Appendix to
Efficiency Costs of Meeting Industry-Distributional Constraints under Environmental Permits and Taxes

A Solution to the Analytical Model

A.1 The market for the final good

A.1.1 Supply

Competitive profit-maximizing behavior by the downstream industry yields

$$P_y \frac{\delta h(.;.)}{\delta V} \frac{\delta v(.;.)}{\delta X} = P_x + T_e \frac{\delta n}{\delta X}, \tag{A-1}$$

$$P_y \frac{\delta h(.;.)}{\delta L_y} = W, \tag{A-2}$$

$$-T_e \frac{\delta n}{\delta g} \frac{\delta g}{\delta C_a} = P_c; \ \ -T_e \frac{\delta n}{\delta g} \frac{\delta g}{\delta Y_a} = P_y, \tag{A-3}$$

$$P_y \frac{\delta h(.;.)}{\delta V} \frac{\delta v(.;.)}{\delta K_y} = R_y. \tag{A-4}$$

The right-hand side of (A-1) indicates that the cost of the intermediate input consists of two parts: the production costs of this input, P_x, and the emission tax levied on the additional emissions generated by the intermediate input.

Loglinearizing the production function of the downstream industry (4) and employing the first-order conditions (A-1), (A-2), (A-3), and (A-4) (and using the fact that the emission function (5) exhibits constant returns to scale), we find

$$y = k_y + (1 - \alpha_v^y)(l_y - v) + (1 - \alpha_k^y)(x - k_y), \tag{A-5}$$

where $\alpha_k^y \equiv R_y K_y / (R_y K_y + P_x X + T_e E + P_c C_a + P_y Y_a)$, $\alpha_v^y \equiv (R_y K_y + P_x X + T_e E + P_c C_a + P_y Y_a)/P_y Y = 1 - (W L_y / P_y Y)$, and $v = \alpha_k^y k + (1 - \alpha_k^y) x$.

With constant-returns-to-scale production and emission functions, the relative change in the output price is a weighted average of the relative changes in the input prices[1]

$$p_y = \alpha_v^y \alpha_k^y r_y + \alpha_x^y p_x + \alpha_e^y t_e, \tag{A-6}$$

where $\alpha_x^y \equiv P_x X / P_y Y$ and $\alpha_e^y \equiv T_e E / P_y Y$ stand for the cost shares of, respectively, the direct production costs of the intermediate good and the emission tax.

Capital supply is given by[2]

$$k_y = \sigma_k^y r_y, \tag{A-7}$$

where σ_k^y stands for the substitution elasticity between the industry-specific capital services in the final goods sector and the capital services in the rest of the economy.

Using (A-4), (A-2), and (A-1) to eliminate P_y and log-linearizing the results, we arrive at the following two equations

$$x - k_y = \sigma_v[r_y - (\alpha_x^y/(\alpha_v^y(1-\alpha_k^y)))p_x - (\alpha_e^y/(\alpha_v^y(1-\alpha_k^y)))t_e], \tag{A-8}$$

$$l_y - v = \sigma_y[\alpha_k^y r_y + (\alpha_x^y/\alpha_v^y)p_x + (\alpha_e^y/\alpha_v^y)t_e], \tag{A-9}$$

where σ_v stands for the substitution elasticity between the intermediate input and capital in the composite $v(.;.)$ while σ_y represents the substitution elasticity between labor and the nest $v(.;.)$ in the production function $h(.;.)$ (see (4)). Substituting (A-7), (A-6), (A-8), and (A-9) into (A-5), we write the supply of the final good in terms of its price, the price of the intermediate good, and the emission tax

$$y = \varepsilon_s^y(p_y - \alpha_x^y p_x - \alpha_e^y t_e) + \{[(1-\alpha_v^y)\sigma_y - \sigma_v]/\alpha_v^y\}p_y, \tag{A-10}$$

where $\varepsilon_s^y \equiv \left[\sigma_k^y + \sigma_v\right] / \left[\alpha_v^y \alpha_k^y\right]$ is the supply elasticity. This supply elasticity becomes infinite if capital (i.e. the "fixed" factor) does not play a role in production (i.e. $\alpha_k^y = 0$ or $\alpha_v^y = 0$), if industry-specific capital is a perfect substitute for capital in the rest of the

---

[1] $p_c = 0$ because P_c is the numeraire. Also $w = 0$ since the downstream and upstream sectors are too small to affect the prices on the labor market. $p_c = 0$ implies that the costs of abatement do not change because, in line with our assumption that the upstream and downstream industries are small compared to the rest of the economy, the share of abatement produced by the downstream industry (i.e. Y_a) in aggregate abatement $g(C_a; Y_a)$ is only infinitely small.

[2] This assumes that all households are well diversified so that income effects can be ignored. Alternatively, one can assume that a share γ_y of capital owners in the downtream industry is completely specialized in this sector (i.e., only derives income from capital in this sector). In that case the elasticity σ_k^y in the following equation is replaced by $(1-\gamma_y)\sigma_k^y + \gamma_y \varepsilon_u$, where ε_u stands for the uncompensated elasticity of aggregate capital supply with respect to the rate of return.

economy (i.e. $\sigma_k^y \Rightarrow \infty$ so that adjustment costs are absent), or if intermediate inputs are a perfect substitute for the imperfectly mobile factor (i.e. capital) (i.e. $\sigma_v \Rightarrow \infty$). In all these cases, the immobile factor does not constrain production of the final good.

In a similar way, we can derive the impact on the demand for the intermediate good (using (A-7), (A-6), and (A-8)) as

$$x = \varepsilon_s^y(p_y - \alpha_x^y p_x - \alpha_e^y t_e) - \sigma_v \left(\frac{\alpha_x^y}{\alpha_v^y(1-\alpha_k^y)} p_x + \frac{\alpha_e^y}{\alpha_v^y(1-\alpha_k^y)} t_e \right) \tag{A-11}$$

In order to find the emissions, we write the emission function (5) as $X = \phi(E, g(C_a; Y_a))$, where $\phi(.,.)$ is homogenous of the first degree in its arguments and log-linearize to arrive at

$$x = e + \frac{\alpha_v^y(1-\alpha_k^y) - \alpha_x^y - \alpha_e^y}{\alpha_v^y(1-\alpha_k^y) - \alpha_x^y}(c_a - e), \tag{A-12}$$

where we have used the fact that Y_a accounts for an infinitely small share of $g(C_a, Y_a)$. Writing (A-3) in terms of $\phi(.,.)$ (i.e. $\frac{d\phi}{dg} / \frac{d\phi}{dE} = \frac{P_c C_a + P_y Y_a}{g P_e}$) and log-linearizing, we arrive at

$$c_a - e = \sigma_e t_e, \tag{A-13}$$

where σ_e represents the substitution elasticity between emissions and aggragate abatement in $\phi(.,.)$. Employing (A-13) to eliminate $(c_a - e)$ from (A-12), we find emissions in terms of the emission tax and the prices of the final and intermediate goods[3]

$$\begin{aligned} e =& x - \frac{\alpha_v^y(1-\alpha_k^y) - \alpha_x^y - \alpha_e^y}{\alpha_v^y(1-\alpha_k^y) - \alpha_x^y}\sigma_e t_e \\ =& \varepsilon_s^y(p_y - \alpha_x^y p_x - \alpha_e^y t_e) - \sigma_v \left(\frac{\alpha_x^y}{\alpha_v^y(1-\alpha_k^y)} p_x + \frac{\alpha_e^y}{\alpha_v^y(1-\alpha_k^y)} t_e \right) \\ & - \frac{\alpha_v^y(1-\alpha_k^y) - \alpha_x^y - \alpha_e^y}{\alpha_v^y(1-\alpha_k^y) - \alpha_x^y}\sigma_e t_e \end{aligned} \tag{A-14}$$

The second term at the first right-hand side of (A-14) shows that the pollution tax reduces emissions per unit of intermediate input. This reduction is especially large if abatement is important (i.e. the cost share of abatement, $\frac{\alpha_v^y(1-\alpha_k^y)-\alpha_x^y-\alpha_e^y}{\alpha_v^y(1-\alpha_k^y)-\alpha_x^y} = \frac{P_c C_a + P_y Y_a}{P_c C_a + P_y Y_a + T_e E}$, is large) and if substitution between abatement and intermediate input is easy (i.e. σ_e is large).

---

[3] Without an initial emission tax, the firm does not abate in the initial equilibrium (i.e. $C_a = Y_a = 0$) so that $\alpha_v^y(1-\alpha_k^y) - \alpha_x^y - \alpha_e^y = 0$. Hence, the relative change in emissions remains finite even though t_e goes to infinity if the initial emission tax is zero.

A.1.2 Demand

Maximization of the utility function (7)yields

$$\frac{\delta g}{\delta Y}\Big/\frac{\delta g}{\delta C} = \frac{P_y}{P_c}. \tag{A-15}$$

Log-linearization of this equation yields the demand function

$$y = -\sigma_g p_y, \tag{A-16}$$

where σ_g represents the substitution elasticity between the final good Y and other consumption goods C in the household sub-utility function $g(.,.)$ (see (7)).

A.1.3 Equilibrium

Equilibrium on the market for the final good implies that demand (i.e. the right-hand side of (A-16) equals the right-hand side of (A-10)). This yields the price of the final good in terms of the demand price of the intermediate good

$$p_y = \frac{\varepsilon_s^y}{\varepsilon_s^y + \varepsilon_d^y}[\alpha_x^y p_x + \alpha_e^y t_e], \tag{A-17}$$

where $\varepsilon_d^y \equiv \sigma_g + [(1 - \alpha_v^y)\ \sigma_y - \sigma_v]/\alpha_v^y$. The final goods sector can shift the entire burden of higher costs (due to either a higher emission tax or a higher price of the intermediate input) forward to consumers if it can as easily substitute away from the intermediate good as the consumers can substitute away from the dirty final good (i.e. $\sigma_y = \sigma_v = \sigma_g$). However, if consumers have more opportunities to substitute away from the final good than final good producers have to substitute away from the intermediate input (i.e. $\sigma_g > \sigma_y = \sigma_v$ so that $\varepsilon_d^y > 0$), the final good industry has to absorb some of the burden of the higher costs of intermediate inputs and emissions. This share becomes larger if a smaller elasticity σ_k^y depresses the supply elasticity $\varepsilon_s^y \equiv \left[\sigma_k^y + \sigma_v\right] / \left[\alpha_v^y \alpha_k^y\right]$.

The impact on the output of the final goods industry is found by substituting (A-17) into (A-16) to eliminate the price of the final good p_y:

$$y = \frac{-\sigma_g \varepsilon_s^y}{\varepsilon_s^y + \varepsilon_d^y}[\alpha_x^y p_x + \alpha_e^y t_e]. \tag{A-18}$$

Higher costs of the intermediate input substantially depress the output of the final good if both the demand elasticity σ_g and the supply elasticity ε_s^y are large.

A.1.4 Distribution

The impact on the producer surplus of the final goods sector is given by [4]

$$\alpha_v^y \alpha_k^y r_y = p_y - \alpha_x^y p_x - \alpha_e^y t_e = -\frac{\varepsilon_d^y}{\varepsilon_s^y + \varepsilon_d^y}[\alpha_x^y p_x + \alpha_e^y t_e], \tag{A-19}$$

where the first equality follows from (A-6) and the second equality by substitution of (A-17) to eliminate p_y. Rentals in the final goods sector thus decline with higher costs of the intermediate input and emissions if the substitution possibilities of consumers exceed those of producers (i.e., $\sigma_g > \sigma_y = \sigma_v$ so that $\varepsilon_d^x > 0$). Rentals increase, however, if capital is a good substitute for the polluting intermediate good (i.e. σ_v is large) while consumers can not easily substitute away from the final good (i.e. σ_g is small) and producers cannot easily substitute labor for the composite $v(K_y; X)$ (i.e. σ_y is small) so that $\varepsilon_d^y \equiv \sigma_g + [(1-\alpha_v^y)\ \sigma_y - \ \sigma_v]/\alpha_v^y < 0$. In this case, the demand for capital rises on account of a positive substitution effect as producers substitute capital (rather than labor) for the polluting input. At the same time, production of the final good does not decline much as households do not respond much to the higher price of the final good. With a substantial positive substitution effect on capital demand thus dominating a small (in absolute value) scale effect on capital demand, the demand for capital rises thereby boosting the rental rate.

A.2 The market for the intermediate good

A.2.1 Demand

Demand for the intermediate good can be written in terms of the price of intermediate goods by substituting (A-17) into (A-11) to eliminate p_y :

$$x^d = -\varepsilon_d^x[p_x + \frac{\alpha_e^y}{\alpha_x^y} t_e], \tag{A-20}$$

where $\varepsilon_d^x \equiv \left[\alpha_x^y \left(\frac{\varepsilon_s^y \varepsilon_d^y}{\varepsilon_s^y + \varepsilon_d^y}\right) + \left(\frac{\alpha_x^y}{\alpha_v^y(1-\alpha_k^y)}\right)\sigma_v\right]$ is the price elasticity of the demand for the intermediate good. A higher price of the intermediate good depresses the demand for the intermediate good through two channels: a negative 'scale' effect on the output of the final goods sector (i.e. the first term in the square brackets at the right-hand side of the definition of the demand elasticity)) and a negative substitution effect (i.e. the second term in the square brackets at the right-hand side of the definition of the demand elasticity)).

---

[4] Producers optimally set the capital stock according to (A-4). Accordingly, the envelope theorem implies that a change in the capital stock does not directly affect the producer surplus.

A.2.2 Supply

Loglinearizing the production function of the upstream industry (1), we find

$$x^s = k_x + (1 - \alpha_k^x)(l_x - k_x), \tag{A-21}$$

where $\alpha_k^x \equiv R_x K_x / P_x X$ stands for the share of capital in output of the upstream sector. With a constant-returns-to-scale production function, the relative change in the output price is a weighted average of the relative changes in the input prices (note that wages do not change)

$$p_x = \alpha_k^x r_x. \tag{A-22}$$

Capital supply is given by[5]

$$k_x = \sigma_k^x r_x, \tag{A-23}$$

where σ_k^x stands for the substitution elasticity between the industry-specific capital services in the intermediate goods industry and the capital services in the rest of the economy.

Using (2) and (3) to eliminate P_x and log-linearizing the results, we arrive at

$$l_x - k_x = \sigma_x r_x, \tag{A-24}$$

where σ_x stands for the substitution elasticity between the two inputs in the production of the intermediate good.

Substituting (A-23), (A-24), and (A-22) into (A-21) to eliminate k_x, $(l_x - k_x)$, and r_x, we write the supply of the final good in terms of its price and the demand price of the intermediate good

$$x^s = \varepsilon_s^x p_x, \tag{A-25}$$

where $\varepsilon_s^x \equiv \left[\sigma_k^x + (1 - \alpha_k^x)\sigma_x\right] / \alpha_k^x$ denotes the supply elasticity. This elasticity becomes infinite if capital (i.e. the 'fixed' factor) does not play a role in production (i.e. $\alpha_k^x = 0$), if capital is a perfect substitute for capital in the rest of the economy (i.e. $\sigma_k^x \Rightarrow \infty$ so that adjustment costs are absent), or if mobile labor is a perfect substitute for the imperfectly mobile factor (i.e. capital) (i.e. $\sigma_x \Rightarrow \infty$). In all these cases, the immobile factor does not constrain production of the final good.

---

[5] This assumes that all households are well diversified so that income effects can be ignored. Alternatively, one can assume that a share γ_x of capital owners in the upstream industry is completely specialized in this sector (i.e. only derives income from capital in this sector). In that case the elasticity σ_k^x in the following equation is replaced by $(1 - \gamma_x)\sigma_k^x + \gamma_x \varepsilon_u$, where ε_u stands for the elasticity of aggregate capital supply with respect to the rate of return.

A.2.3 Equilibrium

The demand for the intermediate good is given by (A-20). The supply is given by (A-25). Setting demand equal to supply, we arrive at

$$p_x = -\left(\frac{\varepsilon_d^x}{\varepsilon_d^x + \varepsilon_s^x}\right)\frac{\alpha_e^y}{\alpha_x^y} t_e, \tag{A-26}$$

and

$$\alpha_x^y p_x + \alpha_e^y t_e = \left(\frac{\varepsilon_s^x}{\varepsilon_d^x + \varepsilon_s^x}\right)\alpha_e^y t_e. \tag{A-27}$$

Demand bears most of the emission tax burden (i.e. the demand price rises substantially—as indicated by the sign of $\alpha_x^y p_x + \alpha_e^y t_e$—while the supply price P_x does not decline much) if demand is inelastic compared to supply (i.e. if ε_d^x is small compared to ε_s^x).

We can now write the reduced form for the price of the final good p_y. Substitution of (A-27) into (A-17) yields

$$p_y = \left(\frac{\varepsilon_s^x}{\varepsilon_d^x + \varepsilon_s^x}\right)\left(\frac{\varepsilon_s^y}{\varepsilon_s^y + \varepsilon_d^y}\right)\alpha_e t_e. \tag{A-28}$$

The effects on the output of the upstream sector are given by (substitute (A-27) into (A-20) to eliminate p_x)

$$x = -\left(\frac{\varepsilon_s^x \varepsilon_d^x}{\varepsilon_d^x + \varepsilon_s^x}\right)\frac{\alpha_e^y}{\alpha_x^y} t_e. \tag{A-29}$$

Output of the intermediate good falls substantially on account of the emission tax if both the demand and supply elasticities are large. This is the case if capital is mobile and demand for the final good is elastic. Moreover, input substitution between capital and the dirty intermediate input in the downstream industry increases the decline in output of the intermediate goods industry.

A.2.4 Emission reductions

The impact on pollution is found by substituting (A-29) into the first equality in (A-14) to eliminate x

$$e = -\left[\left(\frac{\varepsilon_s^x \varepsilon_d^x}{\varepsilon_d^x + \varepsilon_s^x}\right)\frac{\alpha_e^y}{\alpha_x^y} + \sigma_e \frac{\alpha_v^y(1-\alpha_k^y) - \alpha_x^y - \alpha_e^y}{\alpha_v^y(1-\alpha_k^y) - \alpha_x^y}\right] t_e. \tag{A-30}$$

Inverting this equation, we can write the tax rate in terms of the pollution reduction. In this way, we can write the results in terms of the required reduction in pollution rather than the tax rate. Hence, we can alternatively parameterize environmental policy by changes in the pollution tax t_e or by changes in emission permits e. In particular, we can relate the required cost increase (as a ratio of the initial price of the output of the downstream industry) $\alpha_e^y t_e$ to the required emission cut $a = -e$:

$$\alpha_e^y t_e = \kappa a, \tag{A-31}$$

where $\kappa \equiv 1/\left[\left(\frac{\varepsilon_s^x \varepsilon_d^x}{\varepsilon_d^x + \varepsilon_s^x}\right)\frac{1}{\alpha_x^y} + \sigma_e \frac{\alpha_v^y(1-\alpha_k^y)-\alpha_x^y-\alpha_e^y}{\alpha_e^y[\alpha_v^y(1-\alpha_k^y)-\alpha_x^y]}\right]$. The denominator in this definition of κ includes the various channels through which emission can be cut, namely (i) abatement (which is the second term in the denominator), (ii) output of the final good (which is implicit in the first term between square brackets in the definition of ε_d^x (i.e $\varepsilon_d^x \equiv \left[\alpha_x^y\left(\frac{\varepsilon_s^y \varepsilon_d^y}{\varepsilon_s^y + \varepsilon_d^y}\right) + \left(\frac{\alpha_x^y}{\alpha_v^y(1-\alpha_k^y)}\right)\sigma_v\right]$) and thus affects the first term in the denominator of (A-31)), and (iii) input substitution between capital and the intermediate input in the downstream industry (this is implicit in the second term between square brackets in the definition of ε_d^x). The emission cost increase $\alpha_e^y t_e$ required to attain a certain emission cut $(-e)$ falls as these three channels become more effective.

A.3 Distributional impacts

We now analyze the distributional impacts of the environmental policy. The non-environmental welfare impacts consist of the change in the after-tax producer surplus in the upstream industry (PSX), the change in the after-tax producer surplus in the downstream industry (PSY), and the change in non-environmental (after-tax) consumer surplus (NCS). It will be convenient to express these three components of non-environmental welfare relative to $P_y Y$, the initial before-tax value of the output of the downstream industry Y. We can express these changes as

$$psx \equiv \frac{dPSX}{P_y Y} = (1-T)[\alpha_x^y p_x + \pi_x] = (1-T)[\alpha_x^y \alpha_k^y r_x + \pi_x], \tag{A-32}$$

$$psy \equiv \frac{dPSY}{P_y Y} = (1-T)[p_y - \alpha_x^y p_x - \alpha_e^y t_e + \pi_y] = (1-T)[\alpha_v^y \alpha_k^y r_y + \pi_y], \tag{A-33}$$

$$ncs \equiv \frac{dNCS}{P_y Y} = -(1-T)[p_y + (t/\beta)], \tag{A-34}$$

where π_i denotes lump-sum compensation (which is assumed to be taxed at the factor tax T) to sector $i; i = x, y$ (expressed relative to $P_y Y$) and $t \equiv dT/(1-T)$. $\beta \equiv P_y Y/Q$, where Q is aggregate factor income (before tax). This share goes to zero in our model in which the downstream and upstream sectors are very small compared to the rest of the economy.[6]

To arrive at the reduced-form equations, we substitute (A-26), (A-27), and (A-28) (and using (A-31) to eliminate $\alpha_e^y t_e$) into the second right-hand sides of (A-32) and (A-33):

$$psx/(1-T) \equiv -\left(\frac{\varepsilon_d^x}{\varepsilon_d^x + \varepsilon_s^x}\right)\kappa a + \pi_x, \tag{A-35}$$

$$psy/(1-T) = -\left(\frac{\varepsilon_s^x}{\varepsilon_d^x + \varepsilon_s^x}\right)\left(\frac{\varepsilon_d^y}{\varepsilon_s^y + \varepsilon_d^y}\right)\kappa a + \pi_y. \tag{A-36}$$

Setting these equations equal to zero, we find π_x and π_y required to ensure equity value neutrality in both sectors. The first terms at the right-hand sides of these expressions show which shares of the emission cost increase is born by the upstream and downstream industries, respectively.

We compute the compensation ratios as the share of gross revenue that needs to be paid in gross compensation π_i, i.e. $\theta_i \equiv \pi_i/(\alpha_e^y(t_e+e))$ $(i = x, y)$. This yields (by setting (A-35) and (A-36) equal to zero and solving for π_i)

$$\theta_x = \left(\frac{\varepsilon_d^x}{\varepsilon_d^x + \varepsilon_s^x}\right) / \left[1 - \alpha_e^y/\kappa\right], \tag{A-37}$$

$$\theta_y = \left(\frac{\varepsilon_s^x}{\varepsilon_d^x + \varepsilon_s^x}\right)\left(\frac{\varepsilon_d^y}{\varepsilon_s^y + \varepsilon_d^y}\right) / \left[1 - \alpha_e^y/\kappa\right]. \tag{A-38}$$

A.4 Welfare and equity value neutrality

To find *ncs*, we derive t/β from the government budget constraint. This latter constraint is given by

$$\xi + (1-T)\pi = \alpha_e^y(t_e(1-T) + e) + Tq + (1-T)(t/\beta), \tag{A-39}$$

where $q \equiv [\alpha k + (1-\alpha)l]/\beta$ is the change in aggregate factor supply measured relative to the initial output of the downstream industry (α is the share of capital income in aggregate value added and k and l represent aggregrate capital and labor supply, respectively), ξ stands for the change in government spending (expressed relative to the

---

[6] Also the relative change in the factor tax, t, goes to zero. However, the ratio t/β in (A-34) is well defined.

initial output of the downstream industry Y)[7] and $\pi \equiv \pi_x + \pi_y$. The first term $(1 - T)$ at the right-hand side of this equation follows from the non-profit constraint, which implies that a higher pollution tax implies lower factor income (and thus lower factor tax revenue since factor income is taxed at rate T).

Substituting (A-39) into (A-34) to eliminate t, we find for the overall non-environmental welfare effect $\psi \equiv psx + psy + ncs$ (using the first equalities after the definitions in (A-32), (A-33), and (A-34):

$$\psi = \alpha_e^y e + Tq - \xi, \tag{A-40}$$

where we also ignore the welfare effects of higher government spending (just as we ignore the welfare effects of better environmental quality as a result of less pollution).

Aggregate factor supply is

$$q\beta = -\varepsilon_u[t + \beta\alpha_e^y t_e] + \varepsilon_I[\pi\beta], \tag{A-41}$$

where ε_I is the income elasticity of aggregate factor supply. Using (A-39) to eliminate t from this equation, we establish

$$q(1-T) = \left(\frac{1}{1 - \varepsilon_u[T/(1-T)]}\right)[\varepsilon_u\left(\alpha_e^y e - \xi\right) - \varepsilon_c \pi(1-T)]. \tag{A-42}$$

Substitution of (A-42) into (A-40) yields

$$\psi = \lambda\left(\alpha_e^y e - \xi\right) - \mu\pi(1-T), \tag{A-43}$$

where we have used the definitions of λ and μ (see (9) and (10)).

We are now ready to combine the efficiency results with those for equity by exploring the efficiency costs of equity value neutrality (EVN). If EVN is imposed in the downstream industry, we find from (A-35) that $\pi_y = \left(\frac{\varepsilon_s^x}{\varepsilon_d^x + \varepsilon_s^x}\right)\left(\frac{\varepsilon_d^y}{\varepsilon_s^y + \varepsilon_d^y}\right)\kappa a$. Substitution of π_y for π into (A-43) and setting $\xi = 0$ and $-e = a$ yields (11). Similarly, EVN in the upstream industry requires $\pi_x = \left(\frac{\varepsilon_d^x}{\varepsilon_d^x + \varepsilon_s^x}\right)\kappa a$ (from (A-36)), which yields (14).

---

[7] Even though in the policy experiments government spending does not change, we include ξ in order to show that λ measures the cost of financing additional government spending.

B Numerical Model Documentation

We begin with an overview of the numerical model and then describe the specific functions depicting the behavior of agents. We next discuss the solution mechanism used to find equilibrium follows and describe the computations used for welfare analysis under the various modeled policies. We conclude with a summary of the equations used in the computation.

B.1 Overview

The model includes first a representative household which supplies factors of production and consumes final output. There are two factors (capital and labor) and three goods: one intermediate good and two final goods consumed by the household. Details of production and consumption decisions are provided below. There is a government which supplies a transfer to the household and which must balance its budget. One of the final goods includes a pollution externality, which can be abated either through substitution or through end-of-pipe treatment. Various policy instruments are available to control the pollution externality, and the model will analyze the effects of these on different sectors in the economy. Equilibrium is reached via an iterative process on prices such that all of the factor and goods markets clear and budget constraints are satisfied.

Factors: The household elastically supplies capital and labor to the production sector. Labor, L, is assumed to be perfectly mobile across industries, while capital, K, is not. The specification of capital immobility is described in detail in section B.2.3. Aggregate factor supplies are determined by the solution to the household's utility maximization problem and prices (see section B.3).

Goods: There is one intermediate good, X, which can be thought of as an intermediate energy input to production. There are two final goods, C and Y, which represent a clean and a polluting final good respectively. The clean good represents the majority of consumption and is produced using only capital and labor. The polluting good, Y, is produced using the intermediate good X as well as the two factors. Household demand for C and Y is derived from utility, while production is assumed to be competitive and exhibit constant returns to scale.

Emissions: Emissions are produced from the consumption of the intermediate good X and can be abated both through substitution away from X or through the use of an end-of-pipe treatment technology specified in section B.2.2. In the policy scenarios, emissions will be controlled by government regulation. The government issues a fixed number of tradeable permits, which are either auctioned, grandfathered, or distributed via some combination of the two.

Government: In the benchmark scenario, the government levies distortionary taxes on labor and capital and returns the revenue via a transfer to households. In policy scenarios, the government may also raise revenue through auctioned emissions permits. Depending on the scenario, the government may return the additional revenue through the transfer, or recycle it via a cut in the labor and capital tax rates keeping the transfer fixed.

Household: The household supplies factors of production and consumes final goods in accordance with the utility function given below. The level of pollution is assumed to enter household utility in an additively separable way, and so does not affect household supply and demand decisions. The numerical model estimates the gross effects of various policies, so no pollution damage function is included in this document.

B.2 Production

The production functions for the three industries, production of pollution, and distribution of capital used in the numerical model are described in this section.

B.2.1 Production Functions

All production is constant returns to scale, and given by constant elasticity of substitution (CES) production functions. The production of the intermediate good, X, is given by:

$$X = \gamma_x \left[\alpha_{x_k} K_x^{\frac{\sigma_x - 1}{\sigma_x}} + \alpha_{x_l} L_x^{\frac{\sigma_x - 1}{\sigma_x}}\right]^{\frac{\sigma_x}{\sigma_x - 1}} \tag{B-1}$$

where the elasticity of substitution between the two factors is given by σ_x. The model calibrates the α_x's (share parameters) and γ_x (scale parameter) such that in the benchmark simulation the desired amount of each factor is used in producing the benchmark level of X.

Production of the final good C is similar, employing capital and labor in the CES production function:

$$C = \gamma_c \left[\alpha_{c_k} K_c^{\frac{\sigma_c - 1}{\sigma_c}} + \alpha_{c_l} L_c^{\frac{\sigma_c - 1}{\sigma_c}} \right]^{\frac{\sigma_c}{\sigma_c - 1}} \tag{B-2}$$

Again, the scale and share parameters are calibrated to benchmark levels of output and factor consumption in the industry. The elasticity of substitution among inputs, σ_c, is given in the model input as specified in the paper.

Production of the polluting good, Y, is somewhat more complicated as it involves the usual two factors as well as the intermediate good. Production is given by a nested CES function as follows:

$$Y = \gamma_y \left[\alpha_{y_v} V_y^{\frac{\sigma_y - 1}{\sigma_y}} + \alpha_{y_l} L_y^{\frac{\sigma_y - 1}{\sigma_y}} \right]^{\frac{\sigma_y}{\sigma_y - 1}} \tag{B-3}$$

where

$$V = \gamma_v \left[\alpha_{v_k} K_v^{\frac{\sigma_v - 1}{\sigma_v}} + \alpha_{v_x} X_v^{\frac{\sigma_v - 1}{\sigma_v}} \right]^{\frac{\sigma_v}{\sigma_v - 1}} \tag{B-4}$$

Note that together there are three inputs to the production of Y: K, L, and X. In the inner nest (given by V) the inputs K and X substitute for one another with elasticity σ_v. This combination is then used in the outer level, where V and L are substituted with elasticity σ_y. As above (although again more complicated in this case) the model is calibrated such that the benchmark levels of the three inputs combine to produce the benchmark level of Y. The elasticities are specified as input to the model.

B.2.2 Pollution Generation

Pollution in the model comes from the consumption of the intermediate good X and allows for the possibility of end-of-pipe treatment using a combination good, denoted G_a. Total pollution (emissions) are then given as:

$$E = \alpha_e \left[1 + \beta_e \left(\frac{G_a}{X} \right)^{\rho_e} \right]^{\frac{-1}{\rho_e}} \cdot X \tag{B-5}$$

The parameters β_e and ρ_e are given as model inputs and calibrated to reflect the desired ease of end-of-pipe treatment. G_a is again the amount of the composite good G used for end-of-pipe treatment. Finally, α_e is a scale parameter calibrated to produce the desired emissions in the benchmark. Note that emissions are linear in the production of X but not in the use of end-of-pipe treatment.

B.2.3 Capital Transformation

In contrast to labor, capital is not permitted to flow freely among sectors. The aggregate capital supplied by the household is given by K^s (see section B.3 for a discussion of factor supply) and will be divided among the three sectors according to the price of capital in each and the costs of moving capital among sectors. In order to allocate capital among sectors the agent is modeled as maximizing:

$$P_{k_x}K_x + P_{k_y}K_y + P_{k_c}K_c \tag{B-6}$$

subject to the constraint that

$$\gamma_k \left[\alpha_k K_x^{\frac{\sigma_k - 1}{\sigma_k}} + \beta_k K_y^{\frac{\sigma_k - 1}{\sigma_k}} + (1 - \alpha_k - \beta_k) K_c^{\frac{\sigma_k - 1}{\sigma_k}} \right]^{\frac{\sigma_k}{\sigma_k - 1}} \leq K^s \tag{B-7}$$

where K_x, K_y, and K_c are the quantities of capital supplied to each sector and K^s is the aggregate supply from the household. Prices, P_k, are set according to market demand and supply. The parameter σ_k governs the ease with which capital may be substituted among uses, while the remaining parameters are calibrated such that in the benchmark case the equality $K_x + K_y + K_c = K^s$ holds. As capital demands differ from the benchmark, some distortion will occur such that $K_x + K_y + K_c < K^s$. The difference between total capital supply and the sum of individual sector supplies is accounted as a loss due to friction.

B.3 Household Behavior

The household maximizes the utility function given below, which yields a set of demand functions for the final goods and supply functions for the factors of production. These supply and demand functions are determined by the parameters of the utility function, prices, and income as described in the following.

B.3.1 Utility

Household utility is a function of capital and labor supply, consumption, and emissions. Again, emissions are assumed to enter separably and are not included here. Utility is of the nested CES form:

$$U = \left(\alpha_{u_g} G^{\frac{\sigma_u - 1}{\sigma_u}} + \alpha_{u_z} Z^{\frac{\sigma_u - 1}{\sigma_u}}\right)^{\frac{\sigma_u}{\sigma_u - 1}} \tag{B-8}$$

where G and Z are the CES functions:

$$G = \left(\alpha_{g_c} C^{\frac{\sigma_g - 1}{\sigma_g}} + \alpha_{g_y} Y^{\frac{\sigma_g - 1}{\sigma_g}}\right)^{\frac{\sigma_g}{\sigma_g - 1}} \tag{B-9}$$

$$Z = \left(\alpha_{z_l} \ell_l^{\frac{\sigma_z - 1}{\sigma_z}} + \alpha_{z_k} \ell_k^{\frac{\sigma_z - 1}{\sigma_z}}\right)^{\frac{\sigma_z}{\sigma_z - 1}} \tag{B-10}$$

The G (goods) nest is of the standard CES form where the elasticity of substitution between the clean good C and the polluting good Y is given by σ_u. The α's are calibrated as before to yield the desired benchmark proportions of C and Y in consumption. The Z (factors) nest is composed of two symmetric contributors to utility, ℓ_l and ℓ_k. The first of these is leisure, defined in the usual way such that $L^s = \bar{L} - \ell_l$ where L^s is the total labor supply and $\bar{L}$ is the household's total endowment of time. The second item, ℓ_k, is the leisure analog of capital and is defined by the relation: $K^s = \bar{K} - \ell_k$. where $\bar{K}$ can be thought of as a measure of potential capital. Finally, σ_z is a parameter input that determines the relative elasticity between the supply of labor and capital.

In the outer nest given in (B-8) the overall elasticity of substitution between factors and goods is set using the parameter σ_u. This elasticity, combined with the parameters used for $\bar{L}$ and $\bar{K}$, will determine the overall elasticities of labor and capital supply in the model. As usual, the two α parameters are calibrated so that the proportions of total income devoted to Z and G will match the benchmark inputs.

B.3.2 Household Budget Constraint

The utility function above is maximized subject to the following budget constraint, producing goods demand and factor supplies at a given set of prices. The budget constraint can be written as:

$$P_c C + P_y Y \leq P_k^{index}(1 - \tau_k)(\bar{K} - \ell_k) + (1 - \tau_L)(\bar{L} - \ell_l) + gov + permits \tag{B-11}$$

where:

P_k^{index}	A weighted average of capital prices less the capital lost to friction according to (B-6) and (B-7) above.
gov	The government transfer to the household, may vary as described in section B.4.
$permits$	The value of emissions permits grandfathered to the firm, we assume that the household owns the firms and so receives these as a lump sum transfer. See equation (B-13) below.

Prices P and taxes τ are given as solution parameters in the model (see subsection B.5.2). The components of the government transfers to the household and via permits to the firm are described in more detail in section B.4.

B.4 Government Budget and Policies

The government in the simulation model has several sources of revenue and makes transfers to the households that are accounted as described in this section. The overall government budget (revenue and transfers) must balance for each scenario, but the government's choice of tax and permit instruments and the various components of the transfer depend on the policy being modeled.

The government levies distortionary taxes on the two factors, labor and capital. In the benchmark, these tax rates are fixed and no other policy is undertaken. In this case, the size of transfers, $gov_{benchmark}$, is determined simply by the factor taxes as:

$$gov_{benchmark} = \tau_k(P_{k_x}K_x + P_{k_c}K_c + P_{k_y}K_y) + \tau_L L \tag{B-12}$$

where the K and L terms are benchmark factor demands and the τ's are the benchmark distortionary tax rates.

When an emissions policy is put into place, the government also receives revenue from the auctioned emissions permits. We model two policies that balance the government budget in this case. The first, a lump sum transfer, implies simply that benchmark capital and labor taxes remain fixed while the size of the transfer, gov, is allowed to vary such that the budget balances. Alternatively, the additional revenue may be recycled. In this case, the model holds the transfer fixed such that $gov_{real} = gov_{benchmark}$ but varies the factor taxes (in the same proportion) such that the government budget balances. The transfer is fixed in real terms, using the ideal price index for goods given in (B-19).

In addition, the model allows for partial grandfathering of emissions permits. This is accomplished by transferring a fraction of permit revenue from the government to the firms (equivalently, to the household). This amount is indicated by *permits* in (B-11) above. It should be noted that the amount transferred is assumed taxable at the capital tax rate such that:

$$permits = (1 - \tau_k) \cdot compensation\_ratio \cdot P_e E \tag{B-13}$$

where the fraction of inframarginal permits grandfathered, *compensation_ratio*, is set by the solution mechanism such that the lost value of capital in either or both of the X and Y industries is exactly offset. P_e is the price of emissions, making the term P_eE the gross government receipts from auctioned emissions permits.

As an example of the process described above, consider the government budget constraint under Policy 2 from the paper. In this scenario, the budget is balanced by recycling revenue to the factor taxes, and the compensation ratio is set to offset losses in capital value to the Y industry. Therefore in equilibrium:

$$gov_{real} + permits = \tau_k^*(P_{k_x}K_x + P_{k_c}K_c + P_{k_y}K_y) + \tau_L^* L + P_e E \tag{B-14}$$

where gov_{real} is fixed according to benchmark transfers adjusted for prices, *permits* is defined as above such that its value offsets capital losses to Y in equilibrium, and the τ^* terms are set by the government such that the equality in (B-14) holds. Note that the price of emissions permits, P_e, is also simultaneously determined such that the desired emissions quota is reached. See section B.5.2 describing the solution mechanism for details.

B.5 Equilibrium

This section first provides an overview of the equilibrium conditions for the numerical model, and then provides a more detailed description of the markets and algorithm used.

B.5.1 Definition

Equilibrium is reached when prices are such that all output and factor markets clear and all budget constraints are satisfied. Since production in the economy is assumed to be competitive and constant returns to scale, the model assumes that production will meet demand at the cost-minimizing price of production (determined by factor prices as described below in B.5.2). What remains, then, is that all of the factor markets clear.

Since capital is not fully mobile, there will be one market for each type of capital, and a fourth market for labor such that:

$$K_x^d = K_x^s \tag{B-15a}$$

$$K_c^d = K_c^s \tag{B-15b}$$

$$K_y^d = K_y^s \tag{B-15c}$$

$$L_x^d + L_c^d + L_y^d = L^s \tag{B-15d}$$

The household problem is solved such that the household income and capital transformation budget constraints always hold. By Walras' law, then, if the budget constraint and three of the markets in (B-15) hold, the fourth market must also clear. In the algorithm subsection, then, note that we have omitted the market for labor. The model instead solves the three capital markets, and then checks the solution to verify that the labor market indeed clears. This also serves as a useful check for any accounting leakages in the model.

B.5.2 Algorithm

In addition to the markets defined above, of course, the model must also be solving for a set of policy constraints. This subsection describes the main solution algorithm employed by the model, including a complete list of constraints for Policy 2.

Solving for equilibrium is an iterative process, with each iteration started from the set of prices and values in the table below. Note that depending on the policy chosen, a different set of parameters may be used. We again use Policy 2 as an example:

Parameter	**Description**
P_{k_x}	Price of capital for good X
P_{k_c}	Price of capital for good C
P_{k_y}	Price of capital for good Y
$\tau_{multiplier}$	Multiplier for factor taxes, for government budget balance
P_e	Price of emissions permits, to match permit demand with the emissions quota
compensation_ratio	Fraction of gross permit revenue returned to firms, for profit neutrality in Y industry

The first three prices correspond to the three capital markets in (B-15), and, together with labor, will drive goods prices and demands. The next item, $\tau_{multiplier}$, is used to balance the government budget. Recall from (B-14) that the government must set τ^* to balance the budget, this multiplier adjusts benchmark taxes, τ, implying a particular guess for τ^*. The fifth item, P_e, is a guess for the price of emissions permits. Finally, the last item is the fraction of gross permit revenue given back to the Y industry to compensate it for loss from the emissions policy

Using the above list of prices as a starting point, the model determines all other prices and demands in the system–an outline of this process follows: The price of goods X and C follows from the solution to the cost minimization analog to the production functions (B-1) and (B-2). Note that the price of labor is normalized to 1. Solving for the price of Y is considerably more difficult since it will include the price of emissions permits and involves a choice of end-of-pipe treatment. Given prices, the production function (B-3), and the emissions function (B-5), it turns out that there is no closed form solution to the cost minimization problem for Y. In words, this is because the price of Y depends on the price of end-of-pipe treatment, which depends simultaneously on the price of Y (recall that end-of-pipe treatment uses the composite good G composed of C and Y). To solve this, a simple search algorithm is used where a price for Y is guessed, and then updated until convergence based on the end-of-pipe treatment chosen. The solution will satisfy:

$$P_y = P_{k_y}\frac{K_y}{Y} + \frac{L_y}{Y} + P_x\frac{X_y}{Y} + P_e\frac{E(G_a)}{Y} + P_G\frac{G_a}{Y} \tag{B-16}$$

where P_G is a function of P_c and P_y, and K, L, X, and G_a are chosen to minimized P_y.

Once goods prices have been determined, household demands follow easily from the first order conditions maximizing utility (B-8) subject to the household budget, (B-11). Similarly, the aggregate supply of labor and capital is given from the utility maximization problem. Recall though, that aggregate capital supply must still be broken down into supply to each sector using (B-6) and (B-7).

Having determined factor supplies and goods demand, the remaining computations are all fairly straightforward. Government income is accounted using factor taxes given by the benchmark τ multiplied by $\tau_{multiplier}$, and the value of grandfathered permits is determined by the compensation ratio. Finally, the total emissions demanded can be determined directly from equation (B-5).

After computing all of these values, the algorithm must iteratively update the set of prices in the table above until equilibrium is reached. It does this using a derivative search based on Newton's method, solving the following system of equations (which are the equilibrium conditions for Policy 2). The table below corresponds to the one above, with the parameter list in the left column matching:

Parameter	**Equilibrium Condition**
P_{k_x}	$K_x^d - K_x^s = 0$
P_{k_c}	$K_c^d - K_c^s = 0$
P_{k_y}	$K_y^d - K_y^s = 0$
$\tau_{multiplier}$	$gov\_income - gov\_expenditure = 0$
P_e	$E - E_{quota} = 0$
$compensation\_ratio$	$\underbrace{(1-\tau_k^*)(P_{k_y}K_y + permits)}_{policy} - \underbrace{P_{k_y}K_y}_{benchmark} = 0$

The first three conditions, factor markets, are as in (B-15). Government income and expenditure correspond to the right and left sides of (B-14) respectively. E is determined from the emissions function, while E_{quota} is set exogenously as the policy emissions quota. The term labeled *policy* in the final condition refers to the value of capital in the policy case plus the value of compensation, captured as before in the variable *permits*. The term labeled *benchmark* refers to the value of capital in the benchmark (without policy) case. For simplicity the price deflator has been omitted, but note that all prices used for the capital compensation adjustment are kept in real terms.

B.6 Welfare Analysis

The equivalent variation (EV) is calculated for each policy scenario as a measure of gross welfare change (not including environmental benefits). This is defined as the income change in the benchmark case that would create the same utility change as the policy. The numerical computation for this is relatively straightforward; the price of utility in the benchmark (in terms of total income) is computed using the ideal price index given by:

$$P_u = (\alpha_{u_z}^{\sigma_u} P_z^{1-\sigma_u} + \alpha_{u_g}^{\sigma_u} P_g^{1-\sigma_u})^{\frac{1}{1-\sigma_u}} \tag{B-17}$$

where the nested price-indices for factors, P_z, and goods, P_g, are given by:

$$P_z = \left(\alpha_{z_k}^{\sigma_z} \left((1-\tau_k) P_k^{index} \right)^{1-\sigma_z} + \alpha_{z_l}^{\sigma_z} (1-\tau_l)^{1-\sigma_z} \right)^{\frac{1}{1-\sigma_z}} \tag{B-18}$$

$$P_g = \left(\alpha_{g_c}^{\sigma_g} P_c^{1-\sigma_g} + \alpha_{g_y}^{\sigma_g} P_y^{1-\sigma_g}\right)^{\frac{1}{1-\sigma_g}} \tag{B-19}$$

The equivalent variation is then given simply as:

$$EV = P_u^{benchmark} \cdot U^{policy} - TI^{benchmark} \tag{B-20}$$

where the first term determines how much it would cost to achieve the policy level of utility in the benchmark, and the second term is just total income in the benchmark. The difference is the equivalent variation as defined above.

B.7 Summary of Computed Equations

This section contains the first-order conditions used to derive supply and demand in the equilibrium process above. The subsections roughly correspond to the order in which the model solves the various markets.

B.7.1 Production

Producer demands for inputs and output prices for the final good C and the intermediate good X are very similar; the problem for good C is shown. Producers are assumed to solve their cost minimization problem, which can be expressed in per unit terms as follows (recall that production is constant returns to scale):

$$\min_{k_c, l_c} \; P_{k_c} k_c + P_l l_c \tag{B-21}$$

$$s.t. \quad \gamma_c \left[\alpha_{c_k} k_c^{\frac{\sigma_c - 1}{\sigma_c}} + \alpha_{c_l} l_c^{\frac{\sigma_c - 1}{\sigma_c}}\right]^{\frac{\sigma_c}{\sigma_c - 1}} \geq 1 \tag{B-22}$$

$$k_c \geq 0 \tag{B-23}$$

$$l_c \geq 0 \tag{B-24}$$

where k_c and l_c are defined as demands of capital and labor per unit C.

Taking first order conditions and combining (assuming an interior solution) gives the CES factor demand functions:

$$k_c = \frac{1}{\gamma_c}\left[\alpha_{c_k} + \alpha_{c_l}\left(\frac{\alpha_{c_k}}{\alpha_{c_l}}\frac{P_l}{P_{k_c}}\right)^{\sigma_c - 1}\right]^{\frac{\sigma_c}{\sigma_c - 1}} \tag{B-25}$$

$$l_c = \frac{1}{\gamma_c}\left[\alpha_{c_k}\left(\frac{\alpha_{c_k}}{\alpha_{c_l}}\frac{P_l}{P_{k_c}}\right)^{\sigma_c - 1} + \alpha_{c_l}\right]^{\frac{\sigma_c}{\sigma_c - 1}} \tag{B-26}$$

Since production is competitive and constant returns to scale, the price of good C is simply:

$$P_c = P_{k_c}k_c + P_l l_c \tag{B-27}$$

The problem for good X is analogous.

Solving the producer problem for the polluting good Y, however, is considerably more problematic. We divide the problem into two, solving first for the cost minimizing input mix to V, the inner CES nest, and then more simply for the combination of V and L to make the final good Y. (see equation (B-3)) The per-unit cost minimization problem for the inner nest, V, is given as:

$$\min_{k_v, x_v, g_a} \quad P_k k_v + P_x x_v + P_g g_a + \tau e_v \tag{B-28}$$

$$s.t.\ \gamma_v\left[\alpha_{v_k}^{\frac{\sigma_v - 1}{\sigma_v}} k_v + \alpha_{v_x}^{\frac{\sigma_v - 1}{\sigma_v}} x_v\right]^{\frac{\sigma_v}{\sigma_v - 1}} \geq 1 \tag{B-29}$$

$$e_v = \alpha_e\left[1 + \beta_e\left(\frac{g_a}{x_v}\right)^{\rho_e}\right]^{\frac{-1}{\rho_e}} x_v \tag{B-30}$$

$$k_v \geq 0 \tag{B-31}$$

$$x_v \geq 0 \tag{B-32}$$

$$g_a \geq 0 \tag{B-33}$$

where lowercase letters are again per unit of production: k_v, x_v, g_a, and e_v are capital, X, G_a, and E per unit of V. The first order condition on x_v can be rearranged to give the following shadow price for x:

$$\hat{P}_x = \overbrace{P_x}^{1} + \left\{ \overbrace{\tau_e\alpha_e\left[1+\beta_e\left(\frac{g_a}{x_v v}\right)^{\rho_e}\right]^{\frac{-1}{\rho_e}}}^{2} + \overbrace{\tau_e\alpha_e\beta_e\left(\frac{g_a}{x_v v}\right)^{\rho_e}\left[1+\beta_e\left(\frac{g_a}{x_v v}\right)^{\rho_e}\right]^{\frac{-(1+\rho_e)}{\rho_e}}}^{3} \right\} \tag{B-34}$$

The terms of (B-34) can be interpreted as the marginal cost due to

1. purchasing x
2. increasing emissions due to increasing x (holding $\frac{g_a}{x}$ constant)
3. increasing emissions due to decreasing the ratio $\frac{g_a}{x}$

The factor demands for x_v and k_v are then as usual, except that the shadow price of X is used:

$$k_v = \frac{1}{\gamma_v}\left[\alpha_{v_k} + \alpha_{v_x}\left(\frac{\alpha_{v_k}}{\alpha_{v_x}}\frac{\hat{P}_x}{P_{k_v}}\right)^{\sigma_v - 1}\right]^{\frac{\sigma_v}{\sigma_v - 1}} \tag{B-35}$$

$$x_v = \frac{1}{\gamma_v}\left[\alpha_{v_k}\left(\frac{\alpha_{v_k}}{\alpha_{v_x}}\frac{\hat{P}_x}{P_{k_v}}\right)^{\sigma_v - 1} + \alpha_{v_x}\right]^{\frac{\sigma_v}{\sigma_v - 1}} \tag{B-36}$$

As mentioned, there is no closed form solution for the amount of end-of-pipe treatment chosen in the production of V. The first order condition on g_a from equation (B-28) can be reduced only to:

$$\frac{g_a}{x_v} \equiv \left(\frac{P_g}{\tau\alpha_e\beta_e}\right)^{\frac{1}{\rho_e - 1}}\left[1+\beta_e\left(\frac{g_a}{x_v}\right)^{\rho_e}\right]^{\frac{1+\rho_e}{\rho_e(\rho_e - 1)}} \tag{B-37}$$

The model iterates on this equation, solving for the ratio $\frac{g_a}{x_v}$. Once k_v, x_v, and g_a are determined, emissions per unit V, and hence the total shadow price of using V in Y, can be found.

With the price of V and K_y in hand, solving for the inputs to the outer nest of the Y production function is straightforward and analagous to the standard CES functions above. Notice, however, that the solution to (B-28) depends on knowing the price of

G_a, which depends in turn on the price of Y. Therefore, an iterative process is again used where a ”guess” for the price of Y (and therefore G_a) is made in order to solve (B-28). This then feeds into the problem below (given in (B-38)), producing an updated guess for the price of Y. This is iterated until convergence is achieved.

Given the shadow price of V above, the cost minimization problem for Y can be written:

$$\min_{v_y, l_y} \quad P_v v_y + P_l l_y \tag{B-38}$$

$$s.t. \quad \lambda_y \left\{ 1 - \gamma_y \left[\alpha_{y_v} v_y^{\frac{\sigma_y - 1}{\sigma_y}} + \alpha_{y_l} l_y^{\frac{\sigma_y - 1}{\sigma_y}} \right]^{\frac{\sigma_y}{\sigma_y - 1}} \right\} \leq 1 \tag{B-39}$$

where lower case again indicates unit demands, and an interior solution is assumed. Per-unit factor demands for labor and the sub-good V are given as usual by:

$$v_y = \frac{1}{\gamma_y} \left[\alpha_{y_v} + \alpha_{y_l} \left(\frac{\alpha_{y_v}}{\alpha_{y_l}} \frac{P_l}{P_v} \right)^{\sigma_y - 1} \right]^{\frac{\sigma_y}{\sigma_y - 1}} \tag{B-40}$$

$$l_y = \frac{1}{\gamma_y} \left[\alpha_{y_v} \left(\frac{\alpha_{y_v}}{\alpha_{y_l}} \frac{P_l}{P_v} \right)^{\sigma_y - 1} + \alpha_{y_l} \right]^{\frac{\sigma_y}{\sigma_y - 1}} \tag{B-41}$$

B.7.2 Capital transformation

Agents determine the percentage of capital to allocate to a sectors based on its returns and the mobility of capital:

$$\tau_k^* = \tau_k^{benchmark} - (\tau_{multiplier} - 1) \tag{B-42}$$

$$k_y^s = \frac{\overline{k}}{\gamma_k \left[\alpha_k \left(\frac{\tau_k^* P_{k_x}}{\tau_k^* P_{k_y}} \frac{\beta_k}{\alpha_k} \right)^{1 - \sigma_k} - \beta_k - (1 - \alpha_k - \beta_k) \left(\frac{\tau_k^* P_{k_c}}{\tau_k^* P_{k_y}} \frac{\beta_k}{1 - \alpha_k - \beta_k} \right)^{1 - \sigma_k} \right]^{\frac{\sigma_k}{\sigma_k - 1}}} \tag{B-43}$$

$$k_x^s = \left(\frac{\tau_k^* P_{k_x}}{\tau_k^* P_{k_y}} \frac{\beta_k}{\alpha_k} \right)^{-\sigma_k} k_y^s \tag{B-44}$$

$$k_c^s = \left(\frac{\tau_k^* P_{k_c}}{\tau_k^* P_{k_y}} \frac{\beta_k}{1 - \alpha_k - \beta_k} \right)^{-\sigma_k} k_y^s \tag{B-45}$$

B.7.3 Household Problem

The functions for utility —$(B-8),(B-9),(B-10)$— along with income, determine the supply of K and L and the demand for C and Y.

To calculate income, some preliminary calculations of prices are necessary

$$\tau_l^* = \tau_l^{benchmark} + (\tau_{multiplier} - 1) \tag{B-46}$$

$$P_k^{index} = (1-\tau_k^*)k_c^s P_{k_c} + (1-\tau_k^*)k_y^s P_{k_y} + (1-\tau_k^*)k_x^s P_{k_x} \tag{B-47}$$

$$P_z = (\alpha_{z_k}^{\sigma_z} {P_k^{index}}^{1-\sigma_z} + \alpha_{z_l}^{\sigma_z}(1-\tau_l)^{1-\sigma_z})^{\frac{1}{1-\sigma_z}} \tag{B-48}$$

$$P_g = (\alpha_{g_c}^{\sigma_g} P_c^{1-\sigma_g} + \alpha_{g_y}^{\sigma_g} P_y^{1-\sigma_g})^{\frac{1}{1-\sigma_g}} \tag{B-49}$$

$$P_u = (\alpha_{u_z}^{\sigma_u} P_z^{1-\sigma_u} + \alpha_{u_g}^{\sigma_u} P_g^{1-\sigma_u})^{\frac{1}{1-\sigma_u}} \tag{B-50}$$

Household income is made up of transfers, labor income, and capital income. Where applicable, income from grandfathered permits is added.

$$transfers = P_g * transfers_{real} \tag{B-51}$$

$$I_0 = \bar{K} P_k^{index} + \bar{L} P_l (1-\tau_l) + transfers + permits(1-\tau_k^*) \tag{B-52}$$

where

- $\bar{K}$ is the amount of potential capital available
- $\bar{L}$ is the amount of potential labor available
- $transfers_{real}$ are observed/benchmark transfers

Income can be decomposed into “spending” on Z (i.e. the value of potential labor and capital consumed by the household rather than supplied to the market) and into spending on G (goods) From the utility function, the “demand” for (i.e. spending on) Z can be calculated as below and the spending on G as the residual:

$$I_z = \frac{I_0(\alpha_{u_z} P_g)^{\sigma_u} P_z}{(\alpha_{u_z} P_g)^{\sigma_u} P_z + (\alpha_{u_g} P_z)^{\sigma_u} P_g} \tag{B-53}$$

$$I_g = I_0 - Z \tag{B-54}$$

Supply of capital & labor Similarly, spending on Z can be decomposed into spending on leisure and spending on unused capital. The residual between spending on a resource and the total amount available is supplied to the market.

$$\ell_l = \frac{I_z(\alpha_{z_l} P_k^{index})^{\sigma_z}(1-\tau_l)}{(\alpha_{z_l} P_k^{index})^{\sigma_z}(1-\tau_l) + (\alpha_{z_k}(1-\tau_l))^{\sigma_z} P_k^{index}} \tag{B-55}$$

$$L^s = \bar{L} - \frac{\ell_l}{1-\tau_l} \tag{B-56}$$

$$\ell_k = I_z - \ell_l \tag{B-57}$$

$$K^s = \bar{K} - \frac{\ell_k}{P_k^{index}} \tag{B-58}$$

K^s can be decomposed into sector-specific capital supply:

$$K_c^s = k_c^s K^s \tag{B-59}$$

$$K_y^s = k_y^s K^s \tag{B-60}$$

$$K_x^s = k_x^s K^s \tag{B-61}$$

Consumer Demand for C and Y Consumer spending on G can be decomposed into spending on C and Y

$$I_c = \frac{I_g(\alpha_{g_c} P_y)^{\sigma_g} P_c}{(\alpha_{g_c} P_y)^{\sigma_g} P_c + (\alpha_{g_y} P_c)^{\sigma_g} P_y} \tag{B-62}$$

$$C_c^d = \frac{I_c}{P_c} \tag{B-63}$$

$$Y_c^d = \frac{I_g - I_c}{P_y} \tag{B-64}$$

Total demand for C and Y The producer demand for Y will be the portion of G_a that comes from Y. This portion will be calculated based on the percentage of Y in the consumer portion of G: $\frac{Y_c}{G_c} = \frac{Y_c}{C_c+Y_c}$. G_a is calculated based on g_a, the per-unit-of-V use of abatement, multiplied by V, which in turn is calculated as $v_y^d(Y_c^d + Y_p^d)$:

$$G_a = \frac{g_a v_y Y_c^d}{1 - \frac{g_a v_y Y_c^d}{C_c^d + Y_c^d}} \tag{B-65}$$

$$Y_a^d = G_a \frac{Y_c}{C_c^d + Y_c^d} \tag{B-66}$$

$$C_a^d = G_a - Y_a^d \tag{B-67}$$

$$C^d = C_c^d + C_a^d \tag{B-68}$$

$$Y^d = Y_c^d + Y_a^d \tag{B-69}$$

B.7.4 Excess Demand

We are now able to calculate the last variables needed to determine the excess demands in subsection B.5.2

Demand for K

$$K_c^d = C^d k_c^d \tag{B-70}$$

$$K_x^d = X^d k_x^d \tag{B-71}$$

$$K_y^d = V^d k_y^d \tag{B-72}$$

$$\tag{B-73}$$

Government income & expenditure Government income is calculated as the sum of emission tax revenue, labor tax, and capital tax, adjusted for the grandfathered permit revenue

$$\begin{aligned} gov\_income = {} & P_e E + L^s \tau_l + \tau_k^* K_c^s P_{k_c} + \tau_k^* K_y^s P_{k_y} + \tau_k^* K_x^s P_{k_x} \\ & - (1 - \tau_k^*) permits \end{aligned} \tag{B-74}$$

$$gov\_expend = transfers \tag{B-75}$$

Emissions

$$E = V e^v \tag{B-76}$$

Profit

$$\pi_c = (1 - \tau_k^*) P_{k_c} K_c \tag{B-77}$$

$$\pi_x = (1 - \tau_k^*) P_{k_x} K_x \tag{B-78}$$

$$\pi_y = (1 - \tau_k^*)(P_{k_y} K_y + permits) \tag{B-79}$$