Appendix to:
Costs of Alternative Environmental Policy Instruments in the Presence of Industry Compensation Requirements

A. Lans Bovenberg
Lawrence H. Goulder
Mark R. Jacobsen

Appendix: Numerical Model

This appendix starts with an overview of the numerical model. It then presents the equations describing the behavior of agents. The third section presents the solution method used to obtain the general equilibrium under each policy simulated, as well the calculations employed to determined welfare impacts. It concludes with a summary of the equations used in the computation.

1 Overview

The model includes a representative household that supplies factors of production and consumes final output. There are two factors of production (capital and labor) and three produced goods: an intermediate good and two final goods consumed by the household. There is a government that supplies a transfer to the household and that must balance its budget.

Production of one of the final goods generates emissions of a pollutant. These emissions can be abated either through input substitution or through end-of-pipe treatment.

We consider four policy instruments to control the pollution externality: 1) an emissions tax, 2) a tax on the intermediate (fuel) input, 3) a performance standard and 4) a technology mandate. We analyze the effects of the policy on the economy and its incidence on firms. We allow two policy mechanisms for compensating firms: 1) a lump sum tax credit and 2) marginal cuts in the capital tax rates for a specific industry. Equilibrium is reached via an iterative process on prices such that all of the factor and goods markets clear and budget constraints are satisfied.

Factors: The household elastically supplies capital and labor to the production sector. Labor, L, is assumed to be perfectly mobile across industries, while capital,

K, is not. The specification of capital immobility is described in detail in section 2.3. Aggregate factor supplies are determined by the solution to the household's utility maximization problem and prices (see section 3).

Goods: There is one intermediate good, X, which can be thought of as an intermediate energy input to production. There are two final goods, C and Y, which represent a clean and a polluting final good respectively. The clean good represents the majority of consumption and is produced using only capital and labor. The polluting good, Y, is produced using the intermediate good X as well as the two factors. Household demand for C and Y is derived from utility, while production is assumed to be competitive and exhibit constant returns to scale.

Emissions: Emissions are produced from the consumption of the intermediate good X and can be abated both through substitution away from X or through the use of an end-of-pipe treatment technology specified in section 2.2. In the policy scenarios, emissions will be controlled by government regulation.

Government: In the benchmark scenario, the government levies distortionary taxes on labor and capital and returns the revenue via a transfer to households. In policy scenarios, the government may also raise revenue through emissions taxes and fuel taxes. The policy scenarios also introduce two additional government outlays: to model a performance standard, we combine an emissions tax with a revenue neutral subsidy to the Y industry. The technology mandate involves an emissions tax with a revenue neutral subsidy to the X industry.

Finally, the government can compensate industry for losses incurred by the policy in two ways: First is a lump sum tax credit to the affected industries. Second is a cut in the marginal factor tax rate on capital for a specific industry. In all cases, the real value of the government transfer to the households is kept fixed by adjusting overall factor tax rates.

Household: The household supplies factors of production and consumes final goods in accordance with the utility function given below. The level of pollution is assumed to enter household utility in an additively separable way, and so does not affect household supply and demand decisions. The numerical model estimates the gross effects of various policies, so no pollution damage function is included in this document.

2 Production

The production functions for the three industries, production of pollution, and distribution of capital used in the numerical model are described in this section.

2.1 Production Functions

All production is constant returns to scale, and given by constant elasticity of substitution (CES) production functions. The production of the intermediate good, X, is given by:

$$X = \gamma_x \left[\alpha_{x_k} K_x^{\frac{\sigma_x - 1}{\sigma_x}} + \alpha_{x_l} L_x^{\frac{\sigma_x - 1}{\sigma_x}} \right]^{\frac{\sigma_x}{\sigma_x - 1}} \tag{1}$$

where the elasticity of substitution between the two factors is given by σ_x. The model calibrates the α_x's (share parameters) and γ_x (scale parameter) such that in the benchmark simulation the desired amount of each factor is used in producing the benchmark level of X.

Production of the final good C is similar, employing capital and labor in the CES production function:

$$C = \gamma_c \left[\alpha_{c_k} K_c^{\frac{\sigma_c - 1}{\sigma_c}} + \alpha_{c_l} L_c^{\frac{\sigma_c - 1}{\sigma_c}} \right]^{\frac{\sigma_c}{\sigma_c - 1}} \tag{2}$$

Again, the scale and share parameters are calibrated to benchmark levels of output and factor consumption in the industry. The elasticity of substitution among inputs, σ_c, is given in the model input as specified in the paper.

Production of the polluting good, Y, is somewhat more complicated as it involves the usual two factors as well as the intermediate good. Production is given by a nested CES function as follows:

$$Y = \gamma_y \left[\alpha_{y_v} v_y^{\frac{\sigma_y - 1}{\sigma_y}} + \alpha_{y_l} L_y^{\frac{\sigma_y - 1}{\sigma_y}} \right]^{\frac{\sigma_y}{\sigma_y - 1}} \tag{3}$$

where

$$v = \gamma_v \left[\alpha_{v_k} K_v^{\frac{\sigma_v - 1}{\sigma_v}} + \alpha_{v_x} X_v^{\frac{\sigma_v - 1}{\sigma_v}} \right]^{\frac{\sigma_v}{\sigma_v - 1}} \tag{4}$$

Note that together there are three inputs to the production of Y: K, L, and X. In the inner nest (given by v) the inputs K and X substitute for one another with elasticity σ_v. This combination is then used in the outer level, where v and L are substituted with elasticity σ_y. As above (although again more complicated in this case) the model is calibrated such that the benchmark levels of the three inputs combine to produce the benchmark level of Y. The elasticities are specified as input to the model.

2.2 Pollution Generation

Pollution in the model comes from the consumption of the intermediate good X and allows for the possibility of end-of-pipe treatment using a combination good, denoted G_a. Total pollution (emissions) are then given as:

$$E = \gamma_e \left[1 + \beta_e \left(\frac{G_a}{X} \right)^{\rho_e} \right]^{\frac{-1}{\rho_e}} \cdot X \tag{5}$$

The parameters β_e and ρ_e are given as model inputs and calibrated to reflect the desired ease of end-of-pipe treatment. G_a is again the amount of the composite good G used for end-of-pipe treatment. Finally, γ_e is a scale parameter calibrated to produce the desired emissions in the benchmark. Note that emissions are linear in the production of X but not in the use of end-of-pipe treatment.

2.3 Capital Transformation

In contrast to labor, capital is not permitted to flow freely among sectors. The aggregate capital supplied by the household is given by K^s (see section 3 for a discussion of factor supply) and will be divided among the three sectors according to the price of capital in each and the costs of moving capital among sectors. In order to allocate capital among sectors the agent is modeled as maximizing:

$$(1 - T + S_{kx}) R_x K_x + (1 - T + S_{ky}) R_y K_y + (1 - T) R_c K_c \tag{6}$$

subject to the constraint that

$$\gamma_k \left[\alpha_k K_x^{\frac{\sigma_k - 1}{\sigma_k}} + \beta_k K_y^{\frac{\sigma_k - 1}{\sigma_k}} + (1 - \alpha_k - \beta_k) K_c^{\frac{\sigma_k - 1}{\sigma_k}} \right]^{\frac{\sigma_k}{\sigma_k - 1}} \leq K^s \tag{7}$$

where K_x, K_y, and K_c are the quantities of capital supplied to each sector and K^s is the aggregate supply from the household. Capital rental prices, R, are set according to market demand and supply. The parameter σ_k governs the ease with which capital may be substituted among uses, while the remaining parameters are calibrated such that in the benchmark case the equality $K_x + K_y + K_c = K^s$ holds. When capital demands differ from the benchmark, some distortion will occur such that $K_x + K_y + K_c < K^s$.

The difference between total capital supply and the sum of individual sector supplies is accounted as a loss due to friction.

3 Household Behavior

The household maximizes the utility function given below, which yields a set of demand functions for the final goods and supply functions for the factors of production. These supply and demand functions are determined by the parameters of the utility function, prices, and income as described in the following.

3.1 Utility

Household utility is a function of capital and labor supply, consumption, and emissions. Again, emissions are assumed to enter separably and are not included here. Utility is of the nested CES form:

$$U = \left(\alpha_g G^{\frac{\sigma_u - 1}{\sigma_u}} + \alpha_z Z^{\frac{\sigma_u - 1}{\sigma_u}} \right)^{\frac{\sigma_u}{\sigma_u - 1}} \tag{8}$$

where G and Z are the CES functions:

$$G = \left(\alpha_{gy} Y_h^{\frac{\sigma_g - 1}{\sigma_g}} + \alpha_{gc} C_h^{\frac{\sigma_g - 1}{\sigma_g}} \right)^{\frac{\sigma_g}{\sigma_g - 1}} \tag{9}$$

$$Z = \left(\alpha_{zl} \ell_l^{\frac{\sigma_z - 1}{\sigma_z}} + \alpha_{zk} \ell_k^{\frac{\sigma_z - 1}{\sigma_z}} \right)^{\frac{\sigma_z}{\sigma_z - 1}} \tag{10}$$

The G (goods) nest is of the standard CES form where the elasticity of substitution between the clean good C and the polluting good Y is given by σ_u. The α's are calibrated as before to yield the desired benchmark proportions of C and Y in consumption. The Z (factors) nest is composed of two symmetric contributors to utility, ℓ_l and ℓ_k. The first of these is leisure, defined in the usual way such that $L^s = \bar{L} - \ell_l$ where L^s is the total labor supply and $\bar{L}$ is the household's total endowment of time. The second item, ℓ_k, is the leisure analog of capital and is defined by the relation: $K^s = \bar{K} - \ell_k$. where $\bar{K}$ can be thought of as a measure of potential capital. Finally, σ_z is a parameter input that determines the relative elasticity between the supply of labor and capital.

In the outer nest given in (8) the overall elasticity of substitution between factors and goods is set using the parameter σ_u. This elasticity, combined with the parameters used for $\bar{L}$ and $\bar{K}$, will determine the overall elasticities of labor and capital supply in the model. As usual, the two α parameters are calibrated so that the proportions of total income devoted to Z and G will match the benchmark inputs.

3.2 Household Budget Constraint

The utility function above is maximized subject to the following budget constraint, producing goods demand and factor supplies at a given set of prices. The household budget constraint can be written as:

$$P_c C_h + P_y Y_h \leq P_k^{index}(\bar{K} - \ell_k) + (1-T)(\bar{L} - \ell_l) + \Lambda + (1-T)\Pi \tag{11}$$

where:

P_k^{index}	A weighted average of capital prices less the capital lost to friction according to (7) above. Note that the capital prices are net of taxes T and include the subsidies S_{kx} and S_{ky} if present.
Λ	The government transfer to the household.
Π	The value of the lump sum tax credit given to the firm, we assume that the household owns the firms and so receives this as a lump sum transfer.

Prices P and the factor tax rate T are given as solution parameters in the model (see section 5.2). The components of the government transfers to the household are described in more detail in section 4 below.

4 Government Budget and Policies

The government in the simulation model has several sources of revenue and makes transfers to the households that are accounted as described in this section. The overall government budget (revenue and transfers) must balance for each scenario, but the government's choice of tax and permit instruments and the various components of the transfer depend on the policy being modeled.

The government levies a distortionary tax T on the two factors, labor and capital. In the benchmark, this tax rate is fixed and no other policy is undertaken. In this case, the size of transfers, $\Lambda_{benchmark}$, is determined simply by the benchmark factor taxes as:

$$\Lambda_{benchmark} = T(R_x K_x + R_c K_c + R_y K_y) + TL \tag{12}$$

where the K and L terms are benchmark factor demands and T is the benchmark distortionary tax rate.

The government budget is always balanced by adjusting the overall factor tax rate T such that the real value of the benchmark transfer is fixed using the ideal price index for goods given in (18). Under the various policy scenarios, the government has both additioanl sources of revenue and additional obligations. The emissions and fuel taxes raise revenue, while the compensation to firms (in either the lump sum or sector-specific tax cut forms) increase revenue demand.

As an example of the process described above, consider the government budget constraint under a performance standard with industry compensation via a lump sum tax credit. The government budget is, in equilibrium:

$$\Lambda + \Pi(1 - T) + SY = T(L + R_x K_x + R_c K_c + R_y K_y) + T_e E \tag{13}$$

where Λ is fixed according to benchmark transfers adjusted for prices, Π is defined as above such that its value offsets capital losses to X and Y in equilibrium, and T is set by the government such that the equality in (13) holds. Note that the price of emissions permits, T_e, and the offsetting subsidy S are also simultaneously determined such that the desired emissions goal is reached. See section 5.2 describing the solution mechanism for details.

5 Equilibrium

This section first provides an overview of the equilibrium conditions for the numerical model, and then provides a more detailed description of the markets and algorithm used.

5.1 Definition

Equilibrium is reached when prices are such that all output and factor markets clear and all budget constraints are satisfied. Since production in the economy is assumed to be competitive and constant returns to scale, the model assumes that production will meet demand at the cost-minimizing price of production (determined by factor prices as described below in 5.2). What remains, then, is that all of the factor markets clear. Since capital is not fully mobile, there will be one market for each type of capital, and a fourth market for labor such that:

$$K_x^d = K_x^s \tag{14a}$$

$$K_c^d = K_c^s \tag{14b}$$

$$K_y^d = K_y^s \tag{14c}$$

$$L_x^d + L_c^d + L_y^d = L^s \tag{14d}$$

The household problem is solved such that the household income and capital transformation budget constraints always hold. By Walras' law, then, if the budget constraint and three of the markets in (14) hold, the fourth market must also clear. In the algorithm subsection, then, note that we have omitted the market for labor. The model instead solves the three capital markets, and then checks the solution to verify that the labor market indeed clears. This also serves as a useful check for any accounting leakages in the model.

5.2 Algorithm

In addition to the markets defined above, of course, the model must also be solving for an emissions policy constraint and, in the case of the command-and-control policies, for the revenue neutral subsidy. This subsection describes the main solution algorithm employed by the model including, as an example, the complete list of constraints for the performance standard with lump sum compensation.

Solving for equilibrium is an iterative process, with each iteration started from the set of prices and values in the table below. Note that a different set of parameters is involved for alternate policies.

Parameter	Description
R_x	Price of capital for good X
R_c	Price of capital for good C
R_y	Price of capital for good Y
T	Overall factor taxes, for government budget balance
T_e	Emissions tax, to match desired reduction in emissions.
S	Subsidy to the output of Y, solved to model a performance standard by making the policy revenue neutral.
Π	Size of lump sum tax credit returned to firms, for profit neutrality in the X and Y industries.

The first three prices correspond to the three capital markets in (14), and, together with labor, will drive goods prices and demands. The next item, T, is used to balance the government budget. Recall from (13) that the government must set T to balance the budget. The fifth item, T_e, is a guess for the level of the emissions tax to achieve a target level of emissions reductions. S is the revenue neutral subsidy required for modeling the performance standard (as discussed in the main text). Finally, the last item is the size of the lump sum tax credit given back to the X and Y industries to compensate for losses from the emissions policy.

Using the above list of prices as a starting point, the model determines all other prices and demands in the system–an outline of this process follows: The price of goods X and C follows from the solution to the cost minimization analog to the production functions (1) and (2). Note that the price of labor is normalized to 1. Solving for the price of Y is considerably more difficult since it will include the price of emissions permits and involves a choice of end-of-pipe treatment. Given prices, the production function (3), and the emissions function (5), it turns out that there is no closed form solution to the cost minimization problem for Y. This is because the price of Y depends on the price of end-of-pipe treatment, which depends simultaneously on the price of Y (recall that end-of-pipe treatment uses the composite good G composed of C and Y). To solve this, a simple search algorithm is used where a price for Y is guessed, and then updated until convergence based on the end-of-pipe treatment chosen. The solution will satisfy:

$$P_y = R_y \frac{K_y}{Y} + \frac{L_y}{Y} + P_x \frac{X_y}{Y} + T_e \frac{E(G_a)}{Y} + P_G \frac{G_a}{Y} \tag{15}$$

where P_G is a function of P_c and P_y, and K, L, X, and G_a are chosen to minimized P_y.

Once goods prices have been determined, household demands follow easily from the first order conditions maximizing utility (8) subject to the household budget, (11). Similarly, the aggregate supply of labor and capital is given from the utility maximization problem. Recall though, that aggregate capital supply must still be broken down into supply to each sector using (6) and (7).

Having determined factor supplies and goods demand, the remaining computations are all fairly straightforward. Government income is accounted using factor taxes given by T, and the needed tax credit is computed using changes in the price of sector-specific capital. Finally, the total emissions demanded can be determined directly from equation (5).

After computing all of these values, the algorithm must iteratively update the set of prices in the table above until equilibrium is reached. It does this using a derivative search based on Newton's method, solving the following system of equations. The table below corresponds to the one above, with the parameter list in the left column matching:

Parameter	**Equilibrium Condition**
R_x	$K_x^d - K_x^s = 0$
R_c	$K_c^d - K_c^s = 0$
R_y	$K_y^d - K_y^s = 0$
T	$gov\_income - gov\_expenditure = 0$
T_e	$E - E_{goal} = 0$
S	$SY - T_e E = 0$
Π	$\underbrace{(1-T)(R_y K_y + \Pi)}_{policy} - \underbrace{(1-T)R_y K_y}_{benchmark} = 0$

The first three conditions, factor markets, are as in (14). Government income and expenditure correspond to the right and left sides of (13) respectively. E is determined from the emissions function, while E_{goal} is set exogenously as the policy emissions goal. The revenue neutral subsidy S produces a policy equivalent to a performance standard. Finally, the term labeled *policy* in the last condition refers to the value of capital in the policy case plus the value of compensation, captured as before in the variable Π. The term labeled *benchmark* refers to the value of capital in the benchmark (without policy) case. Note that the values of T and R_y are as in the policy and benchmark solutions, respectively. For simplicity the price deflator has been omitted, but note that all prices used for the capital compensation adjustment are kept in real terms.

6 Welfare Analysis

The equivalent variation (EV) is calculated for each policy scenario as a measure of gross welfare change (not including environmental benefits). This is defined as the income change in the benchmark case that would create the same utility change as the policy. The numerical computation for this is relatively straightforward; the price of utility in the benchmark (in terms of total income) is computed using the ideal price index given by:

$$P_u = (\alpha_z^{\sigma_u} P_z^{1-\sigma_u} + \alpha_g^{\sigma_u} P_g^{1-\sigma_u})^{\frac{1}{1-\sigma_u}} \tag{16}$$

where the nested price-indices for factors, P_z, and goods, P_g, are given by:

$$P_z = \left(\alpha_{zk}^{\sigma_z} \left(P_k^{index}\right)^{1-\sigma_z} + \alpha_{zl}^{\sigma_z} (1-T)^{1-\sigma_z}\right)^{\frac{1}{1-\sigma_z}} \tag{17}$$

$$P_g = \left(\alpha_{gc}^{\sigma_g} P_c^{1-\sigma_g} + \alpha_{gy}^{\sigma_g} P_y^{1-\sigma_g}\right)^{\frac{1}{1-\sigma_g}} \tag{18}$$

The equivalent variation is then given simply as:

$$EV = P_u^{benchmark} \cdot U^{policy} - TI^{benchmark} \tag{19}$$

where the first term determines how much it would cost to achieve the policy level of utility in the benchmark, and the second term is just total income in the benchmark. The difference is the equivalent variation as defined above.

7 Summary of Computed Equations

This section contains the first-order conditions used to derive supply and demand in the equilibrium process above. The subsections roughly correspond to the order in which the model solves the various markets.

7.1 Production

Producer demands for inputs and output prices for the final good C and the intermediate good X are very similar; the problem for good C is shown. Producers are assumed to solve their cost minimization problem, which can be expressed in per unit terms as follows (recall that production is constant returns to scale):

$$\min_{k_c, l_c} \; R_c k_c + P_l l_c \tag{20}$$

$$s.t. \quad \gamma_c \left[\alpha_{c_k} k_c^{\frac{\sigma_c - 1}{\sigma_c}} + \alpha_{c_l} l_c^{\frac{\sigma_c - 1}{\sigma_c}} \right]^{\frac{\sigma_c}{\sigma_c - 1}} \geq 1 \tag{21}$$

$$k_c \geq 0 \tag{22}$$

$$l_c \geq 0 \tag{23}$$

where k_c and l_c are defined as demands of capital and labor per unit C.

Taking first order conditions and combining (assuming an interior solution) gives the CES factor demand functions:

$$k_c = \frac{1}{\gamma_c} \left[\alpha_{c_k} + \alpha_{c_l} \left(\frac{\alpha_{c_k}}{\alpha_{c_l}} \frac{P_l}{R_c} \right)^{\sigma_c - 1} \right]^{\frac{\sigma_c}{\sigma_c - 1}} \tag{24}$$

$$l_c = \frac{1}{\gamma_c} \left[\alpha_{c_k} \left(\frac{\alpha_{c_k}}{\alpha_{c_l}} \frac{P_l}{R_c} \right)^{\sigma_c - 1} + \alpha_{c_l} \right]^{\frac{\sigma_c}{\sigma_c - 1}} \tag{25}$$

Since production is competitive and constant returns to scale, the price of good C is simply:

$$P_c = R_c k_c + P_l l_c$$

The problem for good X is analogous.

Solving the producer problem for the polluting good Y, however, is considerably more problematic. We divide the problem into two, solving first for the cost minimizing input mix to v, the inner CES nest, and then more simply for the combination of v and L to make the final good Y. (see equation (3)) The per-unit cost minimization problem for the inner nest, v, is given as:

$$\min_{k_v,x_v,g_a} \quad R_y k_v + P_x x_v + P_g g_a + T_e e_v \tag{26}$$

$$s.t.\ \gamma_v \left[\alpha_{v_k}^{\frac{\sigma_v - 1}{\sigma_v}} k_v + \alpha_{v_x}^{\frac{\sigma_v - 1}{\sigma_v}} x_v \right]^{\frac{\sigma_v}{\sigma_v - 1}} \geq 1 \tag{27}$$

$$e_v = \alpha_e \left[1 + \beta_e \left(\frac{g_a}{x_v} \right)^{\rho_e} \right]^{\frac{-1}{\rho_e}} x_v \tag{28}$$

$$k_v \geq 0 \tag{29}$$

$$x_v \geq 0 \tag{30}$$

$$g_a \geq 0 \tag{31}$$

where lowercase letters are again per unit of production: k_v, x_v, g_a, and e_v are capital, X, G_a, and E per unit of V. The first order condition on x_v can be rearranged to give the following shadow price for x:

$$\hat{P}_x = \overbrace{P_x}^{1} + \left\{ \overbrace{T_e \alpha_e \left[1 + \beta_e \left(\frac{g_a}{x_v v} \right)^{\rho_e} \right]^{\frac{-1}{\rho_e}}}^{2} + \overbrace{T_e \alpha_e \beta_e \left(\frac{g_a}{x_v v} \right)^{\rho_e} \left[1 + \beta_e \left(\frac{g_a}{x_v v} \right)^{\rho_e} \right]^{\frac{-(1+\rho_e)}{\rho_e}}}^{3} \right\} \tag{32}$$

The terms of (32) can be interpreted as the marginal cost due to

1. purchasing x
2. increasing emissions due to increasing x (holding $\frac{g_a}{x}$ constant)
3. increasing emissions due to decreasing the ratio $\frac{g_a}{x}$

The factor demands for x_v and k_v are then as usual, except that the shadow price of X is used:

$$k_v = \frac{1}{\gamma_v} \left[\alpha_{v_k} + \alpha_{v_x} \left(\frac{\alpha_{v_k}}{\alpha_{v_x}} \frac{\hat{P}_x}{P_{k_v}} \right)^{\sigma_v - 1} \right]^{\frac{\sigma_v}{\sigma_v - 1}} \tag{33}$$

$$x_v = \frac{1}{\gamma_v} \left[\alpha_{v_k} \left(\frac{\alpha_{v_k}}{\alpha_{v_x}} \frac{\hat{P}_x}{P_{k_v}} \right)^{\sigma_v - 1} + \alpha_{v_x} \right]^{\frac{\sigma_v}{\sigma_v - 1}} \tag{34}$$

As mentioned, there is no closed form solution for the amount of end-of-pipe treatment chosen in the production of v. The first order condition on g_a from equation (26) can be reduced only to:

$$\frac{g_a}{x_v} \equiv \left(\frac{P_g}{T_e \alpha_e \beta_e}\right)^{\frac{1}{\rho_e - 1}} \left[1 + \beta_e \left(\frac{g_a}{x_v}\right)^{\rho_e}\right]^{\frac{1+\rho_e}{\rho_e(\rho_e - 1)}} \tag{35}$$

The model iterates on this equation, solving for the ratio $\frac{g_a}{x_v}$. Once k_v, x_v, and g_a are determined, emissions per unit v, and hence the total shadow price of using v in Y, can be found.

With the price of v and K_y in hand, solving for the inputs to the outer nest of the Y production function is straightforward and analagous to the standard CES functions above. Notice, however, that the solution to (26) depends on knowing the price of G_a, which depends in turn on the price of Y. Therefore, an iterative process is again used where a "guess" for the price of Y (and therefore G_a) is made in order to solve (26). This then feeds into the problem below (given in (36)), producing an updated guess for the price of Y. This is iterated until convergence is achieved.

Given the shadow price of v above, the cost minimization problem for Y can be written:

$$\min_{v_y, l_y} \quad P_v v_y + P_l l_y \tag{36}$$

$$s.t. \quad \lambda_y \left\{1 - \gamma_y \left[\alpha_{y_v} v_y^{\frac{\sigma_y - 1}{\sigma_y}} + \alpha_{y_l} l_y^{\frac{\sigma_y - 1}{\sigma_y}}\right]^{\frac{\sigma_y}{\sigma_y - 1}}\right\} \leq 1 \tag{37}$$

where lower case again indicates unit demands, and an interior solution is assumed. Per-unit factor demands for labor and the sub-good v are given as usual by:

$$v_y = \frac{1}{\gamma_y}\left[\alpha_{y_v} + \alpha_{y_l}\left(\frac{\alpha_{y_v}}{\alpha_{y_l}} \frac{P_l}{P_v}\right)^{\sigma_y - 1}\right]^{\frac{\sigma_y}{\sigma_y - 1}} \tag{38}$$

$$l_y = \frac{1}{\gamma_y}\left[\alpha_{y_v}\left(\frac{\alpha_{y_v}}{\alpha_{y_l}} \frac{P_l}{P_v}\right)^{\sigma_y - 1} + \alpha_{y_l}\right]^{\frac{\sigma_y}{\sigma_y - 1}} \tag{39}$$

7.2 Capital transformation

Agents determine the percentage of capital to allocate to the sectors based on the rental prices and the mobility of capital:

$$k_y^s = \frac{\overline{k}}{\gamma_k \left[\alpha_k \left(\frac{(1-T+S_{kx})R_x}{(1-T+S_{ky})R_y}\frac{\beta_k}{\alpha_k}\right)^{1-\sigma_k} + \beta_k + (1-\alpha_k-\beta_k)\left(\frac{(1-T)R_c}{(1-T+S_{ky})R_y}\frac{\beta_k}{1-\alpha_k-\beta_k}\right)^{1-\sigma_k}\right]^{\frac{\sigma_k}{\sigma_k-1}}} \tag{40}$$

$$k_x^s = \left(\frac{(1-T+S_{kx})R_x}{(1-T+S_{ky})R_y}\frac{\beta_k}{\alpha_k}\right)^{-\sigma_k} k_y^s \tag{41}$$

$$k_c^s = \left(\frac{(1-T)R_c}{(1-T+S_{ky})R_y}\frac{\beta_k}{1-\alpha_k-\beta_k}\right)^{-\sigma_k} k_y^s \tag{42}$$

7.3 Household Problem

The functions for utility —(8),(9),(10)— along with income, determine the supply of K and L and the demand for C and Y.

To calculate income, some preliminary calculations of prices are necessary

$$P_k^{index} = (1-T)k_c^s R_c + (1-T+S_{ky})k_y^s R_y + (1-T+S_{kx})k_x^s R_x \tag{43}$$

$$P_z = (\alpha_{z_k}^{\sigma_z} {P_k^{index}}^{1-\sigma_z} + \alpha_{z_l}^{\sigma_z}(1-T)^{1-\sigma_z})^{\frac{1}{1-\sigma_z}} \tag{44}$$

$$P_g = (\alpha_{g_c}^{\sigma_g} P_c^{1-\sigma_g} + \alpha_{g_y}^{\sigma_g} P_y^{1-\sigma_g})^{\frac{1}{1-\sigma_g}} \tag{45}$$

$$P_u = (\alpha_z^{\sigma_u} P_z^{1-\sigma_u} + \alpha_g^{\sigma_u} P_g^{1-\sigma_u})^{\frac{1}{1-\sigma_u}} \tag{46}$$

Household income is made up of transfers, net labor income, and net capital income. Where applicable, income from the tax credit for industry compensation is added.

$$I_0 = \bar{K} P_k^{index} + \bar{L} P_l (1-T) + P_g \Lambda + \Pi(1-T) \tag{47}$$

where

- $\bar{K}$ is the amount of potential capital available
- $\bar{L}$ is the amount of potential labor available
- Λ are observed/benchmark transfers

Income can be decomposed into "spending" on Z (i.e. the value of potential labor and capital consumed by the household rather than supplied to the market) and into spending on G (goods) From the utility function, the "demand" for (i.e. spending on) Z can be calculated as below and the spending on G as the residual:

$$I_z = \frac{I_0(\alpha_z P_g)^{\sigma_u} P_z}{(\alpha_z P_g)^{\sigma_u} P_z + (\alpha_g P_z)^{\sigma_u} P_g} \tag{48}$$

$$I_g = I_0 - Z \tag{49}$$

Supply of capital & labor Similarly, spending on Z can be decomposed into spending on leisure and spending on unused capital. The residual between spending on a resource and the total amount available is supplied to the market.

$$\ell_l = \frac{I_z(\alpha_{z_l} P_k^{index})^{\sigma_z}(1-T)}{(\alpha_{z_l} P_k^{index})^{\sigma_z}(1-T) + (\alpha_{z_k}(1-T))^{\sigma_z} P_k^{index}} \tag{50}$$

$$L^s = \bar{L} - \frac{\ell_l}{1-\tau_l} \tag{51}$$

$$\ell_k = I_z - \ell_l \tag{52}$$

$$K^s = \bar{K} - \frac{\ell_k}{P_k^{index}} \tag{53}$$

K^s can be decomposed into sector-specific capital supply:

$$K_c^s = k_c^s K^s \tag{54}$$

$$K_y^s = k_y^s K^s \tag{55}$$

$$K_x^s = k_x^s K^s \tag{56}$$

Consumer Demand for C and Y Consumer spending on G can be decomposed into spending on C and Y

$$I_c = \frac{I_g(\alpha_{g_c} P_y)^{\sigma_g} P_c}{(\alpha_{g_c} P_y)^{\sigma_g} P_c + (\alpha_{g_y} P_c)^{\sigma_g} P_y} \tag{57}$$

$$C_h^d = \frac{I_c}{P_c} \tag{58}$$

$$Y_h^d = \frac{I_g - I_c}{P_y} \tag{59}$$

Total demand for C and Y The producer demand for Y will be the portion of G_a that comes from Y. This portion will be calculated based on the percentage of Y in the consumer portion of G: $\frac{Y_h}{G_h} = \frac{Y_h}{C_h+Y_h}$. G_a is calculated based on g_a, the per-unit-of-V use of abatement, multiplied by V, which in turn is calculated as $v_y^d(Y_h^d + Y_a^d)$:

$$G_a = \frac{g_a v_y Y_h^d}{1 - \frac{g_a v_y Y_h^d}{C_h^d + Y_h^d}} \tag{60}$$

$$Y_a^d = G_a \frac{Y_h}{C_h^d + Y_h^d} \tag{61}$$

$$C_a^d = G_a - Y_a^d \tag{62}$$

$$C^d = C_h^d + C_a^d \tag{63}$$

$$Y^d = Y_h^d + Y_a^d \tag{64}$$

7.4 Excess Demand

We are now able to calculate the last variables needed to determine the excess demands in subsection 5.2.

7.4.1 Demand for K

$$K_c^d = C^d k_c^d \tag{65}$$

$$K_x^d = X^d k_x^d \tag{66}$$

$$K_y^d = V^d k_y^d \tag{67}$$

7.4.2 Government income & expenditure

Government income is calculated as the sum of emission tax revenue, labor tax, and capital tax, adjusted for any lump sum tax credit to firms.

$$\begin{aligned} gov\_income = {} & T_e E + L^s T + T K_c^s R_c + (T - S_{ky}) K_y^s R_y + (T - S_{kx}) K_x^s R_x \\ & - (1 - T)\Pi \end{aligned} \tag{68}$$

$$gov\_expend = P_g \Lambda \tag{69}$$

7.4.3 Emissions

$$E = V e^v \tag{70}$$

7.4.4 Industry Profit including Compensation

$$\pi_c = (1 - T)R_c K_c \tag{71}$$

$$\pi_x = (1 - T + S_{kx})R_x K_x + (1 - T)\Pi_x \tag{72}$$

$$\pi_y = (1 - T + S_{ky})R_y K_y + (1 - T)\Pi_y \tag{73}$$