

Competition in Persuasion

Matthew Gentzkow, *Stanford University and NBER**
Emir Kamenica, *University of Chicago*

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Abstract

We study symmetric information games where a number of senders choose what information to communicate. We show that the impact of competition on information revelation is ambiguous in general. We identify a condition on the information environment (i.e., the set of signals available to each sender) that is necessary and sufficient for equilibrium outcomes to be no less informative than the collusive outcome, regardless of preferences. The same condition also provides an easy way to characterize the equilibrium set and governs whether introducing additional senders or decreasing the alignment of senders' preferences necessarily increases the amount of information revealed.

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1 Introduction

Does competition increase the amount of information revealed? A long tradition in political and legal thought holds that the answer is yes. This view has motivated protection of freedom of speech and freedom of the press, media ownership regulation, the adversarial judicial system, and many other policies.¹

Economic theory suggests several mechanisms by which competition can increase information revelation. Milgrom and Roberts (1986) study verifiable message games and point out that if in each state there is a sender who wishes that state to be known, then full revelation is the unique equilibrium. Shin (1998) shows that two adversarial senders who convey verifiable messages about their independent signals always generate more information than a single signal directly observed by the receiver. In a cheap talk setting, Battaglini (2002) establishes that it is generically possible to construct a full revelation equilibrium when there are two senders and uncertainty is multidimensional.² These results, however, do not imply that competition must increase information in all settings, as the following example makes clear.

Example. There are two pharmaceutical companies $j = 1, 2$ each of which produces a drug, and a unit mass of potential consumers indexed by i . For a given consumer i , drug j may have either low or high efficacy, which we represent by $\omega_{ij} \in \{l, h\}$, with $Pr(\omega_{ij} = h) = 0.2$, distributed independently across consumers and drugs. All consumers prefer high efficacy and are otherwise indifferent between the drugs, but they differ in their outside options: half always buy whichever drug has the higher expected efficacy, while the other half buys the better drug only if its $Pr(\omega_{ij} = h)$ is greater than 0.5. The share of these two types is independent of ω_{ij} .

Each firm j simultaneously chooses one of two disclosure policies: a completely uninformative signal (*null*), or a fully informative one (*reveal_j*) that allows each consumer i to determine ω_{ij} for that firm's drug. The firms maximize the share of consumers buying their drug. We can represent this situation as the following normal form game:

| | | |
|---------------------------|-------------|---------------------------|
| | <i>null</i> | <i>reveal₂</i> |
| <i>null</i> | .25, .25 | .40, .20 |
| <i>reveal₁</i> | .20, .40 | .34, .34 |

¹See Gentzkow and Shapiro (2008) and references cited therein.

²This is the case even though the two senders observe the same signal so that in the absence of conflict the second sender adds no value to the receiver.

This is a Prisoner’s Dilemma. Revealing information is beneficial for the firms’ joint profits as it increases expected profits from the consumers who buy only if $Pr(\omega_{ij} = h) > 0.5$. (Those consumers never buy unless given information.) But revealing information is unilaterally unattractive since it disadvantages the revealing firm in the competition for the consumers who always buy. The uninformative signal is thus a dominant strategy and the unique equilibrium yields no information. In contrast, if the firms were to collude and maximize the sum of their payoffs, they would choose $(reveal_1, reveal_2)$.

In this example, competition between the firms decreases information revelation and lowers consumer welfare. Note that the situation would be very different if firms could disclose information not only about their own drug, but also about their competitor’s drug—i.e., if each firm could choose $reveal_1$ and/or $reveal_2$. In this case, each firm would prefer to unilaterally disclose the efficacy of their competitor’s drug, and full revelation would be an equilibrium.

The main contribution of this paper is to identify a necessary and sufficient condition on the information environment (i.e., the set of signals available to each sender) under which competition cannot decrease information revelation. We consider a setting where senders with a common prior simultaneously conduct costless, publicly observed experiments (“signals”) about an unknown state of the world. The information revealed by these signals can be succinctly summarized by the induced distribution of posterior beliefs (Blackwell 1953). We refer to this distribution of beliefs as the *outcome* of the game. We allow senders to have arbitrary utility functions over outcomes.

We say that an information environment is *Blackwell-connected* if for any profile of others’ strategies, each sender has a signal available that allows her to unilaterally deviate to any feasible outcome that is more informative. Note that the environment in the example above is not Blackwell-connected, because starting from $(null, null)$, firm 1 cannot unilaterally deviate to induce the more informative outcome produced by $(null, reveal_2)$. The modified game where firms can disclose information about their competitors is Blackwell-connected. More generally, whenever the information environment is Blackwell-connected, any individual sender can generate as much information as all senders can do jointly.

Our main result shows that every pure-strategy equilibrium outcome is no less informative than the collusive outcome (regardless of preferences) if and only if the information environment is Blackwell-connected. Moreover, when the environment is Blackwell-connected and all senders have access to the same set of signals, there is a simple way to characterize the set of pure-strategy equilibrium outcomes.

Building on the example above, we might ask more generally whether a joint venture between two pharmaceutical companies that allows them to coordinate decisions about clinical trials would result in consumers becoming more or less informed about the quality of the firms' drugs. If each firm can commission a range of clinical trials about the efficacy of both drugs, the environment is Blackwell-connected, and our result implies that the joint venture could only reduce consumers' information. In contrast, if each firm can conduct clinical trials only about its own drug, the information environment is not Blackwell-connected, so there are some demand systems for which the joint venture will make consumers more informed. We develop this example in more detail below.

We also analyze two other notions of increased competition: adding senders and decreasing the alignment of senders' preferences. When considering these additional comparative statics, we restrict our attention to situations where each sender has access to the same set of signals and we focus on minimally informative, or *minimal*, equilibria. (These equilibria have some desirable properties we discuss below.) We find that if the environment is Blackwell-connected, neither introducing additional senders nor increasing preference misalignment can decrease the informativeness of minimal equilibria.

To simplify our main comparative statics, we assume that the collusive outcome is unique, and we focus on minimal equilibria when we vary the number of senders or preference alignment. In Section 7, we drop these assumptions and state our comparative statics using set comparisons. We also show that the Blackwell-connectedness condition can be significantly weakened if we restrict attention to monotone preferences. Finally, we briefly discuss mixed strategy equilibria and non-Blackwell information orders.

Our work connects to several strands of existing literature. First, our analysis relates to work on multi-sender communication (e.g., Milgrom and Roberts 1986; Krishna and Morgan 2001; Battaglini 2002).³ Our model differs from this literature in three ways. First, senders' information in our model is endogenous, but we abstract from incentive compatibility issues in disclosure, such as those that arise in cheap talk models (e.g., Crawford and Sobel 1982) and persuasion games with verifiable information (e.g., Grossman 1981; Milgrom 1981; Milgrom and Roberts 1986; Bull and Watson 2004; Kartik and Tercieux 2012). Second, we

³Bhattacharya and Mukherjee (2013) analyze multiple-sender persuasion games when there is uncertainty about whether each sender is informed. Under the assumption that senders' preferences are single-peaked and symmetric, they geometrically characterize the equilibrium strategies. They establish that receiver's payoff may be maximized when senders have identical, extreme preferences rather than opposed ones. Chen and Olszewski (2014) analyze a model of debate in which two senders with opposed preferences try to convince a receiver. They take senders' information as exogenous and do not consider comparative statics with respect to the extent of competition. Banerjee and Somanathan (2001) show how the relative homogeneity of preferences within a group affects the ability of individuals to communicate information to a leader.

focus on comparative statics that hold for any preferences, whereas most existing papers focus on identifying specific preferences under which full revelation is possible. Finally, we consider several different notions of competition — comparing non-cooperative equilibria to collusion, varying the set of senders, and varying the alignment of senders’ preferences.

Second, our work relates to research on advocacy. Dewatripont and Tirole (1999) consider a principal who employs agents to gather costly information. Effort exerted to gather information is unobservable and experts’ wages are contingent only on the principal’s decision. The authors establish that employing two advocates with opposed interests is preferable to employing a single unbiased agent; it either generates more information or yields less rent to the employee(s). The main force behind this result is moral hazard in generating costly information; in our setting, information is costless so this force does not play a role. Shin (1998) considers a setting where two advocates get two independent draws of a signal whereas an unbiased investigator gets a single draw. In contrast, we analyze a broader set of environments including those where competition does not change the set of feasible signals.

Third, our analysis is related to a recent literature that examines situations with *ex ante* symmetric information and multiple senders. Gentzkow and Kamenica (2015) analyze a very specific environment where each sender can conduct any experiment whatsoever, including an experiment whose realizations are arbitrarily correlated with the realization of other senders’ experiments.⁴ Their analysis draws on the algebraic structure of that particular setting. Brocas *et al.* (2012) and Gul and Pesendorfer (2012) examine settings where two senders with exactly opposed interests provide costly signals about a binary state of the world.⁵

2 Model

2.1 Setup

Let Ω be the state space, with a typical state denoted ω . There are n senders indexed by i who share a common prior μ_0 . The senders play a simultaneous-move game in which each sender i chooses a signal

⁴Moreover, in contrast to this paper, Gentzkow and Kamenica (2015) assume that senders observe the outcomes of their experiments privately and can potentially withhold information they have learnt.

⁵Ostrovsky and Schwarz (2010) examine a model where schools choose how much information to provide about their students’ abilities so as to maximize the students’ job placement. They focus on a different question than we do – they examine whether the amount of information revealed depends on how students’ abilities are distributed across schools.

$\pi_i \in \Pi_i$. Throughout the paper we focus on pure-strategy equilibria. We denote a strategy profile by $\pi = (\pi_1, \dots, \pi_n)$. Let $\Pi \equiv \times \Pi_i$ and $\Pi_{-i} \equiv \times_{j \neq i} \Pi_j$. We refer to Π as the *information environment*.

For some results, we will focus on situations where *each sender has access to the same set of signals*, i.e., where $\Pi_i = \Pi_j$ for any pair i and j .

A *signal* π is a random variable whose distribution may depend on ω . Given a set of signals P , let $\langle P \rangle$ denote the distribution of beliefs of a Bayesian with prior μ_0 who observes the realization of all signals in P .⁶ We say that $\pi = \pi'$ if $\langle \{\pi, \pi'\} \rangle = \langle \pi \rangle = \langle \pi' \rangle$. Note that in this notation, saying that $\pi = \pi'$ does not only mean that observing π yields as much information as observing π' ; it also means that the information in the two signals is mutually redundant. Two i.i.d. signals are not the same signal, whereas two identically distributed and perfectly correlated signals are.

A sender's payoff is a function of the aggregate information revealed jointly by all the signals. We summarize this information by the distribution of beliefs $\langle \pi \rangle$. Sender i 's payoff given distribution of beliefs $\tau = \langle \pi \rangle$ is denoted by $v_i(\tau)$. In Subsection 2.2, we discuss a range of models nested by this specification.

If π is a Nash equilibrium, we say $\langle \pi \rangle$ is an *equilibrium outcome*. If π^c solves $\max_{\pi \in \Pi} \sum_i v_i(\langle \pi \rangle)$, we say that $\langle \pi^c \rangle$ is a *collusive outcome*. All of our results remain true (with near-identical proofs) if we define collusive outcomes based on other aggregations of senders' preferences.⁷ We say an outcome τ is *feasible* if there exists $\pi \in \Pi$ such that $\tau = \langle \pi \rangle$.

Let \geq denote the Blackwell (1953) order on distributions of beliefs. That is, $\tau \geq \tau'$ if τ is a mean preserving spread of τ' , in which case we say that τ is *more informative* than τ' . It is immediate that if $P' \subseteq P$, we have $\langle P \rangle \geq \langle P' \rangle$. If $\tau \not\geq \tau'$, we say that τ' is *no less informative* than τ . If $\tau \geq \tau'$ or $\tau' \geq \tau$, we say that τ and τ' are *comparable*.

We assume that no sender is forced to provide information, so that each Π_i includes the *null signal* $\underline{\pi}$ s.t. $\langle P \cup \underline{\pi} \rangle = \langle P \rangle$ for any P . We also make a technical assumption that the set of equilibrium outcomes is non-empty and compact. This could be guaranteed by assuming that each $v_i(\langle \pi \rangle)$ is jointly continuous in

⁶Formally, $\langle \cdot \rangle$ is a function from the set of all subsets of $\cup_i \pi_i$ to $\Delta(\Delta(\Omega))$; i.e., it maps any set of signals into a distribution over posterior beliefs. We will abuse notation by writing $\langle \pi \rangle$ for $\langle \{\pi\} \rangle$ and writing $\langle \pi \rangle$ for $\langle \{\pi_i\}_{i=1}^n \rangle$.

⁷Consider any function V such that if $v_i(\tau') \geq v_i(\tau'')$ for all i and at least one inequality is strict, then $V(\tau') > V(\tau'')$. If we define a collusive outcome to be a τ that maximizes V , all of our results remain true. For example, we could assume that $V(\tau) = \sum \lambda_i v_i(\tau)$ to reflect that some senders' have more influence in determining the collusive outcome. Or, we could assume that $V(\tau) = \prod \max(v_i(\tau) - v_i^d, 0)$ to reflect that senders reach the collusive outcome through Nash bargaining where sender i 's outside option is v_i^d .

In Section 7, where we drop the assumption that the collusive outcome is unique, we could also simply define a collusive outcome as any Pareto-undominated outcome. That definition, however, would be problematic under the assumption that the collusive outcome is unique.

all components of π and that each Π_i satisfies regularity conditions.⁸

Finally, to ease exposition we assume that the collusive outcome is unique. This will be true generically.⁹ When we say a result holds “regardless of preferences,” we mean that it holds for any preferences consistent with the uniqueness of the collusive outcome. In Section 7, we relax the uniqueness assumption and state our results using orders on sets.

2.2 Interpretation

One natural interpretation of our model is a case where the senders wish to influence the action $a \in A$ of a receiver with a utility function $u(a, \omega)$. Advertisers might want to influence a customer’s purchases, political candidates might want to influence a citizen’s vote, or firms might want to influence an investor’s allocation of resources. In these cases, the sender preferences $v_i(\tau)$ would be derived from a more primitive preference $\tilde{v}_i(a, \omega)$ over the receiver’s action and the state:

$$v_i(\tau) = E_\tau \left[E_\mu \tilde{v}_i(a^*(\mu), \omega) \right] \quad (1)$$

where $a^*(\mu)$ is the action chosen by the receiver given belief μ , the outer expectation is taken over beliefs μ in the support of τ , and the inner expectation is taken over states in the support of μ .¹⁰

Our framework also allows for circumstances where the receiver’s action depends on τ as well as μ . Suppose, for instance, that sender i is a seller and the receiver is a potential buyer who must pay a cost to visit the seller’s store. The seller chooses information π_i that will be revealed about ω , the buyer’s value for the seller’s good. If the receiver observes the choice of π_i (but not its realization) before deciding whether to visit the store, then a^* depends on τ as well as μ .

Finally, our model applies to settings with multiple receivers who may engage in strategic interaction with each other. For instance, suppose sender i is an auctioneer and there are a number of bidders. The sender chooses information π_i that will be revealed about ω , the bidders’ values for the good (e.g., as in Milgrom

⁸Namely, that it is a non-empty compact subset of a locally compact Hausdorff topological vector space.

⁹If we perturb any set of preferences replacing $v_i(\tau)$ with $v_i(\tau) + \varepsilon$ where ε has an atomless distribution, the collusive outcome will be unique almost surely.

¹⁰This microfoundation puts some restrictions on the induced preferences over informational outcomes: not every v_i can be generated by choosing a suitable u and \tilde{v}_i . This is particularly important to keep in mind when interpreting the results that a given property of the information environment is necessary for comparative statics to hold for all possible v_i . It is an open question of whether there is a weaker condition on the information environment that is necessary for comparative statics to hold for any v_i generated by some u and \tilde{v}_i . Also, note that continuity of u and \tilde{v}_i are not sufficient to guarantee continuity of v_i when a^* is not single-valued.

and Weber 1982). Our model covers both the case of common values (where ω is one-dimensional) and the case of private values (where ω is the vector of valuations). It also applies both to the case where the bidders observe the same signal realizations (in which case $a^*(\mu)$ is the equilibrium vector of actions given the commonly held posterior μ) and the case where each bidder observes an independent draw of the signal realizations (in which case the equilibrium actions may be a function of τ as well as μ).

2.3 Discussion of Assumptions

Our model makes several important assumptions. First, we assume that the information generated is publicly observed, and so abstract from incentive issues in disclosure, such as those that arise in cheap talk or verifiable message settings. In Kamenica and Gentzkow (2011), we discuss the kinds of real-world settings where this assumption is suitable.

Second, an important feature of our model is that senders do not have any private information at the time they choose their signal. If they did, the choice of the signal could convey information even conditional on the signal realization, which would substantially complicate the analysis.¹¹

Third, we assume that signals are costless.¹² Were we to relax this assumption, there would be an additional force by which competition could decrease information revelation: a classic public goods problem. Suppose for instance that all senders have the same preferences $v_i(\tau) \equiv v(\tau)$ and prefer more information to less: $\tau > \tau' \implies v(\tau) > v(\tau')$. Then, if generating information is privately costly, non-cooperative strategic behavior would reduce provision of information relative to social optimum. Ruling out costs is clearly a restrictive assumption, but the fact that we allow for arbitrary Π_i 's means that our framework does allow for cases where some signals are prohibitively costly to generate. The possibility that Π_i can vary across senders means that some senders might have an advantage in accessing certain kinds of information.

Fourth, we restrict our attention to situations where senders move simultaneously. The impact of competition on the amount of information revealed can be quite different if information is provided sequentially. Even if the information environment is Blackwell-connected and every sender is always given additional opportunities to reveal more information, competition does not necessarily lead to more information revelation. For example, there can be subgame perfect equilibria where each sender remains quiet because he

¹¹Perez-Richet and Prady (2012) and Rosar (2014) consider related models where senders are privately informed.

¹²An implication is that a sender cares only about the aggregate information revealed. In this sense, our model is related to the literature on aggregate games (e.g., Martimort and Stole 2012), though the preferences we consider are even starker than in that literature as we assume each sender cares only about the aggregate and not about her own contribution to it.

knows that if he provides information he would like revealed, that will in turn cause further revelations by others, resulting in an undesirable outcome.¹³

Finally, our model implicitly assumes that no sender can drown out the information provided by others, say by sending many useless messages. This is the basic import of the fact that $P' \subseteq P$ implies that $\langle P \rangle \geq \langle P' \rangle$. One interpretation of this assumption is that receiver is a classical Bayesian who can costlessly process all information she receives. This means that, from receiver's point of view, the worst thing that any sender can do is to provide no information.

3 The information environment

Definition. Π is *Blackwell-connected* if for all i , $\pi \in \Pi$, and $\pi_{-i} \in \Pi_{-i}$ such that $\langle \pi \rangle \geq \langle \pi_{-i} \rangle$, there exists a $\pi_i \in \Pi_i$ such that $\langle \pi \rangle = \langle \pi_{-i} \cup \pi_i \rangle$.

In other words, an information environment is Blackwell-connected if, given any strategy profile, each sender can unilaterally deviate to induce any feasible outcome that is more informative. We illustrate this definition with several examples.

Example 1. (*Number of draws*) Given a signal π , each sender i chooses the number of independent draws from π to generate. Signals from distinct senders are uncorrelated conditional on the state, so if sender i chooses n_i draws, aggregate information is simply $\sum_i n_i$ independent draws from π .

Example 2. (*Precisions*) Suppose $\Omega \subset \mathbb{R}$ and let π^h be a normal signal with precision h —i.e., π^h generates a signal realization s with distribution $\mathcal{N}\left(\omega, \frac{1}{h}\right)$. Each sender chooses the precision $h_i \in \mathbb{R}_+$ of her signal. (We interpret the signal with zero precision as the uninformative signal $\underline{\pi}$.) Signals from distinct senders are uncorrelated conditional on the state, so aggregate information is a normal signal with precision $\sum_i h_i$.

Example 3. (*Partitions*) Each sender chooses a partition of Ω . Observing the realization of a signal means learning which element of the partition the state is in (as in Aumann 1976).

Example 4. (*Facts*) There is a set F of facts about ω and revealing any one of these facts generates an *i.i.d.* signal. Each sender i chooses a subset $F_i \subset F$ of facts to reveal. The outcome is determined by the total number of facts that are revealed, i.e., by the cardinality of $\cup F_i$.

¹³Li and Norman (2015) provide additional analysis of the sequential-move version of our model. Brocas *et al.* (2012) and Gul and Pesendorfer (2012) consider dynamic environments with costly information provision by conflicting parties.

Example 5. (*All-or-nothing*) Each sender has access to only two signals, $\underline{\pi}$ that reveals nothing and $\bar{\pi}$ that fully reveals the state of the world.

All of these information environments are Blackwell-connected.

A key implication of an environment being Blackwell-connected is that an individual sender can unilaterally provide as much information as several senders can do jointly. Say that *individual feasibility equals aggregate feasibility* if for any sender i , $\{\langle \pi \rangle | \pi \in \Pi_i\} = \{\langle \pi \rangle | \pi \in \Pi\}$.¹⁴

Remark 1. If the information environment is Blackwell-connected, then individual feasibility equals aggregate feasibility.

This follows from the observation that for any $\pi \in \Pi$ and any sender i , Π being Blackwell-connected means there is a $\pi_i \in \Pi_i$ s.t. $\langle \pi_i \cup \underline{\pi} \rangle = \langle \pi \rangle$ where $\underline{\pi} = (\underline{\pi}_1, \dots, \underline{\pi}_i) \in \Pi_{-i}$. Remark 1 implies that the information environment from the example in the introduction cannot be Blackwell-connected because each firm can generate information only about the efficacy of its own drug. More generally, whenever a firm can only reveal information about itself (e.g., as in Admati and Pfleiderer 2000), the environment is not Blackwell-connected. Also, consider the *Number of draws* environment, but modify it so that each sender can generate no more than some fixed number of draws. Remark 1 implies that this modified information environment is not Blackwell-connected.

Remark 1 identifies the most important feature of Blackwell-connectedness. When the environment is Blackwell-connected, no sender has an advantage in generating specific type of information. Consequently, eliminating senders does not reduce the set of feasible outcomes. Such removals only impact information revelation by changing equilibrium interactions. It is also important to note that Blackwell-connectedness does not merely assume that each sender can provide at least as much information as other senders; it requires the stronger condition that each sender can *precisely* match the informativeness of signals provided by others. Hence, even environments where each sender can generate a fully informative signal are not necessarily Blackwell-connected.

Having individual feasibility equal aggregate feasibility is thus a necessary condition for Blackwell-connectedness, but it is not quite sufficient. An information environment can also fail to be Blackwell-

¹⁴Note that if each sender has access to the same set of signals, then individual feasibility necessarily equals aggregate feasibility, but the converse does not hold. In the *Number of draws* and *Precisions* environments, for example, individual feasibility equals aggregate feasibility, but it is not the case that each sender has access to the same set of signals. In contrast, *Partitions*, *Facts* and *All-or-nothing* are environments where each sender has access to the same set of signals.

connected if the sets Π_i are insufficiently rich. Suppose, for example, that we modify the *Number of draws* environment so that each sender must generate at least two draws (unless she sends the null signal). Even though each sender can unilaterally generate any feasible outcome, this information environment is not Blackwell-connected: the outcome with three total draws is feasible and more informative than the outcome with two total draws, but if π_{-i} induces the outcome with two total draws, there is no π_i that sender i can choose such that $\pi_{-i} \cup \pi_i$ induces the outcome with three.

4 Competition versus collusion

4.1 Main result

When does competition increase information revelation? In this section, we consider this question by comparing equilibrium outcomes to the collusive outcome. Since this comparison is meaningful only when there are at least two senders, we assume that $n \geq 2$. Recall that we maintain the assumption that the collusive outcome is unique. Our main result is the following:

Proposition 1. *Suppose $n \geq 2$. The collusive outcome cannot be more informative than an equilibrium outcome (regardless of preferences) if and only if the information environment is Blackwell-connected.*

The basic intuition behind this proposition is the following. Let τ^* be some equilibrium outcome and let τ^c denote the collusive outcome. Suppose contrary to the proposition that $\tau^c > \tau^*$. It must be the case that for at least one sender i we have $v_i(\tau^c) > v_i(\tau^*)$; otherwise, it could not be the case that τ^c is collusive and τ^* is not. But, since the environment is Blackwell-connected, this sender i could deviate from the strategy profile that induces τ^* and induce τ^c instead. Hence, τ^* could not be an equilibrium.

The proof of the converse is constructive. If the environment is not Blackwell-connected, there is some strategy profile π^c and some $\pi'_{-i} \in \Pi_{-i}$ such that $\langle \pi^c \rangle \geq \langle \pi'_{-i} \rangle$ but player i cannot induce π^c when others are playing π'_{-i} . Consider a strategy profile π^* where i sends the null signal and others play π'_{-i} . If sender i strictly prefers π^c to π^* and other senders are indifferent, then π^c is collusive, π^* is an equilibrium, and yet $\langle \pi^c \rangle > \langle \pi^* \rangle$.

The environment being Blackwell-connected does not by itself ensure that an equilibrium outcome is comparable to the collusive outcome. It might be the case that the collusive outcome yields information that is more relevant for a particular decision maker than an equilibrium outcome does. In fact, an argument

closely related to the proof of Proposition 1 can be used to show that every equilibrium outcome is more informative than the collusive outcome if and only if the information environment is Blackwell-connected and all feasible outcomes are comparable.

4.2 Illustrations

In this subsection, we illustrate the implications of Proposition 1 with a few examples.

4.2.1 Clinical trials on competitors' drug

We wish to know whether a joint venture between two pharmaceutical companies that allows them to coordinate clinical trials would result in consumers being more or less informed about the firms' drugs.

Suppose that each firm can commission a third-party to conduct a clinical trial, and can freely choose any number of subject groups. Each additional group provides an i.i.d. signal about the quality of both drugs. This environment is Blackwell-connected (and moreover, any two outcomes are comparable.) Hence, Proposition 1 tells us that regardless of the demand structure—the extent of differentiation between the firm's products, consumers' outside options, etc.—the joint venture cannot increase consumers' information about the firms' drugs.

In contrast, suppose that each firm has a comparative advantage in generating certain types of information. For instance, it might be the case that a firm is not allowed to conduct clinical trials about its competitor's drug. Remark 1 tells us that whenever such comparative advantage exists, the information environment is not Blackwell-connected. By Proposition 1, whether a joint venture leads to more or less information will depend on the demand structure. For some demand structures, such as the one in the introductory example, the joint venture could make consumers more informed.

4.2.2 Educating consumers about a new technology

There are two firms, G and N in a food industry. Firm G 's food contains genetically modified organisms (GMOs) whereas firm N 's food does not. Each firm can conduct an investigation into safety of GMOs. An investigation by firm i generates an i.i.d. normal signal about safety with precision $h_i \in [0, H]$ where $H \in [0, \infty]$. Precision of 0 is equivalent to the null signal $\underline{\pi}$. Aggregate information τ_h is determined by $h \equiv h_G + h_N$. Uncertainty about the safety of GMOs reduces the expected demand for firm G 's product and

somewhat increases the expected demand for firm N 's product, but by substantially less. Consequently, $v_G(\tau_h)$ and $v_G(\tau_h) + v_N(\tau_h)$ are increasing in h while $v_N(\tau_h)$ is decreasing in h .

First, consider the case where $H = \infty$, i.e., each firm can convincingly reveal the safety of GMOs on its own. In that case, the collusive outcome is full revelation (τ_∞). Moreover, the environment is Blackwell-connected, so the equilibrium outcome cannot be any less informative: full revelation is also the unique equilibrium outcome. (Clearly, firm G chooses the fully revealing signal.)

In contrast, suppose that H takes on some finite value. This implies that no amount of available evidence will completely eliminate consumers' uncertainty. more importantly, it also means that firm G cannot unilaterally reveal as much information as firms G and N can do together. Hence, the environment is not Blackwell-connected, and we cannot be certain that equilibria will be more informative than the collusive outcome. In fact, the unique equilibrium profile is $h_G = H$ and $h_N = 0$, strictly less informative than the collusive $h_G = h_N = H$. Firm G would like to bribe firm N to reveal information, but contractual incompleteness prevents it from doing so. This can happen whenever the environment is not Blackwell-connected.

4.2.3 Dislike of partial information

There are two facts about the world θ and γ which jointly determine the state of the world. There are four possible outcomes, determined by which facts are revealed: $\tau_{\{\theta, \gamma\}}$, $\tau_{\{\theta\}}$, $\tau_{\{\gamma\}}$, and τ_\emptyset where τ_S is the outcome when facts in S are revealed. Each firm i can reveal any or all facts in the set $F_i \subset \{\theta, \gamma\}$. All firms have the same preference over the informational outcome: $v_i(\tau_{\{\theta, \gamma\}}) > v_i(\tau_\emptyset) > v_i(\tau_{\{\theta\}}), v_i(\tau_{\{\gamma\}})$. In other words, all firms would prefer a fully informed public, but partial information is worse than no information.

First, consider the Blackwell-connected environment where $F_i = \{\theta, \gamma\}$ —i.e., any firm can reveal both facts. In this case, the collusive outcome and the unique equilibrium coincide at full information.

In contrast, suppose that both facts can be revealed ($\cup F_i = \{\theta, \gamma\}$), but each individual firm can reveal at most one fact ($|F_i| \leq 1$). In this case, the environment is not Blackwell-connected, and there is an equilibrium outcome τ_\emptyset where nothing is revealed. Hence, competition is bad for information. This example illustrates how equilibrium miscoordination can lead to less information being revealed than would take place under collusion, but only if the environment is not Blackwell-connected.

4.2.4 Racing to release news

There are two media firms, 1 and 2, that report on a breaking news event. The event occurs at time $t = 0$. The truth about the event is $x \sim N(0, 1)$, and firm i 's report is defined by its content $\hat{x}_i \in \mathbb{R}$ and the time $\hat{t}_i \in \{0, 1, 2\}$ when it is published. Each firm employs reporters of uncertain quality $q_i \in \{0, 1\}$, with independent prior probabilities $Pr(q_i = 1) = \frac{1}{4}$.

Each firm wants to convince consumers that its reporters are high quality but also may care about the accuracy of its published report. The key decision for each firm is whether to be fast or thorough. Being fast means the firm can release its report quickly, but the content will be both less accurate and less independent, since the reporters will not have time for fact checking and will have to rely on public information. Being thorough means the content will be more informative, but it will be released with a delay. High quality ($q_i = 1$) reporters have an advantage in investigative reporting, so they are thorough with less delay.

Formally, the state is $\omega = (x, q_1, q_2)$, and Π_i consists of two reporting strategies, *fast* _{i} and *thorough* _{i} . All firms have immediate access to public information $z \sim N(x, 1)$ which is released at time zero. Firms can also ask their reporters to gather additional information $w_i \sim N(x, 1)$ drawn independently across i . High-quality reporters learn and report w_i in one period, while the delay for low-quality reporters is two periods. Choosing signal *fast* _{i} means $\hat{x}_i = z$ and $\hat{t}_i = 0$; choosing *thorough* _{i} means $\hat{x}_i = (z + w_i)/2$ and $\hat{t}_i = 1 + q_i$. Note that being thorough unambiguously makes the outcome more informative since it provides more information both about x and about the reporters' quality.

Firms care about profit but may also have public-interest incentives. The profit motive is shaped by consumers who will in the future buy from whichever firm they believe to be higher quality. The public-interest incentive is served by maximizing the precision of aggregate information available about x . In particular, firm i 's payoffs are

$$v_i(\tau) = E_\tau [4 \cdot \text{demand}_i + \lambda_i \cdot \text{precision}],$$

where $\lambda_i \geq 0$, *demand* _{i} equals 1 if the posterior probability that i is high quality is greater than the corresponding posterior for i 's competitor, 0 if it is less, and $\frac{1}{2}$ if the two posteriors are equal, and *precision* denotes the inverse of the variance of the aggregate error $\frac{\hat{x}_1 + \hat{x}_2}{2} - x$.

How does competition in this market play out? The firms face a tradeoff. By asking its reporters to

be thorough, a firm improves the quality of its information, which serves the public interest. But the firm also reveals its type through the delay in its reporting, which can be a disadvantage in competition with the other firm. To solve the game, note that the expectation of *precision* is 1 when both choose *fast*, 2 when one chooses *fast* and the other chooses *thorough*, and 3 when both choose *thorough*. The expectation of *demand_i* is $\frac{1}{2}$ whenever both firms choose the same signal, $\frac{3}{4}$ if a firm chooses *fast* and its competitor chooses *thorough*, and $\frac{1}{4}$ if a firm chooses *thorough* and its competitor chooses *fast*.¹⁵ We thus have the following two-by-two game, where for simplicity (since the game is symmetric) we write only the payoffs of the row player ($i = 1$):

| | | |
|------------------------------|--------------------------|------------------------------|
| | <i>fast</i> ₂ | <i>thorough</i> ₂ |
| <i>fast</i> ₁ | $2 + \lambda_1$ | $3 + 2\lambda_1$ |
| <i>thorough</i> ₁ | $1 + 2\lambda_1$ | $2 + 3\lambda_1$ |

Note that the unique collusive outcome is for both firms to be thorough whenever $\lambda_i > 0$ for at least one i .

This information environment is not Blackwell-connected: each firm can produce information about the quality of its own reporters but not about the reporters of the other firm; moreover, the two firms together can produce more information about x than either firm can do on its own. Consistent with this, the effect of competition relative to collusion depends on preferences. If both players have strong public interests ($\lambda_1, \lambda_2 > 1$), being thorough is a dominant strategy, so competition has no impact on the outcome. If both have weak public interests ($\lambda_1, \lambda_2 < 1$), competition reduces information: the game is a prisoner's dilemma and *fast* is a dominant strategy for both players. If one player's public interest is strong and the other's is weak, the equilibrium is asymmetric and competition again reduces information.

How could we modify the information environment to make competitive interaction more informative? A natural policy to consider would be an embargo specifying that no reports could be published before period 2 (so even high quality thorough reports are delayed by two periods). This would make *demand_i* independent of firms' reporting strategies. The information environment would still not be Blackwell-connected, however, because the precision of the signal ($thorough_1, thorough_2$) would still be greater than what either firm could produce on their own. Consistent with this, the set of preferences under which competition is harmful shrinks but remains non-empty. For example, if $\lambda_1 > 0$ and $\lambda_2 = 0$, ($thorough_1, fast_2$) is an equilibrium less informative than the collusive outcome.

¹⁵To see this, suppose 1 chooses *fast* and 2 chooses *thorough*. Then 2's quality will be revealed through its delay, while 1's quality will remain uncertain. Since $Pr(q_2 = 1) = \frac{1}{4}$, we have $E_\tau[demand_1] = \frac{3}{4}$ and $E_\tau[demand_2] = \frac{1}{4}$.

If, in addition to prohibiting early reporting, we allow each firm to report both w_1 and w_2 in period 2, the environment becomes Blackwell-connected. In this case, consistent with Proposition 1, no equilibrium outcome can be less informative than the collusive outcome, regardless of the λ_i 's.

5 Characterizing equilibrium outcomes

5.1 General result

While Proposition 1 allows us to assess the effect of competition, determining what actually happens in equilibrium can be challenging. Whether a particular strategy profile π is an equilibrium depends on the full set of outcomes that each player can deviate to, and for each player this set can depend on the signals chosen by her competitors. We do not have a general recipe for solving this fixed point problem.

When the information environment is Blackwell-connected, however, the set of outcomes that a sender can deviate to becomes easy to identify: sender i can induce a feasible outcome τ if and only if $\tau \geq \langle \pi_{-i} \rangle$. The “only if” part of this claim is trivial. The “if” part is equivalent to the environment being Blackwell-connected. Moreover, the “if” part imposes an important restriction on equilibrium outcomes. Say that an outcome τ is *unimprovable for sender i* if for any feasible $\tau' \geq \tau$, we have $v_i(\tau') \leq v_i(\tau)$. If Π is Blackwell-connected, any equilibrium outcome must be unimprovable for all senders. Moreover, if each sender has access to the same set of signals and there are at least two senders, this condition is not only necessary but also sufficient for a given τ to be an equilibrium outcome:

Proposition 2. *Suppose each sender has access to the same set of signals, the information environment is Blackwell-connected, and $n \geq 2$. A feasible outcome is an equilibrium outcome if and only if it is unimprovable for each sender.*

As we already mentioned, the “only if” part of this result follows directly from the environment being Blackwell-connected. The “if” part relies on having multiple senders and on the strong assumption that each sender has access to the same set of signals. Given an outcome τ that is unimprovable for each sender, there must be a π such that $\langle \pi \rangle = \tau$. Then (π, \dots, π) is an equilibrium: the only deviations possible are those that yield more information, but such deviations cannot be profitable.¹⁶

¹⁶When senders do not have access to the same set of signals, there may be an outcome τ that is unimprovable for everyone, but that can only be induced by strategy profiles that allow some senders to deviate to less-informative outcomes; hence, τ might not be an equilibrium outcome.

Proposition 2 provides an easy way to determine the set of equilibrium outcomes. We need only take an intersection of sets. We illustrate this in Figure 1.

We consider a *Facts* environment with a unit measure of facts. Each of the two senders chooses a set of facts $F_i \subset [0, 1]$ to uncover and the outcome is determined by the overall share of facts that are revealed—i.e., by the measure of $F_1 \cup F_2$. Hence, we can represent each feasible τ simply as an element of the unit interval.

Panel (a) shows the graph of a hypothetical $v_1(\tau)$ as well as the outcomes that are unimprovable for sender 1 given those preferences (the thick lines on the x-axis). As the figure shows, this set of unimprovable outcomes is a union of two intervals. The dashed lines show how the set of unimprovable outcomes is identified. Panel (b) depicts the same information for sender 2, with another hypothetical utility function $v_2(\tau)$. In this case, the set of unimprovable outcomes is the union of a single interval and the fully revealing outcome.

Panel (c) depicts how to construct the equilibrium set. The set of unimprovable outcomes for sender 1 (M_1) is shown right above the unimprovable outcomes for sender 2 (M_2). Their intersection, $M = [\tau_1, \tau_2] \cup \{1\}$, is the set of equilibrium outcomes. What is revealed in equilibrium is either all facts, or some share of facts between τ_1 and τ_2 .

We can also illustrate our main comparative statics result in Figure 1. Panel (d) shows collusive preferences, $v_1(\tau) + v_2(\tau)$. The dashed line depicts its argmax, the collusive outcome τ^c . Panel (c) shows that τ^c reveals fewer facts than any equilibrium outcome, consistent with Proposition 1.

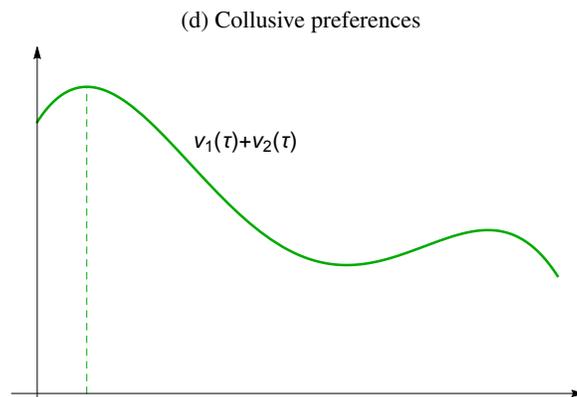
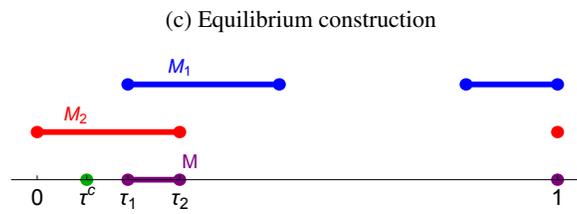
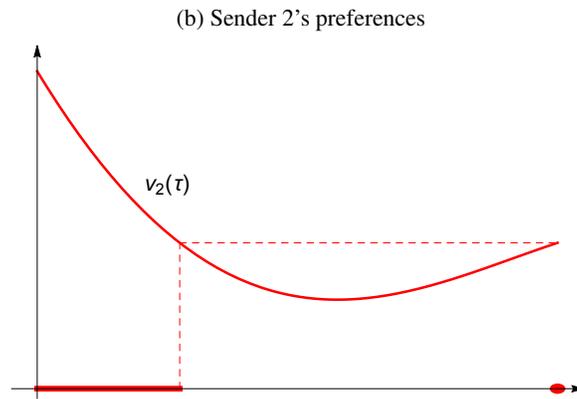
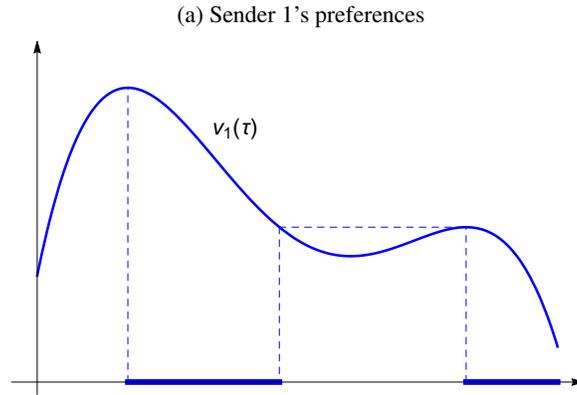
Finally, note that Proposition 2 implies that—when the premises of the Proposition hold—any maximally informative, feasible outcome is an equilibrium outcome. This is the case even if all senders prefer to reveal as little information as possible. For this reason, in Section 6, we focus on minimally informative equilibria, which are always Pareto-preferred by the senders.

5.2 *Partitions* environment with simple persuasion preferences

The analysis in the previous subsection applies regardless of senders' preferences and applies to any Blackwell-connected environment. In this subsection, we examine a special case that allows us to provide a sharper characterization of equilibrium. The sharper characterization requires us to test unimprovability of individual beliefs, rather than full distributions of beliefs.

Suppose that we are in the *partitions* environment (Section 3), where each sender can choose an arbitrary

Figure 1: Characterizing equilibrium outcomes



partition of Ω as in Aumann (1976). Denote an event by $E \subset \Omega$, and denote a partition of Ω that induces outcome τ by ρ_τ . Suppose that senders have what we will call *simple persuasion preferences*: conditional on a realized event E , senders' payoffs are independent of the partition from which it was drawn. That is, we can write $v_i(\tau) = \sum_{E \in \rho_\tau} \mu_0(E) \tilde{v}_i(E)$ for some $\tilde{v}_i(\cdot)$, where $\mu_0(E)$ denotes the prior probability of event E . This will be true, for example, when information only matters to senders via its effect on the action $a \in A$ of a receiver with a utility function $u(a, \omega)$.

The partitions environment is Blackwell-connected and each sender has access to the same set of signals, so Proposition 2 applies: a strategy profile that induces partition ρ_τ is an equilibrium if and only if τ is unimprovable for each sender. But we can go further. Say that *an event E is unimprovable for sender i* if for every partition ρ' of E we have

$$\tilde{v}_i(E) \geq \sum_{E' \in \rho'} \frac{\mu_0(E') \tilde{v}_i(E')}{\mu_0(E)}.$$

In other words, E is unimprovable for sender i if, given that a signal has revealed the state is in E , sender i would rather not provide any further information.

We can then characterize equilibria in terms of unimprovable events. This simplifies the application of the result significantly, because we can analyze the improbability of each belief a strategy profile induces in isolation, rather than having to consider each possible distribution of such beliefs.

Proposition 3. *Suppose $n \geq 2$, each sender chooses an arbitrary partition of Ω , and senders have simple persuasion preferences. Then, a strategy profile is an equilibrium if and only if each event in the induced partition is unimprovable for every sender.*

The argument behind this Proposition is closely related to the proof of Proposition 2.

6 Other notions of increased competition

In this section we consider two other notions of increased competition: adding senders and making senders' preferences less aligned. The most general version of this analysis, which we take up in Section 7.1, requires us to compare sets of equilibria. Here, we simplify the problem by focusing on a particular class of equilibria.

The class we define rules out a certain type of miscoordination among senders. When the antecedent of Proposition 2 is satisfied and τ is an equilibrium outcome, any feasible $\tau' \geq \tau$ is also an equilibrium

outcome. In particular, if the set of feasible outcomes has a maximum element $\bar{\tau}$, this is an equilibrium outcome regardless of senders' preferences. Hence, even if all senders have the exact same preferences and dislike providing any information, revealing all information is still an equilibrium. Such "excessively informative" equilibria clearly rely on miscoordination. More broadly, for any two comparable equilibrium outcomes, the less informative one must be preferred by all senders:

Remark 2. Suppose the information environment is Blackwell-connected. If π and π' are equilibria and $\langle \pi \rangle \geq \langle \pi' \rangle$, then for each sender i , $v_i(\langle \pi' \rangle) \geq v_i(\langle \pi \rangle)$.

To see this, note that Blackwell-connectedness implies sender i can unilaterally deviate from π' to π , so if $v_i(\langle \pi' \rangle)$ were strictly less than $v_i(\langle \pi \rangle)$, π' could not be an equilibrium.

We say that π is a *minimal equilibrium* if π is an equilibrium and there is no equilibrium π' such that $\langle \pi \rangle > \langle \pi' \rangle$. If π is a minimal equilibrium, we say that $\langle \pi \rangle$ is a *minimal equilibrium outcome*. Minimal equilibria rule out equilibrium miscoordination on excessively informative outcomes.

6.1 Adding senders

How do equilibrium outcomes change as we add senders to the game? Intuitively, more senders corresponds to greater competition, and intuition from Milgrom and Roberts (1986) and others suggests this could increase information revelation. We show that this intuition generalizes when our Blackwell-connectedness condition is satisfied and senders have access to the same set of signals. In applications, these results would be relevant to considering the impact of barriers to entry or licensing restrictions.

We consider changing the set of senders from J' to J , where $J' \subset J$. Note that if the information environment is Blackwell-connected when the set of senders is J , it is also Blackwell-connected when the set of senders is $J' \subset J$. Hence, when we say that Π satisfies this property, we mean that it does so when the set of senders is J .

Observe that simply having a Blackwell-connected environment is no longer sufficient for unambiguous comparative statics. For example, suppose we are in the *Number of draws* environment and let τ_m denote the outcome if a total of m independent draws are generated. Suppose sender 1's preferences satisfy $v_1(\tau_0) > v_1(\tau_2) > v_1(\tau_m) \forall m \notin \{0, 2\}$, and sender 2's preferences satisfy $v_2(\tau_1) > v_2(\tau_2) > v_2(\tau_m) \forall m \notin \{1, 2\}$. If senders 1 and 2 are the only senders, any equilibrium outcome must generate strictly more

than two independent draws.¹⁷ But, if we introduce a third sender who is indifferent across all outcomes, we can now support τ_2 as an equilibrium by having senders 1 and 2 generate no draws and sender 3 generate two draws. No sender can then profitably deviate. This is an example of a more general principle that introducing additional senders can *reduce* the set of possible deviations available to the existing senders. Hence, additional senders can expand the set of equilibrium outcomes and make minimal equilibria less informative.

In the special case where senders have access to the same set of signals, however, we can derive unambiguous comparative statics. (Recall that access to the same set of signals means not only that $\langle \Pi_i \rangle = \langle \Pi \rangle$ but that $\Pi_i = \Pi$.)

Proposition 4. *Suppose each sender has access to the same set of signals and the information environment is Blackwell-connected. Regardless of preferences, a minimal equilibrium outcome when the set of senders is J' cannot be more informative than a minimal equilibrium outcome when the set of senders is $J \supset J'$.*

When $|J'| \geq 2$, this result is related to Proposition 2: loosely speaking, adding senders “shrinks” the set of equilibrium outcomes and thus makes minimal equilibria more informative. When $|J'| = 1$, a different argument, more closely related to the proof of Proposition 1, establishes the result.

It is easy to show that the environment being Blackwell-connected is sufficient, but not necessary, for the result. For an extreme example, suppose that all senders in J' can only send the null signal: $\Pi_j = \{\underline{\pi}\} \forall j \in J'$. Then, additional senders cannot make the equilibrium outcome less informative regardless of whether the informational environment is Blackwell-connected or not.

6.2 Reducing alignment of senders’ preferences

Another way to increase competitive pressure is to make senders’ interests less aligned. The literature on advocacy highlights conflict of incentives as a force that can increase information revelation in settings such as judicial or congressional proceedings. We show that this intuition, like the one for adding senders, applies in our setting when the environment is Blackwell-connected and senders have access to the same set of signals.

¹⁷Suppose τ_0 is an equilibrium. Then, sender 2 has a profitable deviation by increasing her number of draws by one. Suppose τ_1 is an equilibrium. Then, sender 1 has a profitable deviation by increasing her number of draws by one. Suppose τ_2 is an equilibrium. It must be the case that sender 2 does not generate any draws; otherwise, she has a profitable deviation by lowering her number of draws by one. Hence, it must be that sender 1 generates both draws, but then she has a profitable deviation to τ_0 .

Given that senders can have any arbitrary state-dependent utility functions, the extent of preference alignment among senders is not easy to parameterize in general. Hence, we consider a specific form of preference alignment. Suppose there are two functions f and g , and two senders j and k , with preferences of the form:

$$v_j(\tau) = f(\tau) + bg(\tau)$$

$$v_k(\tau) = f(\tau) - bg(\tau)$$

while preferences of other senders are independent of b . The parameter $b \geq 0$ thus captures the extent of preference misalignment between the two senders.

Proposition 5. *Suppose $n \geq 2$, each sender has access to the same set of signals, and the information environment is Blackwell-connected. Regardless of preferences, a minimal equilibrium outcome when misalignment is b' cannot be more informative than a minimal equilibrium outcome when misalignment is $b > b'$.*

A detailed proof is in the Appendix. The basic intuition behind Proposition 5 is the following. When preferences are less aligned, there are fewer outcomes that none of the senders wishes to deviate from. Hence, the set of equilibrium outcomes shrinks, and minimal equilibria become more informative. As in the previous subsection, this comparative static only applies to settings where each sender has access to the same set of signals. Otherwise, it is possible for increased misalignment to increase the set of equilibrium outcomes and make minimal equilibria less informative.

Proposition 5 also raises a natural question: in the limit, when senders' preferences become fully opposed, do we necessarily get only the most informative outcomes as equilibria? The answer is indeed affirmative. We say that preferences are *strictly zero sum* if $\sum_i v_i(\tau)$ is a constant and, given any pair of outcomes τ and τ' , it is not the case that all senders are indifferent between τ and τ' . We say an outcome τ is *maximally informative* if there is no feasible τ' such that $\tau' > \tau$. An outcome τ is *the most informative outcome* if for any feasible τ' we have $\tau \geq \tau'$.

Proposition 6. *Suppose $n \geq 2$, each sender has access to the same set of signals, the information environment is Blackwell-connected, and preferences are strictly zero sum. Then, the set of maximally informative*

outcomes is the set of equilibrium outcomes. In particular, if the most informative outcome exists, it is the unique equilibrium outcome.

7 Extensions

7.1 Comparing of sets of outcomes

So far in the paper we made simplifying assumptions that allowed us to avoid the issue of comparative statics on sets. We assumed that the collusive outcome is unique, and in Section 6 we focused on minimal equilibria. In this subsection we will drop these assumptions and analyze comparative statics on sets of outcomes.

Note that it would be too much to expect for *any* equilibrium outcome to be more informative than *any* collusive outcome simply because the two sets may overlap. For example, if all senders are indifferent across all outcomes, every outcome is both an equilibrium outcome and a collusive outcome; hence, it is not the case that every equilibrium is more informative than every collusive outcome.

We will define two orders on sets of outcomes following Topkis (1998). Because these orders are defined on subsets of a lattice, and the set of distributions of posteriors is not always a lattice under the Blackwell order (Müller and Scarsini 2006), we focus on the special case in which any two feasible outcomes are comparable. We say that T is *strongly more informative than* T' if for any $\tau \in T$ and $\tau' \in T'$ such that $\tau' \geq \tau$ we have $\tau \in T'$ and $\tau' \in T$. We say T is *weakly more informative than* T' if for any element of T there is a less informative element of T' , and for any element of T' there is a more informative element of T .

We might hope that Proposition 1 generalizes to the claim that the set of equilibrium outcomes is strongly (or weakly) more informative than the set of collusive outcomes if and only if the information environment is Blackwell-connected. This, however, turns out not to be true in general. Accordingly, we restrict our attention to two natural classes of information environments for which the analogue of Proposition 1 does hold.

Say that *signals are independent* if, given any sender i and any $\pi_a, \pi_b \in \Pi_i$, we have that for all $\pi_{-i} \in \Pi_{-i}$, $\langle \pi_a \rangle \geq \langle \pi_b \rangle \Leftrightarrow \langle \pi_a \cup \pi_{-i} \rangle \geq \langle \pi_b \cup \pi_{-i} \rangle$. In other words, when signals are independent, whether one signal is more informative than another does not depend on what other information is being provided. This will be true, in particular, if the signal realizations are statistically independent conditional on the state.

At the other extreme is a situation where each sender has access to the same set of signals, so one of the senders can always provide information that makes the other's information completely redundant. Note that the *Number of draws* and *Precisions* environment are independent, while *Facts* and *All-or-nothing* are environments where each sender has access to the same set of signals. Restricting our attention to these types of environments, we establish the key result of this subsection:

Proposition 7. *Suppose any two feasible outcomes are comparable. Suppose that each sender has access to the same set of signals or that signals are independent. The set of equilibrium outcomes is strongly more informative than the set of collusive outcomes (regardless of preferences) if and only if the information environment is Blackwell-connected.*

Consider some collusive outcome τ^c and some equilibrium outcome $\tau^* = \langle \pi^* \rangle$ with $\tau^c \geq \tau^*$. To establish Proposition 7, we need to show that, if the environment is Blackwell-connected, it must be the case that τ^* is also a collusive outcome and τ^c is also an equilibrium outcome. The argument for the first part is closely analogous to the proof of Proposition 1. Since $\tau^c \geq \tau^*$, any sender can deviate from τ^* to τ^c , so it must be the case that $v_i(\tau^*) \geq v_i(\tau^c)$ for each sender i . Since τ^c is a collusive outcome, this implies that τ^* must also be a collusive outcome. The second part of the argument, which establishes that τ^c must be an equilibrium, is more involved, and relies on the assumption that each sender has access to the same set of signals or that signals are independent. As the proof in the Appendix shows, in these two classes of environments there must be some strategy profile π^c such that $\tau^c = \langle \pi^c \rangle$ and any outcome that sender i can deviate to from π^c , she can also deviate to from π^* . Therefore, since π^* is an equilibrium, π^c must be one as well.

We also derive full-equilibrium-set analogues of our comparative statics results on adding senders and increasing preference misalignment, though they only hold (i) in the weak set order, (ii) when each sender has access to the same set of signals, and (iii) when the set of feasible outcomes has a maximum element (there exists a feasible $\bar{\tau}$ s.t. for all $\pi \in \Pi$ we have $\bar{\tau} \geq \langle \pi \rangle$).

Proposition 8. *Suppose any two feasible outcomes are comparable. Suppose each sender has access to the same set of signals and the set of feasible outcomes has a maximum element. If the information environment is Blackwell-connected, then:*

(1) *The set of equilibrium outcomes when the set of senders is some J is weakly more informative than the set of equilibrium outcomes when the set of senders is some $J' \subset J$.*

(2) *The set of equilibrium outcomes when the level of misalignment is b is weakly more informative than the set of equilibrium outcomes when the level of misalignment is $b' < b$.*

The basic idea behind Proposition 8 is the following. Since each sender has access to the same set of signals and the environment is Blackwell-connected, the set of equilibrium outcomes is the intersection of unimprovable outcomes for each sender. Hence, the set of equilibrium outcomes shrinks when we add senders or increase misalignment of their preferences. But, Proposition 2 also implies that the maximum element always remains an equilibrium outcome. Hence, loosely speaking, adding senders shrinks the equilibrium set “toward” the most informative equilibrium. The argument is somewhat different when $|J'| = 1$.

7.2 Restricted preferences

Our main results ask when we can be sure competition will not decrease information no matter what the preferences of the senders. If we were willing to limit attention to a more restricted class of sender preferences, we might expect that the Blackwell-connectedness condition could be weakened to something less demanding.

As one example, we consider the case of monotone preferences. We say that preferences are monotone if for each sender i either (i) $v_i(\tau) \geq v_i(\tau')$ for all $\tau \geq \tau'$ or (ii) $v_i(\tau) \leq v_i(\tau')$ for all $\tau \geq \tau'$. It turns out that if we focus on monotone preferences, we can weaken Blackwell-connectedness significantly. As in Section 4, we assume the collusive outcome is unique.

Proposition 9. *Suppose $n \geq 2$ and preferences are monotone. The collusive outcome cannot be more informative than an equilibrium outcome (regardless of preferences) if and only if for every feasible outcome τ , for every sender i , there is a $\pi \in \Pi_i$ such that $\langle \pi \rangle \geq \tau$.*

To understand this condition, recall that one implication of Blackwell-connectedness is that individual feasibility equals aggregate feasibility: for every feasible outcome τ , each sender i can unilaterally induce τ . Proposition 9 implies that—if preferences are monotone—this feasibility condition alone is sufficient; we can do away with the additional implications of Blackwell-connectedness. Moreover, we can weaken the condition to only require that each sender can induce something more informative than τ , rather than requiring that she can induce τ itself.

The intuition for the proof is straightforward. Suppose the collusive outcome τ^c were strictly more informative than an equilibrium outcome τ^* . There must be some sender i who would strictly prefer τ^c to τ^* . In the baseline model, we needed a condition that would guarantee this sender could deviate from the equilibrium strategy profile to induce τ^c ; this is Blackwell-connectedness. With monotone preferences, we know that this sender must also prefer anything more informative than τ^c to τ^* , so we need only guarantee that i can deviate to induce some $\tau' \geq \tau^c$, which our weaker condition guarantees. The proof of the converse is constructive and follows a similar logic to Proposition 1.

7.3 Mixed strategies

Throughout the paper we focus on pure strategy equilibria. This focus has substantive consequences for our results. If senders play mixed strategies, the information environment may effectively cease to be Blackwell-connected. For example, in most of the information environments we discuss in Section 3—e.g., *Number of draws*, *Precisions*, *Partitions*, and *Facts*—we can construct a feasible outcome τ and a mixed strategy profile $\tilde{\pi}_{-i}$ such that $\tau \geq \langle \tilde{\pi}_{-i} \rangle$ but there exists no $\pi_i \in \Pi_i$ such that $\tau = \langle \tilde{\pi}_{-i} \cup \pi_i \rangle$. Without knowing which signal others will generate, sender i cannot “add” a suitable amount of information to induce a particular outcome. Consequently, by Proposition 1, we know that in all of these cases there are preferences such that the collusive outcome is strictly more informative than some mixed strategy equilibrium outcome.

7.4 Non-Blackwell orders

While the Blackwell order is a natural way to present our comparative statics, none of our results rely on use of this particular order. Consider any partial order \geq on the set of outcomes. Say that the information environment is \geq -connected if for all i , $\pi \in \Pi$, and $\pi_{-i} \in \Pi_{-i}$ such that $\langle \pi \rangle \geq \langle \pi_{-i} \rangle$, there exists a $\pi_i \in \Pi_i$ such that $\langle \pi \rangle = \langle \pi_{-i} \cup \pi_i \rangle$. By the exact same arguments as before, we can conclude that every equilibrium outcome is no less informative (under the \geq order) than the unique collusive outcome regardless of preferences if and only if the environment is \geq -connected. Similar analogues apply to the characterization result and the other comparative statics.

For example, given a specific receiver, we could define a complete order on the set of outcomes based on whether receiver’s expected utility is greater. The analogue of Proposition 1 would then be: the receiver is better off in any equilibrium than under the collusive outcome (regardless of the senders’ preferences) if

and only if, given any strategy profile, any sender can unilaterally deviate to induce any feasible outcome that the receiver would prefer.

This observation illustrates how considering a (more) complete order on the space of outcome strengthens the conclusion of the theorem but also stretches the plausibility of the connectedness condition, which then needs to hold for a broader range of outcome pairs.

8 Conclusion

A large body of policy and legal precedent has been built on the view that competition in the “marketplace of ideas” will ultimately lead more truth to be revealed. Existing models of strategic communication with multiple senders have focused on settings such as cheap talk or disclosure where the key issue is the credibility with which senders can communicate what they know, and shown some conditions under which this intuition is valid. The strategic complexity of these settings, however, means that they stop short of full characterizations and consider a limited range of comparative statics (Sobel 2013).

We depart from the literature in setting aside incentive compatibility in communication and focusing instead on senders’ incentives to gather information, assuming that they can commit to communicate it truthfully. In this setting, we show that the impact of competition is ambiguous in general, and that Blackwell-connectedness is the key condition separating cases where competition is guaranteed to be beneficial from those where it is not.

9 Appendix

9.1 Proof of Proposition 1

Proof. Suppose that Π is Blackwell-connected. Suppose τ^* is an equilibrium outcome and τ^c is the collusive outcome. Let π^* be the strategy profile that induces τ^* and π^c the strategy profile that induces τ^c . Suppose, contrary to the claim, that $\tau^c > \tau^*$. Since τ^c is the unique collusive outcome, there is some sender i s.t. $v_i(\tau^c) > v_i(\tau^*)$. Consider $\pi_{-i} = (\pi_1^*, \dots, \pi_{i-1}^*, \pi_{i+1}^*, \dots, \pi_n^*) \in \Pi_{-i}$. We have $\langle \pi^c \rangle \geq \langle \pi^* \rangle \geq \langle \pi_{-i} \rangle$, so the fact that Π is Blackwell-connected implies that there exists a signal $\pi' \in \Pi_i$ s.t. $\langle \pi_{-i} \cup \pi' \rangle = \langle (\pi_1^*, \dots, \pi_{i-1}^*, \pi', \pi_{i+1}^*, \dots, \pi_n^*) \rangle = \tau^c$. Hence, sender i has a profitable deviation, which contradicts the claim that π^* is an equilibrium.

Suppose that Π is not Blackwell-connected. This means that there exist some i , $\pi^c \in \Pi$ and $\pi'_{-i} \in \Pi_{-i}$ such that $\langle \pi^c \rangle \geq \langle \pi'_{-i} \rangle$, but for all $\pi \in \Pi_i$ we have $\langle \pi^c \rangle \neq \langle \pi'_{-i} \cup \pi \rangle$. Moreover, since $\underline{\pi} \in \Pi_i$, we have $\langle \pi^c \rangle > \langle \pi'_{-i} \rangle$. Let $\pi^* = (\pi'_1, \dots, \pi'_{i-1}, \underline{\pi}, \pi'_{i+1}, \pi'_n)$. For all $j \neq i$, let v_j be constant. Let $v_i(\langle \pi^c \rangle) = 1$, $v_i(\langle \pi^* \rangle) = 0$, and $v_i(\tau) = -1$ for $\tau \notin \{\langle \pi^c \rangle, \langle \pi^* \rangle\}$. Then we have that $\langle \pi^c \rangle$ is the unique collusive outcome, $\langle \pi^* \rangle$ is an equilibrium outcome, and yet $\langle \pi^c \rangle > \langle \pi^* \rangle$. \square

9.2 Proof of Proposition 2

Proof. Let \mathcal{P} denote the set of signals available to each sender.

We first show that every equilibrium outcome is unimprovable for every sender. Consider a feasible outcome τ that is improvable for some sender i . Let π^* be a strategy profile that induces τ . Since τ is improvable for sender i , there is a π s.t. $\langle \pi \rangle \geq \tau$ and $v_i(\langle \pi \rangle) > v_i(\tau)$. Consider $\pi^*_{-i} = (\pi_1^*, \dots, \pi_{i-1}^*, \pi_{i+1}^*, \dots, \pi_n^*) \in \Pi_{-i}$. Since $\langle \pi \rangle \geq \tau = \langle \pi^* \rangle \geq \langle \pi^*_{-i} \rangle$ and Π is Blackwell-connected, there exists a signal $\pi' \in \Pi_i$ s.t. $\langle \pi^*_{-i} \cup \pi' \rangle = \langle \pi \rangle$. Hence, π' is a profitable deviation for sender i from π^* , so π^* is not an equilibrium.

Conversely, suppose that some feasible outcome τ is unimprovable for each sender. Let π' be a strategy profile that induces τ . Consider $\underline{\pi} = (\underline{\pi}_1, \dots, \underline{\pi}_n) \in \mathcal{P}^{n-1}$. Since $\langle \pi' \rangle \geq \langle \underline{\pi} \rangle$ and Π is Blackwell-connected, there exists some $\pi^* \in \mathcal{P}$ s.t. $\langle \pi' \rangle = \langle \pi^* \rangle$. Consider a strategy profile $\pi^* = (\pi^*_1, \dots, \pi^*_n)$. Since $n \geq 2$, no sender can deviate except to a more informative outcome. Since τ is unimprovable for each sender, no such deviation is profitable. \square

9.3 Proof of Proposition 3

Proof. Consider some partition ρ such that each of its elements is an unimprovable event for every sender. Let π be a strategy profile where each sender sets $\pi_i = \rho$. Since $n \geq 2$, each sender can only deviate to induce refinements of ρ . But since each element of ρ is unimprovable, no such deviation is profitable. Hence, π is an equilibrium strategy and ρ is an equilibrium partition.

Consider some partition ρ such that some $E \in \rho$ is not unimprovable for some sender i . Suppose π is a strategy profile that induces ρ . Let ρ' be the partition of E such that $\tilde{v}_i(E) < \sum_{E' \in \rho'} \frac{\mu_0(E') \tilde{v}_i(E')}{\mu_0(E)}$. Consider a partition π'_i that groups all states in E according to ρ' and any states not in E are in the same element of π'_i if and only if they are in the same element of π_i . Then, (π'_i, π_{-i}) induces a partition that groups all states outside of E in the same way as ρ but replaces E with ρ' . This partition yields a higher payoff for sender i , so ρ could not have been an equilibrium partition. \square

9.4 Proof of Proposition 4

Proof. First, consider the case where $|J'| = 1$. Let i be the sender in J' and let τ^i be a minimally informative outcome that is unilaterally optimal for sender i . Let τ^* be a minimal equilibrium outcome when the set of senders is J . Suppose that $\tau^i > \tau^*$. Since τ^i is a minimally informative outcome that is unilaterally optimal for sender i , τ^* must not be unilaterally optimal for sender i . (By Remark 1, we know $\tau^* \in \{\langle \pi \rangle | \pi \in \Pi_i\}$ because Π is Blackwell-connected). In other words, $v_i(\tau^i) > v_i(\tau^*)$. But, since Π is Blackwell-connected, given any strategy profile π^* that induces τ^* , there exists a signal $\pi' \in \Pi_i$ that allows sender i to deviate from π^* to induce τ^i . Hence, π^* cannot be an equilibrium.

Now consider the case where $|J'| > 1$. Let τ' be a minimal equilibrium outcome when the set of senders is J' and let τ^* be a minimal equilibrium outcome when the set of senders is J . Suppose that $\tau' > \tau^*$. But, by Proposition 2, the set of equilibrium outcomes when the set of senders is J is a subset of the set of equilibrium outcomes when the set of senders is J' . Hence, τ^* is an equilibrium outcome when the set of senders is J' . Therefore, $\tau' > \tau^*$ contradicts the fact that τ' is a minimal equilibrium outcome. \square

9.5 Proof of Proposition 5

We begin the proof with the following lemma (which will also be useful for the proof of Proposition 8).

Lemma 1. *Suppose each sender has access to the same set of signals. Let T^m be the set of equilibrium outcomes when the level of misalignment is $m \in \{b, b'\}$. If $b > b'$, then $T^b \subset T^{b'}$.*

Proof. Let T_i^m be the set of feasible outcomes that are unimprovable for sender i when the level of misalignment is $m \in \{b, b'\}$. By Proposition 2, $T^m = \cap_i T_i^m$. For $i \notin \{j, k\}$, we have $T_i^b = T_i^{b'}$. Hence, it will suffice to show that $T_j^b \cap T_k^b \subset T_j^{b'} \cap T_k^{b'}$. Consider any $\tau \in T_j^b \cap T_k^b$. Let T' be the set of feasible outcomes that are more informative than τ . Since $\tau \in T_j^b$, we have that $f(\tau) + bg(\tau) \geq f(\tau') + bg(\tau')$ for all $\tau' \in T'$. Since $\tau \in T_k^b$, we have that $f(\tau) - bg(\tau) \geq f(\tau') - bg(\tau')$ for all $\tau' \in T'$. Combining these two inequalities, we get $f(\tau) - f(\tau') \geq b|g(\tau) - g(\tau')| \forall \tau' \in T'$, which in turn implies that $f(\tau) - f(\tau') \geq b'|g(\tau) - g(\tau')| \forall \tau' \in T'$. This last inequality implies $f(\tau) + b'g(\tau) \geq f(\tau') + b'g(\tau')$ and $f(\tau) - b'g(\tau) \geq f(\tau') - b'g(\tau') \forall \tau' \in T'$, so we have $\tau \in T_j^{b'} \cap T_k^{b'}$. \square

With this lemma, the proof of Proposition 5 follows easily:

Proof. Let τ' be a minimal equilibrium outcome when the level of misalignment is b' and let τ be a minimal equilibrium outcome when the level of misalignment is b . Suppose $\tau' > \tau$. By Lemma 1, the set of equilibrium outcomes when the level of misalignment is b is a subset of the set of equilibrium outcomes when the level of misalignment is b' . Hence, τ is an equilibrium outcome when the level of misalignment is b' . Therefore, $\tau' > \tau^*$ contradicts the fact that τ' is a minimal equilibrium outcome when the level of misalignment is b' . \square

9.6 Proof of Proposition 6

Proof. Any maximally informative outcome is vacuously unimprovable for every sender; hence, by Proposition 2, it is an equilibrium outcome. It remains to show that we cannot have an equilibrium outcome that is not maximally informative. Suppose τ^* were such an equilibrium outcome induced by a strategy profile π^* . Consider any $\tau' > \tau^*$. Since preferences are strictly zero sum, there must be a sender i for whom $v_i(\tau') > v_i(\tau^*)$. But since the environment is Blackwell-connected, sender i can deviate from π^* to induce τ' , which contradicts the claim that π^* is an equilibrium. \square

9.7 Proof of Proposition 7

We begin by establishing a key property of settings where each sender has access to the same set of signals or signals are independent. Say that τ' is an *i-feasible deviation from π* if there exists a $\pi' \in \Pi_i$ s.t. $\langle \pi_{-i} \cup \pi' \rangle = \tau'$. Say that Π is *simple* if, given any $\pi \in \Pi$ and any feasible $\tau' \geq \langle \pi \rangle$, there exists a π' s.t. $\langle \pi' \rangle = \tau'$, and for any sender i , the set of *i-feasible deviations from π'* is a subset of *i-feasible deviations from π* . Say that Π is *incrementable* if, given any $\pi \in \Pi$ and any feasible $\tau' \geq \langle \pi \rangle$, there exists a $\pi' \in \Pi$ s.t. $\langle \pi' \rangle = \tau'$ and $\pi'_i \geq \pi_i$ for all i .

Lemma 2. *If Π is Blackwell-connected and each sender has access to the same set of signals, then Π is simple.*

Proof. If $n = 1$, every environment is simple, so suppose $n \geq 2$. Suppose Π is Blackwell-connected and each sender has access to the same set of signals. Let \mathcal{P} denote the set of signals available to each sender. Consider some $\pi \in \Pi$ and some feasible $\tau' \geq \langle \pi \rangle$. Since Π is Blackwell-connected, individual feasibility equals aggregate feasibility (by Remark 1), which implies there exists a $\pi' \in \mathcal{P}$ such that $\langle \pi' \rangle = \tau'$. Let $\pi' = (\pi', \dots, \pi')$. Consider some τ'' , an *i-feasible deviation from π'* . We must have $\tau'' \geq \tau' \geq \langle \pi \rangle$. Since Π is Blackwell-connected, τ'' must be *i-feasible from π* . \square

Lemma 3. *If Π is Blackwell-connected and signals are independent, then Π is simple.*

Proof. Suppose the environment is Blackwell-connected and independent. We first show that Π is incrementable. Consider some $\pi \in \Pi$ and a feasible $\tau' \geq \langle \pi \rangle$. Pick any sender i and consider π_{-i} . We have that $\tau' \geq \langle \pi \rangle \geq \langle \pi_{-i} \rangle$. Since the environment is Blackwell-connected, there is a $\pi' \in \Pi_i$ s.t. $\langle \pi_{-i} \cup \pi' \rangle = \tau'$. Let $\pi' = (\pi_{-i}, \pi')$. By construction, $\pi'_j = \pi_j$ for $j \neq i$, so $\pi'_j \geq \pi_j$ for $j \neq i$. By independence, $\langle \pi_{-i} \cup \pi' \rangle = \tau' \geq \langle \pi \rangle = \langle \pi_{-i} \cup \pi_i \rangle$ implies that $\pi' = \pi'_i \geq \pi_i$.

Next we show that Π is simple. Given a $\pi \in \Pi$ and a feasible $\tau' \geq \langle \pi \rangle$ consider a $\pi' \in \Pi$ s.t. $\langle \pi' \rangle = \tau'$ and $\pi'_j \geq \pi_j$ for all j . (Such a profile exists since the environment is incrementable.) Suppose that some outcome τ^d is an *i-feasible deviation from π'* . This means that $\tau^d \geq \pi'_{-i} \geq \pi_{-i}$. The last inequality follows from the fact that $\pi'_j \geq \pi_j$ for all $j \neq i$ and the fact that the environment is independent. Since $\tau^d \geq \pi_{-i}$ and the environment is Blackwell-connected, we know that τ^d is an *i-feasible deviation from π* . \square

We are now ready to turn to the proof of Proposition 7.

Proof. Suppose that each sender has access to the same set of signals or that signals are independent. Suppose the information environment is Blackwell-connected and any two feasible outcomes are comparable. By Lemmas 2 and 3, we know the environment is simple. Suppose there is some collusive outcome τ^c and some equilibrium outcome τ^* such that $\tau^c \geq \tau^*$. We need to show that τ^* is a collusive outcome and τ^c is an equilibrium outcome. Let π^* be the strategy profile that induces τ^* . Since $\tau^c \geq \tau^*$ and Π is Blackwell-connected, any sender can deviate from π^* to induce τ^c , so it must be the case that $v_i(\tau^*) \geq v_i(\tau^c)$ for each sender i . Since τ^c is a collusive outcome, this implies that τ^* is also a collusive outcome. Moreover, since the environment is simple, there exists a strategy profile π^c such that $\tau^c = \langle \pi^c \rangle$ and any outcome that is i -feasible from π^c is also i -feasible from π^* for every sender i . But then the fact that π^* is an equilibrium implies that π^c is also an equilibrium. This completes the “if” part of the proof.

We now turn to the “only-if” part. Suppose Π is not Blackwell-connected. There exist some i , $\pi^c \in \Pi$, and $\pi'_{-i} \in \Pi_{-i}$ such that $\langle \pi^c \rangle \geq \langle \pi'_{-i} \rangle$, and for all $\pi \in \Pi_i$ we have $\langle \pi^c \rangle \neq \langle \pi'_{-i} \cup \pi \rangle$. Let $\pi^* = (\pi'_1, \dots, \pi'_{i-1}, \underline{\pi}, \pi'_{i+1}, \pi'_n)$. For all $j \neq i$, let v_j be constant. Let $v_i(\langle \pi^c \rangle) = 1$, $v_i(\langle \pi^* \rangle) = 0$, and $v_i(\langle \pi \rangle) = -1$ for $\pi \notin \{\pi^c, \pi^*\}$. Then we have that $\langle \pi^c \rangle$ is a collusive outcome, $\langle \pi^* \rangle$ is a non-collusive equilibrium outcome, and $\langle \pi^c \rangle \geq \langle \pi^* \rangle$. \square

9.8 Proof of Proposition 8

Proof. We first prove part (1). Let τ^* be an equilibrium outcome when the set of senders is some set J and τ' an equilibrium outcome when the set of senders is some $J' \subset J$. We need to show that: (i) there is an equilibrium outcome when the set of senders is J that is more informative than τ' ; and (ii) there is an equilibrium outcome when the set of senders is J' that is less informative than τ^* . Consider the case where $|J'| = 1$. If $J = J'$, the proposition is trivially true, so suppose that $|J| > 1$. By Proposition 2, we know $\bar{\tau}$ is an equilibrium outcome when the set of senders is J , and by definition $\bar{\tau} \geq \tau'$. This establishes claim (i). Now, suppose contrary to the second claim that there is no outcome $\tau'' \leq \tau^*$ that is unilaterally optimal for the sender in J' . Since any two feasible outcomes are comparable, this implies that $\tau' > \tau^*$ and that τ^* is not optimal for the sender in J' . But since Π is Blackwell-connected, the sender in J' can deviate from the strategy profile that induces τ^* and induce τ' instead so τ^* cannot be an equilibrium outcome. This establishes claim (ii).

Now consider the case where $|J'| > 1$. By Proposition 2, we know $\bar{\tau}$ is an equilibrium when the set of

senders is J and by definition $\bar{\tau} \geq \tau'$. That establishes claim (i). Moreover, by Proposition 2 we know τ^* must be an equilibrium when the set of senders is J' , which establishes claim (ii). This completes the proof of part (1).

We now turn to part (2) of the proposition. Let τ^* be an equilibrium outcome when the level of misalignment is some b and τ' an equilibrium outcome when the level of misalignment is some $b' < b$. By Proposition 2, we know $\bar{\tau}$ is an equilibrium when the level of misalignment is b , and by definition $\bar{\tau} \geq \tau'$. By Lemma 1, we know that τ^* is also an equilibrium when the level of misalignment is b' . \square

9.9 Proof of Proposition 9

Proof. Suppose that for every feasible τ and every i there exists $\pi \in \Pi_i$ such that $\langle \pi \rangle \geq \tau$, but contrary to the claim, the collusive outcome τ^c is strictly more informative than $\langle \pi^* \rangle$ for some equilibrium profile π^* . Observe that (i) there must be some sender i such that $v_i(\tau^c) > v_i(\tau^*)$, (ii) there must be some $\tau' \geq \tau^c$ and $\pi' \in \Pi_i$ such that $\tau' = \langle \pi' \rangle$, and (iii) by monotonicity of preferences, i must strictly prefer any outcome more informative than τ^c to τ^* . Suppose that i were to deviate from the proposed equilibrium to play π' rather than π_i^* . Then because $\langle \pi' \rangle \geq \tau^c$, we know $\langle \pi_{-i}^* \cup \pi' \rangle \geq \tau^c$, and so by (iii) this is a profitable deviation, contradicting the claim that π^* is an equilibrium.

For the converse, suppose there exists a feasible τ^c and sender i such that there is no $\pi \in \Pi_i$ with $\langle \pi \rangle \geq \tau^c$. Since $\underline{\pi} \in \Pi_i$, we must have $\tau^c > \langle \underline{\pi} \rangle$. Suppose that $v_i(\tau)$ equals one for all $\tau \geq \tau^c$ and zero elsewhere. Suppose that for all $j \neq i$, $v_j(\tau)$ equals negative one for all $\tau > \tau^c$ and zero elsewhere. Then preferences are monotone, τ^c is the unique collusive outcome, and the strategy profile where all senders choose an uninformative signal is an equilibrium strictly less informative than τ^c . \square

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