpha + \alpha'L$ and

$$Y = \frac{\beta U'}{pU''}(\alpha'\frac{N+1}{N} + \alpha''\frac{L}{N})$$

Then the two roots of L' are:

$$L' = \frac{1}{2X}\{1 + X + Y \pm \sqrt{(1 + X + Y)^2 - 4Y}\}$$

The two roots correspond respectively to the stable and unstable manifold of the steady state. When the price is constant, the roots of the characteristic polynomial $P(\lambda)$ become:

$$\lambda = \frac{1}{2}\{1 + X + Y \pm \sqrt{(1 + X + Y)^2 - 4Y}\}$$

Of course, the Euler approach used in Section 2 and the policy function approach used here are identical. This is easily verified by noting that, along the optimal path,

$$\frac{\partial L^*_{t+1}}{\partial L^*_t} = \frac{\partial L(S_t)}{S_t}\frac{\partial S_t}{\partial L_t} = L'(1 + \alpha + \alpha' L^*_t)$$

It follows from the above that, when the roots of the characteristic polynomial are large in absolute value, so are the roots of L'. Hence, factors that increase the roots of the characteristic polynomial in absolute value raise the variance of L around its steady state. $\square$

Appendix B: A Numerical Model of Livestock Accumulation

A numerical model of livestock accumulation is constructed as follows. Equilibrium value and policy functions are approximated by Chebychev polynomials of degree four (see Judd (1996)). The coefficients of these approximating polynomials are obtained by iterating on a modified Belman equation as follows. An iterative algorithm, coded in Fortran, solves, for various levels S_t of livestock inherited at the beginning of period t, the optimal choice of $L_{i,t}$ at the end of the period. To capture the fact that producers partially ignore externalities, the level of $L_{i,t}$ corresponding to a particular value of S_t is derived using the following first order condition:

$$p_t U'(y + p_t S_t - p_t L_{i,t}) + \beta\tau\frac{\partial V^{(n-1)}(\omega(\bar{L}^u - L_{a,t})L_{i,t}-1)}{\partial S_{t+1}}(\omega(\bar{L}^u - L_{a,t}) - \omega\frac{L_{i,t}}{N}) +$$

$$\beta(1-\tau)\frac{\partial V^{(n-1)}(\omega(\bar{L}^d - L_{a,t})L_{i,t}-1)}{\partial S_{t+1}}(\omega(\bar{L}^d - L_{a,t}) - \omega\frac{L_{i,t}}{N}) = 0 \qquad (20)$$

where $V^{(n)}$ denotes the nth iterate of the value function

$$V^{(n)}(S_t) = \underset{L_{i,t}}{Max}\, U(y + p_t S_t - p_t L_{i,t}^*) + \beta\tau V^{(n-1)}(\omega(\bar{L}^u - L_{a,t})L_{i,t}-1) +$$

$$\beta(1-\tau)V^{(n-1)}(\omega(\bar{L}^d - L_{a,t})L_{i,t}-1) \tag{21}$$

and $L_{a,t}$ is as in Section 2. Prices p_t are assumed constant.[22]

We then iterate on equations 20 and 21 until convergence, using the previous iterate of the policy function to predict the livestock assets of other producers. Upon convergence, the algorithm gives the equilibrium policy and value functions which are then used to simulate the path of livestock holdings, given an arbitrary path of production shocks. The above iterative process, unlike a standard Belman equation problem, is not guaranteed to converge. Only when $n = 1$ are the conditions for the contraction mapping theorem fully satisfied; the problem then boils down to a simple accumulation problem. When externalities are present, the above algorithm sometimes begins to cycle and fails to converge. This does not come as a complete surprise, given the model's propensity to generate cycles in the first place.

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[22] We experimented with a constant elasticity of demand formulation but convergence turned out to be highly problematic.

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Proof of Proposition 1:

Denote the feasibility correspondence defined by the set of inequalities (2) and (3) as

$\Gamma(L_{i,t-1})$. Let $\{x_t\} \equiv \{(c_{i,t}, L_{i,t})\}$ and $v^o \equiv \sum_{t=0}^{\infty} \beta^t U(c_{i,t}^o)$.

Lemma 1:

(1) v^o satisfies the Bellman equation

$$v(L_{i,t-1}) = Sup_{(c_{i,t}, L_{i,t}) \in \Gamma(L_{i,t-1})}[U(c_{i,t}) + \beta v(L_{i,t})] \tag{22}$$
$$for\ all\ L_{i,t-1}\ s.t.\ 0 \leq L_{i,t-1} \leq N\overline{L} - L_t^j\ ;$$

(2) $\{x_t^o\}$ satisfies

$$v^o(L_{i,t-1}^o) = U(c_{i,t}^o) + \beta v^o(L_{i,t}^o)\ ; \tag{23}$$

(3) A solution v to the Bellman equation such that $\lim_{n \to \infty} \beta^n v(x_n) = 0$ satisfies optimi-

zation problem (1-3);

(4) A feasible plan satisfying (23) and $\lim sup_{t \to \infty} \beta^t v^o(x_t^o) \leq 0$ attains the supremum

in optimization problem (1-3).

Proof of Lemma 1: The feasible set is non-empty for any initial capital stock L_0. For

instance the plan whereby $c_{i,t} = y$ for all t and $L_{i,t} = (1 + \alpha(1/N(L_{i,t-1} + L_{t-1}^j)))L_{i,t-1}$ is

always feasible. Since returns to capital and capital assets are bounded and other income

y is constant and finite, for any feasible plan $c_{i,t}$, $\lim_{n \to \infty} \sum_{t=0}^{n} \beta^t U(c_{i,t})$ exists. Conse-

quently assumptions 4.1 and 4.2 of Stokey and Lucas (1989), p. 68, are satisfied. The

lemma thus holds by application of their theorems 4.2, 4.4, 4.3 and 4.5 respectively.□

Lemma 2: Let v satisfy the Bellman equation (22) and let the policy correspondence G be defined as

$$G(L_{i,t-1}) = \{(c_{i,t}, L_{i,t}) \in \Gamma(L_{i,t-1}) : v(L_{i,t-1}) = U(c_{i,t}) + \beta v(L_{i,t})\}.$$

Then v is strictly concave and G is a continuous, single-valued function.

Proof of Lemma 2: The set of feasible values for $c_{i,t}$ and $L_{i,t}$ is a convex subset of R^2. We have already shown that $\Gamma(L_{i,t-1})$ is non-empty. Obviously it is also compact-valued and, by the continuity of $\alpha(L)$, it is continuous. Furthermore, $U(c)$ is bounded and continuous, and $0 < \beta < 1$. Thus assumptions 4.3 and 4.4 of Stokey and Lucas (1989), p. 78, are satisfied. By assumption, $U(c)$ is concave. Since $\alpha'(L) < 0$ and $\alpha''(L) \leq 0$,

$$c_{i,1} \leq y + p[(1 + \alpha(1/N(L_{i,1} + L_1^j))L_{i,1} - L'_{i,1}$$
$$and \ \ c_{i,2} \leq y + p[(1 + \alpha(1/n(L_{i,2} + L_2^j)))L_{i,2} - L'_{i,2}$$

imply that

$$\theta c_{i,1} + (1-\theta)c_{i,2} \leq y + p[(1 + \alpha(1/N(\theta(L_{i,1} + L_1^j) + (1-\theta)(L_{i,2} + L_2^j))))$$
$$(\theta L_{i,1} + (1 - L_{i,2}) - \theta L'_{i,1} - (1-\theta)L'_{i,2}$$

for all $0 \leq \theta \leq 1$. Therefore $\Gamma(x)$ is convex. Thus assumptions 4.7 and 4.8 of Stokey and Lucas (1989), p. 80, are satisfied and the proposition holds by application of their theorem 4.8.□

Proof of Proposition 1: (1) By Lemma 1 and 2, we have shown that assumptions 4.3, 4.4, 4.7 and 4.8 of Stokey and Lucas are satisfied. Furthermore, since $U(c)$ is continuously differentiable, their assumption 4.9 is also satisfied. Consequently the proposition holds by their theorem 4.15, p. 98, appropriately amended since, by non-satiation, the budget constraint is always binding.□

(2) By proposition 1.1, it suffices to show that the optimal path everywhere belongs to the

interior of the feasible set, except for the budget constraint which is always binding.

(a) The budget constraint is always binding by non-satiation.

(b) Since $\lim_{c \to \infty} U(c) = -\infty$, $c_{i,t} > 0$ for all t.

(c) $L_{i,t} = N\overline{L} - L_t^j$ cannot be part of an optimal plan. Suppose the contrary. Then $L_{i,t+1} = 0$. But then utility could be increased by selling capital at time t. Consequently $L_{i,t} < N\overline{L} - L_t^j$ for all t.

(d) If the return to capital at $L = 0$ is higher than $1/\beta$, $L_{i,t} = 0$ cannot be part of an optimal plan whenever $L_t^j = 0$. Suppose the contrary. For this to be optimal, it must be that

$$1 + \alpha(0) < \frac{U'(y + p_t(1 + \alpha(1/N(L_{i,t-1} + L_{t-1}^j)))}{\beta U'(y - p_{t+1}L_{i,t+1})}. \tag{24}$$

Since $U(.)$ is concave, $U'(.)$ is highest when consumption is lowest and vice versa. Consequently, equation (24) is more likely to be satisfied when the numerator of the right hand side is large and the denominator small. The right hand side of (A) is largest when $L_{i,t+1} = 0$ and $p_t(1 + \alpha(1/N(L_{i,t-1} + L_{t-1}^j))) \to 0$. Then it tends toward $1/\beta$ from below. But this contradicts the assumption that $1 + \alpha(0) > 1/\beta$. Thus $L_{i,t} > 0$ at all times.□

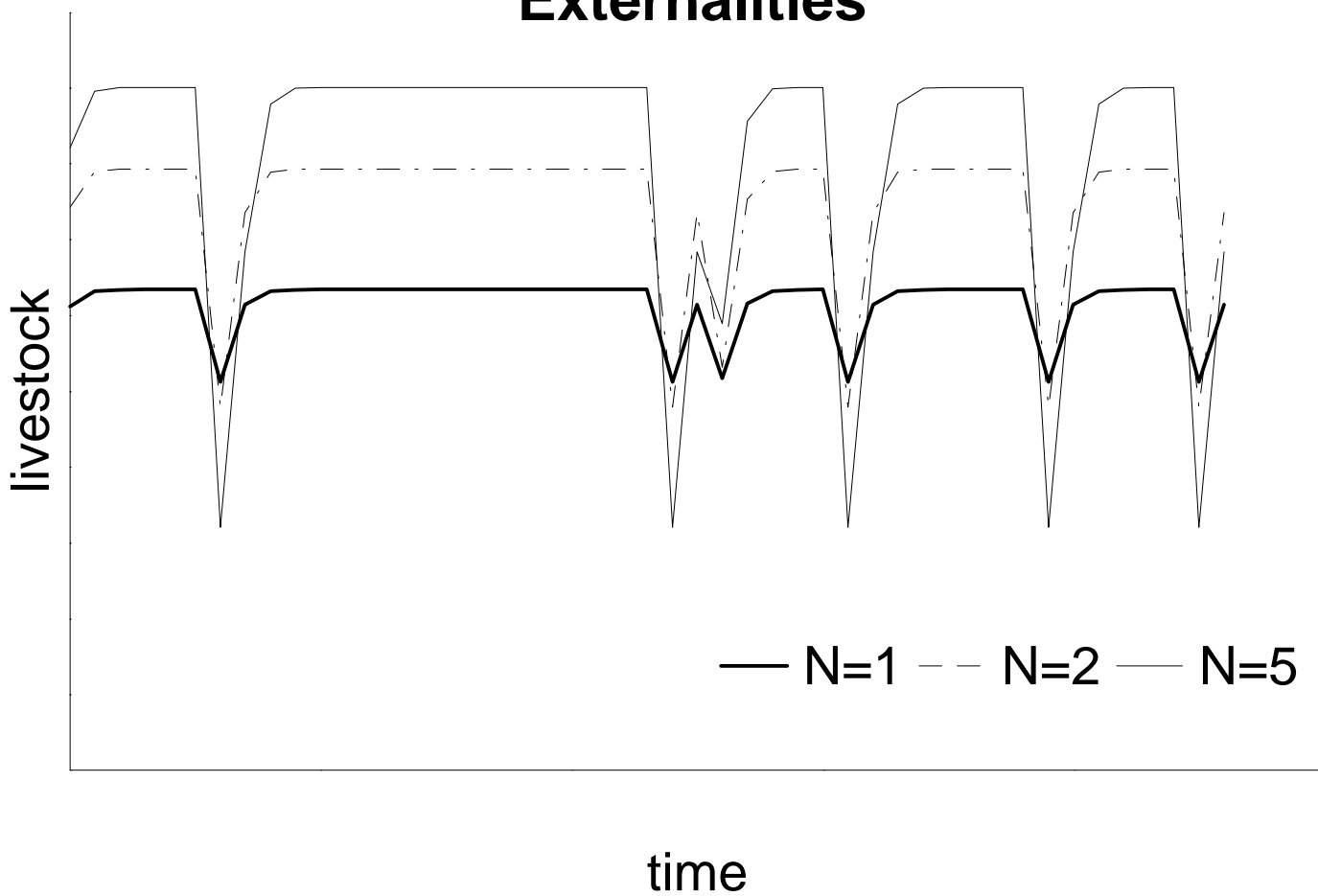# Figure 1. Livestock Cycles and Externalities

livestock

N=1 — — N=2 —— N=5

time

Figure 2. Livestock Cycles and Risk Aversion

— Ψ = 0.5 — Ψ = 1.5

livestock

time