Correcting Estimates of the Total Factor Productivity Residual to Account for Omission of Natural Resource Inputs

(Appendix to "Are We Consuming Too Much?," *Journal of Economic Perspectives*, Summer 2004)

In Column 5 of Table 2 (page 153) of our article we reported estimates of the average annual value of the residual in our set of countries and regions over the period 1970-2000. These were taken from Klenow and Rodrigues-Clare (1997). In estimating the residual, Klenow and Rodrigues-Clare used a specification for the aggregate production function that did not include natural resources as inputs in production. As we noted in Section 2 of our article, such a specification can lead to biased estimates of TFP and, subsequently, of changes in per-capita genuine wealth. In our article we did not make any adjustment for this bias. In this appendix, however, we offer a formula indicating how such a correction could be performed.

The following procedure can correct for the bias.

Klenow and Rodrigues-Clare (1997) applied the following aggregate production function:

$$Y_t = K_t^{\alpha} H_t^{\beta} (A_t L_t)^{1-\alpha-\beta} \tag{B-1}$$

where K is the stock of manufactured capital, H is the stock of human capital, $A^{1-\alpha-\beta}$ is total factor productivity, and L is labor hours. We first derive an expression for the growth rate of total factor productivity residual that emanates from this specification. Equation B-1 can be rewritten as:

$$Y/L = \left(K/L\right)^{\alpha} \left(H/L\right)^{\beta} A^{1-\alpha-\beta} \tag{B-2}$$

$$= \left(K/Y\right)^{\alpha} \left(H/Y\right)^{\beta} \left(Y/L\right)^{\alpha+\beta} A^{1-\alpha-\beta}$$

Then

$$\left(Y/L\right)^{1-\alpha-\beta} = \left(K/Y\right)^{\alpha} \left(H/Y\right)^{\beta} A^{1-\alpha-\beta}$$

or

$$\left(Y/L\right) = \left(K/Y\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(H/Y\right)^{\frac{\beta}{1-\alpha-\beta}} A \tag{B-3}$$

Let the function *g(x)* indicate the growth rate of x. Then

$$g\left(Y/L\right) = \frac{\alpha}{1-\alpha-\beta} g\left(K/Y\right) + \frac{\beta}{1-\alpha-\beta} g\left(H/Y\right) + g(A) \tag{B-4}$$

Define $\tilde{A} \equiv A^{1-\alpha-\beta}$. Note that $\tilde{A}$ is the total factor productivity residual. From (B-4),

$$g\left(\tilde{A}\right) = (1-\alpha-\beta)\, g(A) = (1-\alpha-\beta)\, g\left(Y/L\right) - \alpha\, g\left(K/Y\right) - \beta\, g\left(H/Y\right). \tag{B-5}$$

Suppose the true production relationship is

$$Y_t = K_t^{(1-\nu)\alpha} H_t^{(1-\nu)\beta} (A_t L_t)^{(1-\nu)(1-\alpha-\beta)} R^{\nu},$$

where R represents natural resource contributions to measured output, and ν is the share of R in production. The exponents for the other factors now include the coefficient (1-ν), so that the exponents again sum to 1. The above production function can be rewritten as

$$Y/L = \left(K/L\right)^{(1-\nu)\alpha} \left(H/L\right)^{(1-\nu)\beta} A^{(1-\nu)(1-\alpha-\beta)} \left(R/L\right)^{\nu}$$

$$= \left(K/Y\right)^{(1-\nu)\alpha} \left(H/Y\right)^{(1-\nu)\beta} \left(R/Y\right)^{\nu} \left(Y/L\right)^{(1-\nu)(\alpha+\beta)+\nu} A^{(1-\nu)(1-\alpha-\beta)}$$

Thus:

$$\left(Y/L\right)^{(1-\nu)(1-\alpha-\beta)} = \left(K/Y\right)^{(1-\nu)\alpha} \left(H/Y\right)^{(1-\nu)\beta} \left(R/Y\right)^{\nu} A^{(1-\nu)(1-\alpha-\beta)} \tag{B-6}$$

From (B-6) and the definition of $\tilde{A}$ we can write:

$$g^{c}\left(\tilde{A}\right) = (1-\nu)\left[(1-\alpha-\beta)\, g\left(Y/L\right) - \alpha\, g\left(K/Y\right) - \beta\, g\left(H/Y\right)\right] - \nu\, g\left(R/Y\right) \tag{B-7}$$

where the superscript "*c*" is employed to distinguish the corrected growth rate $g^{c}(\tilde{A})$ from the uncorrected growth rate $g(\tilde{A})$ derived from the original model. Comparing equations (B-7) and (B-5) above, we observe that the corrected growth rate is related to the uncorrected rate according to:

$$g^{c}\left(\tilde{A}\right) = (1-\nu)\, g\left(\tilde{A}\right) - \nu\, g\left(R/Y\right)$$