The Substantial Bias from Ignoring General Equilibrium Effects in Estimating Excess Burden, and a Practical Solution

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ABSTRACT

This paper shows that under typical conditions the simple “excess-burden triangle” formula substantially underestimates the excess burden of commodity taxes. This formula performs poorly because it ignores general equilibrium interactions – most importantly, interactions between the market for the taxed commodity and the labor market. Using analytically tractable and numerically solved general equilibrium models, we show that the simple formula tends to significantly understate excess burden, in some cases by a factor of ten or more. We then derive an implementable alternative to the simple formula. This alternative formula captures interactions that are left out of the simple one, and as a result it is both unbiased and usually more accurate.

Many prior theoretical studies have shown that general equilibrium interactions affect excess burden, but earlier work has not appreciated the bias associated with ignoring these interactions or recognized the quantitative importance of this bias. There are two main sources of our findings. The first is the recognition that the interaction between a new commodity tax and existing labor taxes will tend to be far more important than interactions with other taxes. Second, we focus on the case where the taxed commodity is average in terms of its substitutability with leisure. This greatly simplifies the analysis and allows us to derive an implementable alternative to the simple excess burden formula. One implication of this research is that government programs financed by taxes on particular intermediate or consumer goods must meet a higher benefit hurdle than is often assumed.

I. Introduction

By driving a wedge between marginal benefits and marginal costs, taxes tend to generate distortionary costs or excess burden. The assessment of this excess burden is a principal activity of economists interested in improving tax policies and other government interventions that affect prices or costs.

Nearly forty years ago, Harberger (1964) developed a comprehensive measure of excess burden, which can be written as[1]:

$$EB = -\frac{1}{2} t_k^2 \frac{dX_k}{dt_k} - \sum_{i \neq k} t_i t_k \frac{dX_i}{dt_k}$$

where EB is the excess burden from the imposition of a tax on good k, X_i is the quantity of commodity i, and t_i is the tax on commodity i. Under the assumptions underlying the formula, the tax rate represents marginal distortionary cost – the discrepancy between marginal social value and marginal social cost. Hence, the excess burden of a new tax can be represented as the sum, over all markets, of the product of the prior tax rate in each market and the change in quantity in that market caused by the new tax.

In practice, however, economists rarely use this general formula, in large part because it is difficult – perhaps impossible – to obtain all of the derivatives dX_i/dt_k that would be needed to apply it. Practitioners typically employ the simpler "Harberger triangle" or "excess burden triangle" formula[2]:

$$EB = -\frac{1}{2} t_k^2 \frac{dX_k}{dt_k}$$

This formula ignores all but the first term of the more comprehensive formula. It thus considers only the distortionary impact in the market in which the new tax is imposed.

This paper makes two main contributions related to the approximation of excess burden. First, it shows that, under "typical" conditions defined below, the simpler formula substantially underestimates the excess burden of commodity taxes. While economists have long understood that general equilibrium interactions affect excess burden,[3] they have tended to think that such interactions have little quantitative

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[1] This formula comes from expression (3) in Harberger (1964), with the simplifying assumption that only one tax rate is changed. The general formula for the case of simultaneous changes to several different tax rates is referred to as the "Harberger double-sum," because the excess burden of each tax change is calculated as a sum across all markets, and this is then summed for all of the tax changes.

[2] Recent papers that ignore distortions in other markets in calculating the excess burden of a commodity tax include Fershtman *et al* (1999), Farrell and Walker (1999), Hausman (1997 and 1999), Konig and Ridder (1997), and Poterba (1992). In principle, an alternative to employing the general formula is to build a computable general equilibrium model and ascertain excess burden through simulations. But in many circumstances researchers do not have the time or the resources to construct such a model for the problem immediately at hand.

[3] In addition to Harberger's work, see for example Ramsey (1927), Hotelling (1938), Hicks (1946), Meade (1955), Viner (1950), Corlett and Hague (1953), and Lipsey and Lancaster (1956-57). Several well-known optimal tax

significance. Prior work does not reveal the systematic bias of the simpler formula or establish the large magnitude of that bias.[4] Using analytically tractable and numerically solved general equilibrium models, we show that the simple formula is biased in the sense that, for an average commodity (in terms of its substitutability with leisure), it underestimates excess burden. The error is substantial; in plausible cases the simpler formula underestimates excess burden by a factor of ten or more. In such cases, the new tax creates far more excess burden through general equilibrium interactions than it does through the additional distortion in the market for the taxed good itself.

The second main contribution is to offer an implementable formula that is both unbiased and more accurate. Our formula incorporates general equilibrium interactions that are left out of the simplest excess burden formula, while avoiding many of the information problems posed by the more complicated Harberger formula.

These two contributions stem from two sources. The first is the recognition (derived below) that the interaction between a new commodity tax and the existing labor tax tends to be far more important than interactions with other taxes. It is well-known that broad-based commodity taxes implicitly tax factors of production such as labor. A narrow commodity tax has a similar effect, discouraging labor supply, in addition to its effect in the market for the taxed good itself. Given the large magnitude of pre-existing labor market distortions, even a small additional distortion in the labor market generates a substantial amount of excess burden. Because it ignores labor-market impacts, the simplest formula's assessment of excess burden can be substantially off the mark. Our alternative formula, in contrast, accounts for the labor market impact, and thus provides a close approximation to excess burden even though it ignores effects in other markets.

The other main source of our results is our use of a crucial assumption which, to our knowledge, has not been employed in previous analyses of excess burden: that the taxed commodity is average in terms of its substitutability with leisure. Cross-price elasticities for particular goods are difficult to estimate, but the cross-price elasticity with the average good can be expressed in terms of the consumption share for the taxed good and labor supply elasticities, which are much easier to estimate. Thus, this assumption enables us to provide an implementable formula for excess burden that incorporates labor market effects.

results depend on general-equilibrium interactions. For example, Diamond and Mirrlees's (1971) conclusion that intermediate goods should not be taxed implicitly invokes such interactions: if it were not for general-equilibrium interactions, the optimum would include at least a small tax on every good.

[4] One example of economists' thinking on this issue is the recent interpretive article by Hines (1999). That article offers many illuminating insights about the Harberger triangle formula, and makes clear that the simple formula does not provide a comprehensive measure of excess burden. However, the article does not recognize any general

To the extent that these assumptions hold, our new formula will very closely approximate the true excess burden. If, however, the taxed good is a much stronger or weaker than average leisure substitute, or if tax interactions outside the labor market are important (as would be the case for a good that is a strong complement or substitute for another heavily taxed good), then this will reduce the accuracy of our formula. The simple excess-burden triangle formula, in contrast, usually provides a very poor approximation of excess burden of commodity taxes. It will be accurate only if the labor market distortion is insignificant (if the labor tax rate or the compensated labor supply elasticity is nearly zero) or if the taxed good is a much weaker substitute for leisure than is the average good. In the latter case, the excess-burden triangle formula may be more accurate than our new formula, whereas in the former case, the difference between the two formulas becomes insignificant.

The rest of the paper is organized as follows. The next section develops the basic analytical general equilibrium model and applies that model to arrive at an alternative excess-burden formula.[5] Section III employs numerical simulations to examine the accuracy of the simplest excess-burden formula and our alternative formulas. The final section offers conclusions.

II. The Model

This section develops a simple general equilibrium model to derive formulas for the excess burden of taxes on consumer goods and intermediate goods.[6] It begins by describing the model's assumptions, and deriving an expression for excess burden that is equivalent to the extended formula from Harberger (1964). We then apply a number of simplifying assumptions–that tax interactions outside the labor market are unimportant, that the taxed good is an average substitute for leisure, and that there are no income effects–to derive a much simpler excess burden formula and provide intuition for why tax interactions are important. We then reintroduce income effects to yield an excess-burden formula that is more accurate and yet still easy to implement. Finally, we derive a similar formula for a tax on an intermediate input, rather than a consumer good.

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bias or significant inaccuracy in the use of that formula, giving the impression that the simplest formula generally yields a reasonable estimate of excess burden. This view is typical among economists.

[5] In 1995, revenues from intermediate input and consumer good taxes (excluding general sales taxes) accounted for about six percent of total tax receipts to Federal, State, and local governments in the U.S. (Department of Commerce, *Survey of Current Business*, January/February 1996). Such taxes include excises on alcohol, tobacco, motor vehicles, gasoline, air transportation services, public utilities, communications services, and financial services. These taxes are relatively more important at the State and local levels than at the Federal level.

[6] The model and method of analysis here are similar in some respects to those used in previous studies of second-best issues in environmental regulation (Parry, Williams, and Goulder, 1999) and in international trade (Williams, 1999). This model is distinct, however, in that it (1) disregards environmental externalities and international trade, (2) allows for pre-existing commodity taxes in addition to the pre-existing labor tax, and (3) considers (in sections III and IV) a more general production function.

A. Assumptions

A representative agent model is assumed, where households divide their time endowment (T) between leisure (l) and labor (L), which is the only primary factor of production. Households use their income to purchase each of M consumer goods ($C_1, \ldots, C_M$) in order to maximize the utility function

(1) $$U(l, C_1, \ldots, C_M)$$

which is continuous and quasi-concave.

Each consumer good is produced using labor and N intermediate goods ($I_1, \ldots, I_N$), following

(2) $$C_i = F_i(L_{Ci}, I_{i1}, \ldots, I_{iN})$$

where L_{Ci} is the amount of labor and I_{ij} is the amount of intermediate good j used in production of consumer good i. All production functions are assumed to exhibit constant returns to scale. Intermediate goods are produced exclusively from labor, with units normalized such that one unit of labor input can produce one unit of any intermediate good.[7] Thus, production of intermediate goods follows

(3) $$I_j = L_{Ij}$$

where L_{Ij} is the amount of labor used in production of intermediate good j. The amount of each intermediate good used in production of consumer goods must match the amount produced.

(4) $$I_j = \sum_{i=1}^{M} I_{ij}$$

The household's time constraint is given by

(5) $$T = L + l = \sum_{i=1}^{M} L_{Ci} + \sum_{j=1}^{N} L_{Ij} + l$$

The government provides a lump-sum transfer payment G to the household, financed by taxes on labor (t_L), consumer goods (t_{Ci}), and intermediate goods (t_{Ij}).[8] After normalizing the pre-tax wage to unity, we can write the government budget constraint as

(6) $$G = t_L L + \sum_{i=1}^{M} t_{Ci} C_i + \sum_{j=1}^{N} t_{Ij} I_j$$

and the household budget constraint as

(7) $$(1 - t_L)L + G = \sum_{i=1}^{M} p_{Ci} C_i$$

where p_{Ci} is the price of consumer good i. The first-order conditions for firm profit-maximization give

[7] In Section III, we extend this model to allow for the use of intermediate goods in the production of intermediate goods. This complicates the analysis somewhat, but the results remain unchanged for a tax on a typical good.

[8] As is well known, one of these $N+M+1$ taxes is redundant; the government can produce identical outcomes with only $N+M$ taxes. The intuition behind this paper's results is clearer, though, if we maintain this redundancy.

(8) $$p_{Ci} = t_{Ci} + \frac{1}{\partial F_i / \partial L_{Ci}} = t_{Ci} + \frac{p_{ij}}{\partial F_i / \partial I_{ij}}$$

where p_{Ij} is the price of intermediate good j, given by

(9) $$p_{Ij} = t_{Ij} + 1$$

Households maximize utility (1) subject to their time constraint (5) and budget constraint (7), taking government transfers, prices, and tax rates as given. This yields the first order conditions:

(10) $$U_{Ci} = p_{Ci}\lambda \; ; \; U_l = (1 - t_L)\lambda$$

where λ represents the marginal utility of income. These first-order conditions, together with the other equations given thus far, implicitly define the demand functions[9]:

(11) $$C_i(t_{C1} \ldots t_{CM}, t_{I1} \ldots t_{IN}, t_L, Y_G); \;\; I_j(t_{C1} \ldots t_{CM}, t_{I1} \ldots t_{IN}, t_L, Y_G); \;\; l(t_{C1} \ldots t_{CM}, t_{I1} \ldots t_{IN}, t_L, Y_G)$$

where Y_G is lump-sum income from the government. In this model the government's only spending is in the form of this lump-sum transfer; there is no spending on public goods. Hence Y_G is equal to G.

B. Effects of a Tax on a Consumer Good

Here we examine the effect of a tax on a single consumer good k. Totally differentiating utility with respect to t_{Ck}, substituting in the consumer first-order conditions, and dividing through by λ yield

(12) $$\frac{1}{\lambda}\frac{dU}{dt_{Ck}} = \sum_{i=1}^{M} p_{Ci}\frac{dC_i}{dt_{Ck}} + (1 - t_L)\frac{dl}{dt_{Ck}}$$

Totally differentiating the production equation (2) for each consumer good with respect to t_{Ck} and substituting in the equations for consumer good prices (8) and intermediate goods prices (9) give

(13) $$(p_{Ci} - t_{Ci})\frac{dC_i}{dt_{Ck}} = \frac{dL_{Ci}}{dt_{Ck}} + \sum_{j=1}^{N} p_{Ij}\frac{dI_{ij}}{dt_{Ck}}$$

Totally differentiating the household time constraint with respect to t_{Ck} gives

(14) $$0 = \frac{dl}{dt_{Ck}} + \sum_{i=1}^{M}\frac{dL_{Ci}}{dt_{Ck}} + \sum_{j=1}^{N}\frac{dL_{Ij}}{dt_{Ck}}$$

Subtracting (14) from (12), substituting in (13) and canceling terms yield

(15) $$\frac{1}{\lambda}\frac{dU}{dt_{Ck}} = \underbrace{t_{Ck}\frac{dC_k}{dt_{Ck}}}_{dW^P} + \underbrace{\sum_{i \neq k} t_{Ci}\frac{dC_i}{dt_{Ck}} + \sum_{j=1}^{N} t_{Ij}\frac{dI_j}{dt_{Ck}} - t_L\frac{dl}{dt_{Ck}}}_{dW^I}$$

This expression is very similar to the general formula for excess burden obtained by Harberger (1964b) and mentioned in the introduction, differing only in the way it divides the economy's markets into the

[9] The form of these demand functions implies that a partial derivative of the demand function will be an uncompensated demand function. A total derivative, in contrast, will incorporate both a tax rate change and a change in the lump-sum government transfer.

categories of labor, intermediate-input, and consumer-good markets. The first term is the *primary welfare effect* (dW^P): the impact of the tax t_{Ck} on the market for good k. This is the effect captured by the usual formula. It equals the tax rate times the change in consumption of good k. The remaining terms are the *tax-interaction effect* (dW^I): the impact of t_{Ck} on other tax-distorted markets.[10] The three terms within dW^I are analogous to the primary welfare effect, except that they pertain to other consumer goods, to intermediate goods, and to labor, respectively.

It is difficult to obtain all of the partial derivatives that enter this expression. However, by transforming this formula, we can get a sense as to which elements are likely to be most important. Equation (15) can be expressed in terms of elasticities as

$$(16)\qquad \frac{1}{l}\frac{dU}{dt_{Ck}} = \underbrace{\frac{t_{Ck}C_k}{p_{Ck}}e_{Ck}}_{dW^P} + \underbrace{\sum_{i\neq k}\frac{t_{Ci}C_i}{p_{Ck}}e_{CiCk} + \sum_{j=1}^{N}\frac{t_{Ij}I_j}{p_{Ck}}e_{IjCk} - \frac{t_L L}{p_{Ck}}e_{LCk}}_{dW^I}$$

where e_{Ck} is the own-price elasticity of demand for consumer good k, e_{CiCk} is the cross-price elasticity for consumer good i, e_{IjCk} is the cross-price elasticity for intermediate good j, and e_{LCk} is the cross-price elasticity of labor supply, all with respect to the price of consumer good k. When the tax rates are sufficiently low, these elasticities approximate the compensated elasticities, because the increased transfer approximately offsets the income effect of the increased tax rate. Thus, for now, we will follow prior work and treat them as being compensated; this assumption will be relaxed later in the paper.

The above expression suggests which terms in the tax-interaction effect are likely to be the most important. Each term in the tax-interaction effect is a cross-price elasticity term times the revenue from the tax in a particular market, divided by the price of good k. Thus, if the cross-price elasticities are all of roughly equal magnitude, the importance of the interaction term for a given market will be roughly proportional to the tax revenue raised in that market. In most industrialized economies, there are relatively few significant commodity taxes, except for broad consumption taxes (such as state sales taxes in the U.S., or value-added taxes in other nations), whose impact can be captured in this model simply by renormalizing the labor tax.[11] Thus, the terms for interactions with other commodity taxes will typically be unimportant relative to the other terms in equation (16).[12] From now on, we will omit these terms.

[10] These effects have been defined and analyzed in prior literature examining the impacts, in a second-best setting, of environmental taxes (Parry (1995), Goulder *et al*. (1997), and Parry *et al*. (1999)) and of barriers to international trade (Williams (1999)). These earlier studies incorporated environmental impacts and terms-of-trade effects (respectively) in the primary welfare effect. No such impacts occur in the model presented here.

[11] As is well known, in a static setting a labor tax is equivalent to a uniform set of consumer good taxes.

[12] This might suggest that tax interactions will be less important in calculating the excess burden of a factor tax than they are in calculating the excess burden of commodity taxes. However, tax interactions may also be important in calculating the excess burden of factor taxes, because different factor taxes may interact with one another. For

This greatly reduces the information necessary to estimate excess burden, and should still yield an accurate approximation. Only in circumstances where there is a heavily taxed (or subsidized) consumer good that is a strong complement or substitute to the taxed good will these terms contribute importantly to excess burden. In such cases, it is simple to add the interaction terms for particular goods back in, and thus such interactions could be included if the relevant cross-price elasticities can be estimated.

Rewriting the effect of labor-market interaction in terms of the own-price labor supply elasticity (see Appendix A for derivation) and dropping the other interaction terms yields

$$\frac{1}{l}\frac{dU}{dt_{Ck}} = \frac{t_{Ck}C_k}{p_{Ck}} e_{Ck} - \frac{t_L L}{p_{Ck}} e_L s_k (q+1) \tag{17}$$

where e_L is the compensated elasticity of labor supply, s_k is the share of income spent on good k, and

$$q \equiv \frac{e_{C_k L}}{\sum_{i=1}^{M} s_i e_{C_i L}} - 1 \tag{18}$$

Expression (17) reveals the marginal excess burden of a tax on an arbitrary consumer good in a world with pre-existing factor (labor) taxes. This expression helps reveal the first-order importance to excess burden of interactions with the labor market. The first term is the simple Harberger triangle formula; it captures the excess burden from the change in demand for the taxed good itself. The second term represents the interaction with the tax distortion in the labor market. Note that this second term is equal to zero when there is no prior tax on labor, and thus equation (17) collapses to the simple Harberger excess-burden formula. However, when prior labor taxes are non-zero, this second term contributes very importantly to excess burden. And, for a sufficiently small commodity tax rate (and a non-zero labor tax rate), the tax-interaction term will dominate, because the first term is proportional to the commodity tax rate, while the second term (the tax interaction term) is proportional to the labor tax rate.

We were able to express the labor-market interaction in terms of the own-price labor supply elasticity because a compensated increase in the tax rates on all consumption goods is equivalent to a compensated increase in the tax rate on labor. The former increases consumption good prices while the latter decreases the after-tax wage; the resulting effect on the real wage is the same. A tax on one consumer good has a similar effect, but the effect is smaller in keeping with the fact that the taxed good constitutes only a part of overall consumption. Thus, the change in labor supply from a marginal tax increase on the average consumer good is equal to the change from a marginal increase in the labor tax, times the share of the taxed good in total consumption.

The term q expresses whether the taxed good is a stronger or weaker substitute for leisure than

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example, Feldstein (1978) points out that interactions between labor and capital taxes significantly increase the excess burden of taxes on capital.

the average consumer good. When q is positive (negative), the taxed good is a stronger (weaker) leisure substitute than the average consumer good is. An increase in the relative price of the taxed good will produce a smaller decrease in labor supply the smaller the value of q, that is, the weaker is the good's substitutability with leisure. When q is negative, the tax on this good will exacerbate the labor market distortion by less than the equivalent increase in the labor tax (for an incremental increase in t_{Ck} from zero). This is consistent with the prior literature's result[13] that the optimal commodity tax on a relative complement to leisure is higher than the optimal tax on other goods.

It is quite difficult to estimate the cross-price elasticity between a particular good and leisure. Thus, for most practical applications, it will be necessary to assume that $q = 0$, or, in other words, that the taxed good is an average substitute for leisure. We make this assumption for numerical calculations throughout the paper, though all of the analytical derivations still allow q to take on other values. Thus, in the rare cases in which one can estimate the cross-price elasticity between the taxed good and leisure, one can incorporate this information to yield a more accurate excess-burden approximation.

C. Considering Larger Tax Changes

The welfare-change formula derived in the previous section applies only for a marginal change in the tax rate. This section derives an approximation formula for the effects of a larger tax change that expresses the excess burden in terms of observable quantities and estimable parameters.

The usual excess-burden formula for a non-marginal change integrates the welfare change over the change in the tax rate and assumes a linear demand curve, giving a first-order approximation for the welfare effect of the tax – the familiar Harberger triangle. A similar linearity assumption–that the effect on labor is also linear–can be applied here to produce a first-order approximation for the welfare effect including second-best effects. Along with the assumption that revenue from commodity taxes is small relative to the economy (and thus that $Y \approx L$), this yields

$$\text{(20)} \qquad \frac{1}{l}\Delta U \approx s_k Y\left[\frac{t_{Ck}^2}{2p_{Ck}^2}e_{Ck} - \frac{t_{Ck}t_L}{p_{Ck}}e_L(q+1)\right]$$

Equation (20) is similar to (17), and the interpretation of each term is the same. It simply replaces each term from (17) with a first-order approximation to the integral of that term. For a non-linear demand curve, these approximations will introduce some error. However, they will be roughly as accurate for the second-best case as the usual formula is for the first-best case. Moreover, in a second-best setting they are vastly more accurate than the usual formula.

Table 1 compares the excess burden estimates from this formula to those from the usual formula,

[13] See Corlett and Hague (1953), Harberger (1964), and Atkinson and Stiglitz (1972).

for a range of tax rates and demand elasticities. It demonstrates both the potential magnitude of the error introduced by omitting general-equilibrium effects, and the cases in which these effects are likely to be important. The first numbered row of the table presents estimates of excess burden using the simple excess-burden formula, while the second numbered row employs the general equilibrium approximation formula (20) above. Cases 1, 2, and 3 differ in terms of the relative size of the commodity tax (τ_{Ck} / p_{Ck}) and the own-price elasticity of demand (ε_{Ck}). When the commodity tax rate and the own-price elasticity are high, as in Case 1, the bulk of the excess burden is generated in the market for the taxed good itself. Thus, the error from omitting general-equilibrium effects is relatively small, though still not insignificant. In contrast, when the tax rate and elasticity of demand are low, as in Case 3, the excess burden from the labor market interaction can be far greater than that generated in the market for the taxed good itself, and thus the usual formula will dramatically understate excess burden.

D. Incorporating Income Effects

Thus far, we have assumed that all of the elasticities are compensated, or, in other words, that the lump-sum transfer of commodity tax revenue will exactly compensate individuals for the income loss from the higher tax rate. But because the commodity tax involves an excess burden, the transfer is less than the real income loss from the tax – the latter being the tax revenue plus the excess burden. Hence there is an income effect.[14] The income effect and excess burden interact, and must be considered simultaneously This section derives a formula that incorporates this income effect, and examines the significance of this effect for the overall excess burden.

Using the Slutsky equation and the fact that a total derivative will equal the uncompensated derivative plus the income effect of the change in the government transfer, we can rewrite equation (15) as

$$(21)\qquad \frac{1}{\lambda}\frac{dU}{d\tau_{Ck}} = \tau_{Ck}\frac{\partial C_k^C}{\partial \tau_{Ck}} + \sum_{i\neq k}\tau_{Ci}\frac{\partial C_i^C}{\partial \tau_{Ck}} + \sum_{j=1}^{N}\tau_{Ij}\frac{\partial I_j^C}{\partial \tau_{Ck}} - \tau_L\frac{\partial l^C}{\partial \tau_{Ck}} + \left(\sum_{i=1}^{M}\tau_{Ci}\frac{\partial C_i}{\partial Y_G} + \sum_{j=1}^{N}\tau_{Ij}\frac{\partial I_j}{\partial Y_G} - \tau_L\frac{\partial l}{\partial Y_G}\right)\left(\frac{dG}{d\tau_{Ck}} - C_k\right)$$

where the superscript "*C*" denotes a compensated derivative. The first four terms represent the influence on welfare from the substitution effect–the focus of our previous discussion–while the product of the two terms in brackets is the influence of the income effect. For each good (and labor), the income effect equals the derivative of demand with respect to income, times the effective change in real income, which consists of both the change in the government transfer and the loss from the increased price of good *k*. Totally differentiating the government budget constraint (6) with respect to τ_{Ck} and rearranging yield an expression for that effective change in income

[14] See Hines (1999) for further discussion of why income effects appear even when tax revenues are returned.

(22) $$\frac{dG}{d\tau_{Ck}} - C_k = \sum_{i=1}^{M} \tau_{Ci} \frac{dC_i}{d\tau_{Ck}} + \sum_{j=1}^{N} \tau_{Ij} \frac{dI_j}{d\tau_{Ck}} - \tau_L \frac{dl}{d\tau_{Ck}}$$

Notice that the right-hand side of this equation is equivalent to the right-hand side of equation (15); the effective change in income equals the marginal excess burden of the tax. By definition, excess burden is the difference between the cost to the household and the revenue raised. Thus, if there is no excess burden, returning the revenue will exactly compensate the household, and there will be no income effect. But when the marginal excess burden is positive, the marginal revenue will be insufficient to fully compensate the household, yielding an income effect that is proportional to marginal excess burden.[15] Thus, we can replace the last term of (21) with the left-hand side of (15). Rearranging the resulting expression yields

(23) $$\frac{1}{\lambda}\frac{dU}{d\tau_{Ck}} = \left[\tau_{Ck} \frac{\partial C_k^C}{\partial \tau_{Ck}} + \sum_{i \neq k} \tau_{Ci} \frac{\partial C_i^C}{\partial \tau_{Ck}} + \sum_{j=1}^{N} \tau_{Ij} \frac{\partial I_j^C}{\partial \tau_{Ck}} - \tau_L \frac{\partial l^C}{\partial \tau_{Ck}} \right] \Bigg/ \left(1 - \sum_{i=1}^{M} \tau_{Ci} \frac{\partial C_i}{\partial Y_G} - \sum_{j=1}^{N} \tau_{Ij} \frac{\partial I_j}{\partial Y_G} + \tau_L \frac{\partial l}{\partial Y_G} \right)$$

Rewriting this equation in terms of elasticities and then following the same steps (with the numerator) that led from (16) to (17) yield

(24) $$\frac{1}{\lambda}\frac{dU}{d\tau_{Ck}} = \left[\frac{\tau_{Ck} C_k}{p_{Ck}} \varepsilon_{Ck} - \frac{\tau_L L}{p_{Ck}} \varepsilon_L s_k (\theta + 1) \right] \Bigg/ \left(1 - \sum_{i=1}^{M} \frac{\tau_{Ci} C_i}{Y} \varepsilon_{C_iY} - \sum_{j=1}^{N} \frac{\tau_{Ij} I_j}{Y} \varepsilon_{I_jY} - \frac{\tau_L L}{Y} \varepsilon_{LY} \right)$$

where ε_{C_iY} and ε_{I_jY} are the income elasticities of demand for goods C_i and I_j, respectively, and ε_{LY} is the income elasticity of labor supply. Note that each of the income effect terms is proportional to the total tax revenue in a particular market, including the effect in the market for good k. Thus, if the labor tax raises a large majority of government revenue, then omitting the income effects in the other markets will have little effect. Using those two results and following the same steps that led from (17) to (20) yield

(25) $$\frac{1}{\lambda}\Delta U \approx s_k Y \left[\frac{\tau_{Ck}^2}{2 p_{Ck}^2} \varepsilon_{Ck} - \frac{\tau_{Ck} \tau_L}{p_{Ck}} \varepsilon_L (\theta + 1) \right] \Big/ (1 - \tau_L \varepsilon_{LY})$$

This expression is similar in form to (20), but includes an additional income effect term. This tends to reduce excess burden: the income effect leads to an increase in labor supply, which reduces the distortion in the labor market and thus partly offsets the welfare loss resulting from the substitution-effect-induced

[15] This appears to conflict with the well-known result (see Diamond and Mirrlees, 1971, for example) that optimal taxes depend only on compensated elasticities. But the two results are easily reconciled. The optimum equates marginal excess burden across all taxes, which implies that the income effects are also equal across all taxes; thus the optimum depends only on compensated elasticities. It is not that there are no income effects, but merely that the income effect terms drop out of the equations describing the optimum; the income effects still influence excess burden.

changes in goods demand and labor supply.[16] But because the income effect is proportional to excess burden, that welfare gain can never completely offset the loss. Note that incorporating income effects adds little complexity; the only additional information required for the approximation is the income elasticity of labor supply.

The third row of Table 1, labeled "Full General Equilibrium," presents results from the approximation formula with income effects given by (25) above. The difference between the results in this row and those in row 2 indicates the significance of the income effect. In cases 1, 2, and 3, accounting for the income effect reduces the estimated excess burden by about 8 percent. Equation (25) indicates that a smaller value for the tax on labor reduces the importance of the income effect (assuming a negative value for e_{LY}). To get a sense of the magnitudes involved, we consider an additional case – Case 2' – which assumes the same values of the commodity tax and own-price elasticity as in Case 2, but employs a much lower pre-existing tax labor tax (.05, as compared with 0.4 under Case 2). In this case, interactions with the labor market become less important. Thus, incorporating the income effect reduces the estimated excess burden by less than one percent.

E. Effects of a Tax on an Intermediate Good

Here we examine the effect of a tax on an intermediate good. Adopting an approach similar to that used in section B gives

(15') $$\frac{1}{l}\frac{dU}{dt_{Ik}} = \underbrace{t_{Ik}\frac{dI_k}{dt_{Ik}}}_{dW^P} + \underbrace{\sum_{i=1}^{M} t_{Ci}\frac{dC_i}{dt_{Ik}} + \sum_{j \neq k} t_{Ij}\frac{dI_j}{dt_{Ik}} - t_L\frac{dl}{dt_{Ik}}}_{dW^I}$$

As with the consumer good tax, the tax on an intermediate good affects welfare through its impacts on the market for the taxed good, the markets for other taxed goods, and the distorted labor market. The first term on the right-hand side represents the primary welfare effect (the effect on the market for the taxed good), while the remaining terms represent the tax-interaction effect (the effect on the markets for other taxed goods and on the distorted labor market).

Again following the same approach as in section B (see Appendix A for details), we arrive at the following expression for the welfare effect of a tax:

(17') $$\frac{1}{l}\frac{dU}{dt_{Ik}} = \frac{t_{Ik}I_k}{p_{Ik}}e_{Ck} - t_L L\frac{I_K}{Y}e_L(q+1)$$

[16] Interestingly, this means that if the compensated elasticity of labor supply were sufficiently small, general-equilibrium interactions in the labor market could actually produce a welfare gain; as the compensated elasticity goes to zero, the substitution effect in the labor market will disappear, but the income effect would remain. Thus, the usual approximation could actually overstate excess burden. This case seems rather unlikely to appear in

where $Y = (1 - t_L)L + Y_G$ is total household income, and

$$(18')\qquad q \equiv \frac{\sum_{i=1}^{M} e_{C_k L} a_{ik}}{\sum_{i=1}^{M} s_i e_{C_i L}} - 1$$

and a_{ij} is the fraction of total production of intermediate good j that is used to produce consumer good i.

Expression (17') has the same form as the corresponding expression for a consumer good tax (17), incorporating both the direct effect in the market for the taxed good and the interaction with the labor market. However, the expression for q (18') is a bit different from the corresponding expression (18) associated with the tax on the consumer good. As with a tax on a consumer good, the welfare impact of the tax depends in part on how it affects the labor/leisure decision. But because the intermediate good is not consumed by households, the impact on the labor/leisure decision is indirect. Now the labor supply effect of a tax on that good depends on the complementarity or substitutability with leisure of the consumer goods that are produced from the intermediate good. The right-hand side of (18') expresses these indirect complementarity and substitutability elements. If the average good produced from the taxed intermediate (weighted by the amount of the taxed intermediate used in each good) is more of a complement to leisure than the average consumer good (weighted by expenditure share), then q will be negative, and the tax-interaction effect and excess burden from the tax will be relatively low.

The resemblance between (17') and (17) does not imply that intermediate input taxes have the same status as consumer good taxes in an optimal tax system. At the Diamond-Mirrlees optimum, the excess burden per marginal dollar of revenue from *all* taxes (on both intermediate inputs and consumer goods) is exactly the same. However, for intermediate inputs, the optimal tax rate at this optimum is exactly zero, whereas they need not be zero for consumer good taxes. Moreover, the equality of excess burdens at the optimum is consistent with the idea that strictly positive (or negative) taxes on intermediate inputs will have higher excess burdens than the excess burdens associated with the optimal values of consumer good (or intermediate good) taxes.

We can then follow a similar process to that in section D to obtain an implementable formula for large changes, including income effects. This yields

$$(25')\qquad \frac{1}{l}\Delta U \approx \frac{p_{Ik} I_K}{Y} Y \left[\frac{t_{Ik}^2}{2p_{Ik}^2} e_{Ik} - \frac{t_{Ik} t_L}{p_{Ik}} e_L (q+1) \right] \bigg/ \left(1 - t_L e_{LY}\right)$$

This expression is analogous to (25), and each term has the same interpretation. The two terms in square brackets represent linear approximations to the primary and tax-interaction effects, respectively, for a large change in the tax rate. The last term accounts for income effects.

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practice, though, because it requires a large income elasticity of labor supply and a very small compensated elasticity, which together imply a strongly backward-bending uncompensated labor supply curve.

III. How Accurate Are the Excess-Burden Approximations?

A. A Numerical Model

The foregoing theoretical discussion indicates the conceptual superiority of the new formulas for excess burden, but does not fully convey their quantitative significance. To gauge the accuracy of the new formulas relative to the usual formula, we employ a simple numerically-solved general equilibrium model. We solve this model to determine "true" excess burden for a given tax, where the excess burden is measured using the equivalent variation.[17] We then compare this "true" burden with the approximations to excess burden under the usual formula and the new formulas. In contrast with the approximation formulas, the numerical assessments of excess burden avoid linear approximations as well as the assumption of a constant marginal utility of income. In addition, they involve a more complex production system than was employed in deriving the formulas. The numerical simulations both illustrate the relative accuracy of the two formulas and show how their common simplifying assumptions affect their accuracy.

The numerical model has the same formal structure as that of the analytical model but specifies a particular set of goods and particular functional forms. The model distinguishes four intermediate good industries (energy, services, agriculture, and manufactures) and five consumer good industries (consumer services, consumer manufactures, transportation, utilities, and food & tobacco), the last of which is further disaggregated (giving six consumer good industries) to analyze the cigarette tax. Just as in the analytical model, households supply labor to firms and receive income in the form of wages and government transfers, which are funded through commodity taxes and a labor tax. Consumption and leisure are assumed to be separable in the household utility function, implying that all consumer goods are equal substitutes for leisure. The utility function and all production functions follow a constant-elasticity-of-substitution (CES) form.[18]

The model is calibrated to match the United States economy in 1995, using data from the *Survey of Current Business*. We assume an uncompensated labor supply elasticity of 0.05, a compensated elasticity of 0.25, and a labor tax rate of forty percent. In order to be conservative, these labor supply

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[17] We also compute the compensating variation. In every case we consider, the difference between the equivalent variation and compensating variation is less than two percent of either measure. This implies that very little error is introduced by assuming a constant marginal utility of income.

[18] This model is similar to the numerical model used by Parry, Williams, and Goulder (1999) to examine the cost of environmental regulation in the presence of pre-existing tax distortions, but differs in that it involves different industry classifications and does not consider environmental externalities. Details of the model used here appear in Appendix B.

elasticities are somewhat lower than the middle of the range suggested by recent surveys,[19] and thus will lead to somewhat smaller general-equilibrium welfare effects.

We consider the welfare impacts from the imposition of two new taxes: a cigarette tax and a tax on energy use.[20] For each tax, we consider a range of tax rates and elasticities. For the cigarette tax, we use three different estimates for the own-price demand elasticity for cigarette consumption – 0.2, 0.4, and 0.8 – which span a range somewhat broader than most estimates.[21] We consider a low-case tax rate of 14.1% ($0.265 per pack), which is equal to the current federal rate plus the lowest state excise tax; a central-case tax rate of 31.3% ($0.59 per pack), equal to the federal rate plus the average state tax; and a high case rate of 65.3% ($1.23 per pack), equal to the federal rate plus the average state rate, plus the additional charges in the proposed tobacco settlement, which amount to roughly $0.64 per pack of cigarettes sold.

For the energy tax, we use elasticities of 0.6, 0.9, and 1.35, which again is a somewhat broader range than most estimates.[22] We take 5 percent as a central case for the tax rate (which is roughly equivalent to the BTU tax proposed by President Clinton in 1993) and also consider tax rates that are one half and twice that rate. In evaluating each tax, we assume that no other commodity taxes are present.

B. Results

Table 2 indicates the differences in the accuracy of the alternative formulas. For each case, the table shows the "true" excess burden as calculated by the numerical model and the approximated excess burden under both the usual formula and the new formula (with income effects) introduced in this paper. (In applying the new formula, we use equations (25) and (25') for the cigarette tax and energy tax, respectively.) The table also shows the percentage errors under each approximation.[23]

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[19] Russek (1996) and Fuchs, Krueger, and Poterba (1998) provide surveys of labor supply elasticity estimates.. See also Ballard (1999) for a discussion of the relationship between these elasticities, the income elasticity of labor supply, and the implicit endowment of potential labor time. A tax rate of 40 percent is similar to that used in other studies (see Browning (1987) and Lucas (1990), for example).

[20] While both goods involve substantial negative externalities, we ignore such externalities here. The purpose of this exercise is to illustrate the importance of considering pre-existing taxes in estimating the (gross) excess burden, not to consider the benefits provided by these policies.

[21] In each case, the model is calibrated to match a particular demand elasticity. Estimates of the demand elasticity for cigarettes range from 0.25 to 0.7, centering around a value of 0.4. See Congressional Budget Office (1998) for a survey of cigarette demand elasticity estimates.

[22] Most estimates of the demand elasticity for energy use are slightly below 1. See, for example, Jorgenson and Wilcoxen (1990).

[23] In calculating the approximation for excess burden under both the old and new formulas, we use the actual change in consumption of the taxed good, as taken from the numerical model, rather than simply plug in the relevant

This table demonstrates that the usual formula is substantially inaccurate in the presence of a pre-existing labor tax. In all but three cases, it yields an approximation that is less than half the true excess burden of a given tax, and in several cases it yields a value which is less than one-fifth the true cost. This indicates that the downward bias of the simple formula can be dramatic, and that previous studies of the cost of commodity taxes that have used this simple formula may have seriously underestimated the cost of such taxes.

This error arises because the usual formula only considers one component of the excess burden – namely, the deadweight loss in the market for the taxed good – and ignores interactions with other markets. Very little of the error comes from the linear approximation. Indeed, for the usual formula, the linear approximation may help reduce the error in the excess-burden estimate. If supply is linear (e.g., perfectly elastic) and demand curves are convex (as is typically the case), the linear approximation yields a higher estimate than would a method that estimates the excess burden region through integration. Hence the error associated with the linear approximation can partly offset the error from omitting interactions with other markets.

The usual excess-burden formula is most accurate for high tax rates levied against goods with relatively elastic demand. In these cases, the distortion in the market for the taxed good (which is captured by the usual formula) is relatively large compared to the distortion in the labor market (which is ignored by the usual formula), and thus the usual formula captures a relatively large portion of the total excess burden. When the tax rate is low or demand is inelastic, the usual formula is off by much more.

In contrast, the formula that includes income effects provides a very close approximation to the true excess burden. The largest error under this formula is ten percent, and in most cases the formula is off by less than five percent.[24] In practice, errors of such small magnitude will be insignificant relative to uncertainty in estimating the elasticity parameters involved in the calculation.

Figure 1 provides further comparisons of the errors under the different formulas, for the energy tax and cigarette tax, respectively. This figure displays the "true" excess burden and the approximated excess burden from the usual and new formulas, over a range of values for the energy and cigarette tax

elasticity to infer a change in consumption. For a marginal change, of course, either approach would yield the same answer. However, for the large changes we consider, the two yield slightly different answers, because for given utility function parameters, the demand elasticity changes slightly as the tax rate changes.

[24] Our new formula overestimates excess burden in every case considered in both Table 2 and Figure 1, which may seem inconsistent with our characterization of this formula as unbiased. However, this is not due to any inherent bias but rather reflects the linear approximation used, along with the assumptions of the numerical model. Convexity in the demand function will cause a linear approximation to overstate excess burden, while convexity in the supply function will have the opposite effect. Because the numerical model assumes perfectly elastic supply but convex demand, the linear approximation overestimates the excess burden arising from the numerical model.

rates. The various panels in the figure show the results under three different pre-existing labor tax rates: 20, 40 and 60 percent.

This figure demonstrates two important points. First, while the excess burden as measured by the usual formula is roughly proportional to the square of the (consumer or intermediate good) tax rate, the "true" excess burden and the new formula are roughly linear with respect to the tax rate, at least over a reasonable range. This reflects the fact that, over the range of tax changes considered, the newly introduced (cigarette or energy) tax remains a fairly small component of the overall effective tax on labor (the labor tax itself is the more important component). The bulk of the excess burden from the new tax comes from the interaction with the labor market, not the deadweight loss in the market for the taxed good itself! Hence the level of the new tax intermediate or final good tax has relatively little effect on the marginal distortion in the labor market, and the overall excess burden is close to linear with respect to the rate of the new tax.

Second, the relative accuracy of the usual formula as compared with the new formula varies with the pre-existing labor tax rate. As the pre-existing labor tax rate rises, interactions with the tax-distorted labor market become more important, and the error in the usual formula is greater. With a pre-existing labor tax rate of zero, the welfare effect of interactions with the labor market disappears, and the usual formula and the new formula give identical answers. Figure 1 shows that, even for a relatively low pre-existing labor tax rate of twenty percent, the new formula is still far more accurate than the usual formula in measuring excess burden.

IV. Conclusions

The simple "excess-burden triangle" formula yields biased and highly inaccurate estimates of the excess burden of commodity taxes: taxes on consumer goods and intermediate inputs. This poor performance reflects the simple formula's inability to capture general equilibrium interactions with prior tax distortions in other markets–factor markets in particular.

While prior research has recognized the potential of general equilibrium interactions to affect excess burden, economists nevertheless have tended to adopt the simple formula, for two main reasons. First, as a conceptual matter, economists have been unaware of the inherent bias in the simple formula and the quantitative importance of that bias. We have shown that, when the taxed commodity is average in terms of its substitutability with leisure, the simplest formula significantly understates excess burden from a commodity tax. Most of the excess burden is attributable to general equilibrium interactions with factor markets, rather than to distortions generated in the market for the taxed commodity.

A second reason why economists have tended to use the simple formula is the absence of an easily implementable alternative. We derived an alternative formula that takes account of general equilibrium interactions yet remains easy to implement. Our alternative formula is unbiased, and numerical simulations indicate that it is far more accurate than the simple excess-burden formula. For realistic central parameter values and for a wide range of assumed rates for prior taxes, the simple excess-burden formula substantially understates the excess burden of taxes on commodities, capturing less than half of the excess burden. Even when low values (20 percent) are used for prior tax rates on labor, for realistic commodity tax rates the usual formula captures no more than two thirds of the excess burden. When the newly introduced tax is "small," the usual formula can be spectacularly wrong, giving a result less than five percent of the true excess burden. In contrast, for all but very high commodity tax rates, our alternative formula yields estimates that are within five percent of the actual excess burden.

These results reveal the importance of interactions between commodity taxes and factor markets. They have significant implications for future empirical work on the excess burden of particular commodity taxes. Studies using the usual excess-burden approximation will substantially understate the true cost of commodity taxes, in some cases quite dramatically. Our results also have implications for tax policy. They reinforce the notion that broad-based factor taxes tend to be more efficient means of raising revenue than taxes on particular commodities. In addition, to the extent that commodity taxes have higher distortionary costs than previously recognized, any project funded with such a tax will need a much higher level of benefits to pass a benefit-cost test.

Some limitations of the present study deserve mention. First, the general-equilibrium framework considers only one primary factor of production (labor). Since commodity taxes are implicit factor taxes, in assessing excess burden one would want to account for how a commodity tax might augment or reduce inefficiencies in the relative taxation of different factors such as capital and labor.[25] This would require a model with multiple factors. Although considering multiple factors would refine the analysis, we would not expect it to alter the qualitative findings obtained here.

Second, while our analytical model and alternative approximation allow the taxed good to be a complement or a substitute with leisure, in our numerical model the taxed good is defined as an average substitute for leisure. As discussed above, the excess burden would differ if the taxed commodity were an exceptionally strong or weak substitute with leisure. Since the actual degree of complementarity or substitutability with leisure is uncertain, any formula for excess burden must ultimately yield estimates that involve uncertainty. However, this does not alter the fact that, for the typical good, our alternative formula will be more accurate (and often far more accurate) than the usual formula.[26]

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[25] Bovenberg and de Mooij (1998) analyze these issues in the context of assessing the costs of revenue-neutral environmental tax reforms.

[26] The usual formula would provide a good approximation to excess burden if there were no taxes on other commodities and the newly taxed good were a much stronger than average complement with leisure. It could also

Finally, our approximation formula ignores interactions with prior taxes on other commodities. It would not be difficult to add terms accounting for such interactions, but it would be difficult to estimate the cross-price elasticities necessary to calculate those terms, and thus the terms would have little practical value. In the U.S., where commodity taxes play a relatively minor role, ignoring such taxes should not have much quantitative importance.

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give a good approximation if there were another taxed commodity (i.e., consumer good) that represented a strong substitute for the newly taxed commodity. But one could just as easily reverse these examples, giving a situation in which the usual formula is even further off the mark than in the central case we consider.

References

Atkinson, A.B. and J.E. Stiglitz, 1972. “The Structure of Indirect Taxation and Economic Efficiency.” *Journal of Public Economics* 1:97-119.

Auerbach, Alan J., 1985. “The Theory of Excess Burden and Optimal Taxation.” *In* A. J. Auerbach and M. Feldstein, eds., *Handbook of Public Economics*, vol. I. Amsterdam: North-Holland.

Ballard, Charles L., 1999. “How Many Hours Are in a Simulated Day? The Effects of Time Endowment on the Results in Tax-Policy Simulation Models.” Working paper, Department of Economics, Michigan State University, October.

Bovenberg, A. Lans, and Ruud A. de Mooij, 1994. “Environmental Levies and Distortionary Taxation.” *American Economic Review* 84(4):1085-9.

Bovenberg, A. Lans, and Ruud A. de Mooij, 1998. “EnvironmentalTaxes, International Capital Mobility, and Inefficient Tax Systems: Tax Burden vs. Tax Shifting.” *International Tax and Public Finance* 5(1):7-40.

Bovenberg, A. Lans, and F. van der Ploeg, 1994. “Environmental Policy, Public Finance and the Labour Market in a Second-Best World,” *Journal of Public Economics* 55:349-70.

Browning, Edgar K., 1987. “On the Marginal Welfare Cost of Taxation.” *American Economic Review* 77: 11-23.

Congressional Budget Office, 1998. *The Proposed Tobacco Settlement: Issues From a Federal Perspective*

Corlett, W. J., and D. C. Hague, 1953. “Complementarity and the Excess Burden of Taxation” *Review of Economic Studies* 21:21-30.

Diamond, P. A. and J. A. Mirrlees, 1971. “Optimal Taxation and Public Production I-II.” *American Economic Review*, 61:8-27, 61:261-278

Dixit, A. K., 1975. “Welfare Effects of Tax and Price Changes” *Journal of Public Economics* 4:103-123

Fershtman, C., N. Gandal and S. Markovich, 1999. "Estimating the effect of tax reform in differentiated product oligopolistic markets" *Journal of Public Economics* 74:151-170

Farrell, L. and I. Walker, 1999. "The welfare effects of lotto: evidence from the UK" *Journal of Public Economics* 72:99-120

Feldstein, Martin, 1978. "The welfare cost of capital income taxation" *Journal of Political Economy* 86:S29-S51

Fuchs, Victor R., Alan B. Krueger and James M. Poterba, 1998. “Economists’ Views about Parameters, Values and Policies: Survey Results in Labor and Public Economics.” *Journal of Economic Literature* 36(3):1387-1425.

Goulder, Lawrence H., 1995a. "Environmental Taxation and the 'Double Dividend:' A Reader's Guide," *International Tax and Public Finance* 2(2):157-83.

Goulder, Lawrence H., 1995b. "Effects of Carbon Taxes in an Economy with Prior Tax Distortions," *Journal of Environmental Economics and Management*.

Goulder, L. H., I. W. H. Parry, and D. Burtraw, 1997. "Revenue-Raising vs. Other Approaches to Environmental Protection: The Critical Significance of Pre-Existing Tax Distortions." *Rand Journal of Economics* 28:708-731.

Goulder, Lawrence H. and Roberton C. Williams III, 1999. "The Usual Excess-Burden Approximation Usually Doesn't Come Close" NBER working paper no. 7034

Harberger, A. C., 1964a. "Taxation, Resource Allocation, and Welfare" in *The Role of Direct and Indirect Taxes in the Federal Revenue System*, Princeton University Press

Harberger, A. C., 1964b. "The Measurement of Waste" *American Economic Review Papers and Proceedings* 54:58-76

Hausman, J., 1997. "Taxation by Telecommunications Regulation" NBER working paper no. 6260

Hausman, J., 1999. "Efficiency Effects on the U.S. Economy from Wireless Taxation" NBER working paper no. 7281

Hicks, J. R., 1946. *Value and Capital*. Second edition; Oxford: Clarendon Press.

Hines, James R., Jr., 1999. "Three Sides of Harberger Triangles." *Journal of Economic Perspectives*, 13:167-188.

Hotelling, Harold, 1938. "The General Welfare in Relation to Problems of Taxation and of Railway and Utility Rates." *Econometrica*, July.

Jorgenson, Dale W. and Peter J. Wilcoxen, 1990. "Environmental Regulation and U.S. Economic Growth, *The Rand Journal of Economics* 21(2):314-40.

Konig, R. and G. Ridder, 1997. "Rent assistance and housing demand" *Journal of Public Economics* 66:1-31

Lipsey, R. G., and K. Lancaster, 1956-57. "The General Theory of Second Best." *Review of Economic Studies* 24(63):11-32.

Lucas, Robert E., 1990. "Supply-Side Economics: An Analytical Review." *Oxford Economic Papers* 42:293-316.

Meade, J. E., 1955. *Trade and Welfare*, Vol. II. London: Oxford University Press.

Parry, I. W. H., 1995. "Pollution Taxes and Revenue Recycling." *Journal of Environmental Economics and Management*, 29:S64-S77

Parry, I. W. H., R. C. Williams III, and L. H. Goulder, 1999. “When Can Carbon Abatement Policies Increase Welfare? The Fundamental Role of Distorted Factor Markets.” *Journal of Environmental Economics and Management*, 37:52-84.

Poterba, J. 1992. "Taxation and Housing: Old Questions, New Answers", *American Economic Review* 82:237-242

Ramsey, F. P., 1927. “A Contribution to the Theory of Taxation.” *Economic Journal* 37:47-61

Viner, Jacob, 1950. *The Customs Union Issue*. Carnegie Endowment for International Peace.

Williams, R. C., 1999. “Revisiting the Cost of Protectionism: The Role of Tax Distortions in the Labor Market.” *Journal of International Economics*, 47:459-477.

Williams, R. C., 1998. “Optimal Taxes, Tax Interactions, and the Structure of Preferences.” working paper, Stanford University

Appendix A: Analytical Model Derivations

Derivation of Equation (17)

Taking a derivative of the household utility function (1) while holding utility constant, and substituting in the consumer first-order conditions (10) yield

$$\text{(A1)} \qquad \frac{\partial l^C}{\partial t_L} = -\frac{\partial l^C}{\partial(1-t_L)} = \sum_{i=1}^{M} \frac{p_{Ci}}{1-t_L} \frac{\partial C_i^C}{\partial(1-t_L)}$$

and the Slutsky symmetry property gives

$$\text{(A2)} \qquad \frac{\partial l^C}{\partial p_{Ci}} = \frac{\partial C_i^C}{\partial(1-t_L)}$$

Dividing (A2) by (A1) and rearranging yield

$$\text{(A3)} \qquad \frac{\partial l^C}{\partial t_{Ck}} = \frac{\partial l^C}{\partial t_L} (1-t_L) \frac{C_k}{Y} \frac{e_{C_k L}}{\sum_{i=1}^{M} s_i e_{C_i L}}$$

where $e_{C_i L} = \dfrac{\partial C_i^C}{\partial(1-t_L)} \dfrac{(1-t_L)}{C_i}$ and $s_i = \dfrac{p_{Ci} C_i}{\sum_{j=1}^{M} p_{Cj} C_j}$

Starting from (16), dropping the terms for the interactions with other commodities, and substituting in (A3) yield (17).

Derivation of Equation (17')

The derivation of equation (17') for a tax on an intermediate good is similar to the derivation of equation (17). Following the same steps that led to (A3) yields

$$\text{(A3')} \qquad \frac{\partial l^C}{\partial t_{Ik}} = \frac{\partial l^C}{\partial t_L} (1-t_L) \frac{I_k}{Y} \frac{\sum_{i=1}^{M} e_{C_k L}\, a_{ik}}{\sum_{i=1}^{M} s_i e_{C_i L}}$$

where $a_{ij} = \dfrac{I_{ij}}{I_j}$. Dropping the terms from (15') for the interactions with other commodities, substituting in (A3') and rewriting in terms of elasticities yields (17').

Generalizing the Production Function for Intermediate Goods

This section extends the analytical model of Section II by generalizing the production function for intermediate goods to include other intermediate goods as inputs, as well as labor.[27] Thus, the intermediate good production function follows

$$(A4)\qquad I_i = F_{Ii}\left(L_{Ii}, I_{Ii1} \ldots I_{IiN}\right)$$

where I_{iij} is the amount of intermediate good j used in production of intermediate good i. The supply and demand for intermediate goods then follow

$$(A5)\qquad I_j = \sum_{i=1}^{M} I_{ij} + \sum_{i=1}^{N} I_{Iij}$$

and the first-order conditions for firm profit-maximization give the price of intermediate good i as

$$(A6)\qquad p_{Ii} = t_{Ii} + \frac{1}{\partial F_{Ii} / \partial L_{Ii}} = t_{Ii} + \frac{p_{Ij}}{\partial F_{Ii} / \partial I_{Iij}}$$

Following the same set of steps as in Section II.C, but using equations (A4), (A5) and (A6) in place of the corresponding equations (3), (4), and (9) gives the same result as equation (17'), but the expression for q now differs from equation (18')

$$(A7)\qquad q = \frac{\sum_{i=1}^{M} h_{C_k L} \sum_{j=1}^{N} I_{ij} G_j I_k^{-1}}{\sum_{i=1}^{M} s_i e_{C_i L}} - 1$$

where G is the vector of total derivatives of the intermediate good prices with respect to the tax t_{Ik}. G is given by

$$(A8)\qquad G = X^{-1} F$$

where F is the vector of derivatives of intermediate good tax rates with respect to t_{Ik} (where all but the kth element equal zero, and the kth element equals 1) and X is the matrix of intermediate input intensities, given by

$$X_{ij} = O_{ij} - \frac{I_{ij}}{I_i}$$

where O_{ij} equals 1 when $i=j$ and equals zero otherwise.

This expression is more complicated than the analogous expression from the simpler model. As before, the numerator in (29) is a weighted average of the cross-elasticities with leisure for the consumer goods produced using a given intermediate good. Here, though, an intermediate good tax affects not only consumer good prices, but also the prices of other intermediate goods, so that the determination of the impact on consumer good prices is more complicated. Still, it can be shown that

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[27] Goulder and Williams (1999) provides an additional extension to the model from Section II, considering the case in which one or more goods are produced under imperfect competition.

(A9) $$\sum_{i=1}^{M}\sum_{j=1}^{N} I_{ij} G_j I_k^{-1} = 1$$

or, in other words, that the weights in the numerator of (A7) sum to 1. This implies that, just as in the simpler previous model, $q = 0$ for a tax on the average intermediate good (or on any intermediate good if leisure and consumption are weakly separable). Thus, since equation (17') is unchanged, and $q = 0$ for the average good, equations (20') and (25') apply even in this extended model. The more general production specification for intermediate goods does not alter the results in any fundamental way.

Appendix B: The Numerical Model

Except where otherwise noted, i ranges over primary or intermediate inputs L, I_A, I_E, I_M, and I_S, while the subscript j ranges over produced (intermediate or consumer) goods I_A, I_E, I_M, I_S, C_B, C_F, C_M, C_S, C_T, and C_U.

I. Parameters

Firm Behavior Parameters

a_{ij}	distribution parameter for input i in production of good j
s_j	elasticity of substitution in production of good j

Household Behavior Parameters

T	total labor endowment
$a_l, a_{CF}, a_{C_B} \ldots a_{C_U}$	distribution parameters for utility function
s_C	elasticity of substitution between consumer goods
σ_U	elasticity of substitution between goods and leisure

Government Policy Parameters

G	government spending (transfers to households, in nominal terms)
$t_{CB} \ldots t_{CU}, t_{IA} \ldots t_{IS}$	tax rates on consumer and intermediate goods
t_L	tax rate on labor income

II. Endogenous Variables

b_{ij}	use of input i per unit of output of good j
C_B, C_F, C_M, C_S,	aggregate demands for tobacco, food, consumer manufactures, consumer services,
C_T, and C_U	transportation, and utilities
C	aggregate demand for composite consumer good
I_A, I_E, I_M, and I_S	aggregate demand for agriculture, energy, manufactures, and services
X_j	aggregate supply of good j
L	aggregate labor supply
l	leisure or non-market time
p_C	price of composite final good

p_j	price of good *j*
REV	government revenue
U	total consumer utility
X_{ij}	use of good *i* in production of good *j*

III. Equations

Structure of Production

In all industries, output is produced according to:

(B1) $$X_j = \left(\sum_i a_{i,j} X_{i,j}^{\frac{s_j - 1}{s_j}} \right)^{\frac{s_j}{s_j - 1}}, \; i = \{L, I_A, I_E, I_M, I_S\}, \; j = \{I_A, I_E, I_M, I_S, C_B, C_F, C_M, C_S, C_T, C_U\}$$

Profit for industry j is given by

(B2) $$p_j = (p_j - t_j) X_j - \sum_I p_i X_{i,j}$$

Optimal Input Intensities

Differentiating profit with respect to the inputs X_{ij} yields the first order conditions for the optimal input mix:

(B3) $$b_{ij} \equiv \frac{X_{ij}}{X_j} = a_{ij}^{s_j} \left\{ \frac{p_i}{p_j - t_j} \right\}^{-s_j}$$

Differentiating profit with respect to output X_j gives an equation for the competitive price for each good

(B4) $$p_j = t_j + \sum_i b_{ij} p_i$$

Household Utility Function: Labor Supply and Final Good Demands

The representative household's utility function is:

(B5) $$U = U(l, C_B \ldots C_U) = \left(a_l l^{\frac{s_u - 1}{s_u}} + a_C C^{\frac{s_u - 1}{s_u}} \right)^{\frac{s_u}{s_u - 1}}$$

where *l* represents leisure and *C* represents composite consumption:

(B6) $$C = \left(\sum_{j \in B..U} a_{C_j} C_j^{\frac{s_C - 1}{s_C}} \right)^{\frac{s_C}{s_C - 1}}$$

The household budget constraint follows

(B7) $$\sum_{j \in B..U} p_{C_j} C_j = p_L L(1 - t_L) + G$$

where t_L is the tax rate on labor income, T is the total time endowment, and G is government transfers. The household maximizes utility subject to its budget constraint and the time constraint

(B8) $L = T - l$

This maximization yields the following equations which express the household's behavior:

(B9) $$b_{C_k} \equiv \frac{C_k}{C} = \left[a_{C_k} + \sum_{j \neq k} a_{C_j} \left(\frac{a_{C_k} p_{C_j}}{a_{C_j} p_{C_k}} \right)^{1-s_C} \right]^{\frac{s_C}{s_C - 1}}$$

(B10) $$p_C = \sum_{j \in B..U} p_{C_j} b_{C_j}$$

(B11) $$l = \frac{p_L(1-t_L)T + G}{p_L(1-t_L) + p_C \left[\frac{a_l p_C}{a_C p_L (1-t_L)} \right]^{-s_U}}$$

(B12) $$C = p_C^{-1} \left[p_L (1-t_L) L + G \right]$$

Combining (B12) with (B9) and (B10) yields the optimal levels of consumption of each good.

Government

Government revenues finance transfers to households, G. The government budget constraint is

(B13) $$G = t_L L + \sum_{i \in B...U} t_{Ci} C_i + \sum_{i \in A...S} t_{Ii} I_i$$

The level of the labor tax is fixed.

Aggregate Demand and Supply

Aggregate demand for the final goods is determined by the household, through equations (B9), (B10), and (B12)

Aggregate demand for labor and for the intermediate goods is determined from the use of each good in production, yielding

(B14) $$AD_i = \sum_j X_{ij}$$

Since production of all goods follows constant returns to scale, supplies of both final goods and both intermediate goods are determined by demand. Thus

(B15) $$X_{C_i} = C_i$$

(B16) $I_i = AD_{Ii}$ for i ranging over D and N

Solving this last equation simultaneously for all values of i yields aggregate supplies and demands for the intermediate goods.

IV. Equilibrium Conditions

The equilibrium conditions are:

(B17) $L = AD_L$

and the government budget constraint (B13)

To solve the model, we compute the value of t_L and the vector of prices that satisfy (B17) and (B13), using p_L as the numeraire. By Walras's Law, if one of the two equilibrium conditions holds, the other will also hold, so the vector of primary prices that satisfies (B13) also satisfies (B17).

V. Benchmark Data

a) Intermediate Good Production

	Energy	Services	Agriculture	Manufactures
Energy	253800.3	35748.4	12135.2	83751.8
Services	55608.3	1182177.2	48378.1	753981.8
Agriculture	174.6	109776.9	353617.4	32591.6
Manufactures	108723.6	537487.8	58516.9	2017510.8
Labor	79221.2	2239303.1	55472.4	1143765.5
Total Production	497528.0	4104493.4	528120.0	4031601.6

b) Consumer Good Production

	Food, Alcohol	Tobacco	Consumer Services	Consumer Manufactures	Transportation	Utilities
Energy	297.1	0.5	34.6	5571.4	50320.6	55868.1
Services	457622.4	22753.2	835116.3	571872.7	92237.5	84745.9
Agriculture	24721.4	0.5	105.5	7131.1	0.5	0.5
Manufactures	293334.9	22096.4	75867.5	917510.0	0.5	553.2
Total Production	775975.8	44850.6	911123.9	1502085.1	142559.1	141167.7

VI. Calibration of the Model

The share parameters a_{ij} in the production functions are calibrated so that the cost-minimizing set of inputs at baseline prices match the actual input mix from the data. For the cases with a tax on a final good, the production elasticity parameters s_j are calibrated to imply an elasticity of substitution in production of 0.8 for intermediate-good industries and 0.9 for consumption-good industries. For cases with a tax on an intermediate good, these elasticities are calibrated to match a particular elasticity of demand for the intermediate good.

For the utility function, the outer-nest elasticity of substitution σ_U is used to calibrate the uncompensated labor supply elasticity, while the outer-nest share parameters a_l and a_{CF} are used to calibrate the income elasticity of labor supply. This also implies a specific baseline level of leisure consumption. Since leisure is unobserved in the data set, we cannot compare implied leisure consumption this to the actual level of leisure. For the inner nest of the utility function, the share parameters $a_{C_B} \ldots a_{C_U}$ are calibrated based on the shares of each consumption good in the baseline data. For cases with a tax on an intermediate good, the inner-nest elasticity parameter s_C is calibrated to imply an elasticity of substitution among consumption goods of 0.85. For cases with a tax on a final good, this parameter is calibrated to match a particular own-price elasticity of demand for the taxed good.

Table 1

Measures of Efficiency Cost

	Case 1	Case 2	Case 3	Case 2'
Assumptions				
t_{Ck}/p_{Ck} (tax rate on good k)	0.5	0.2	0.1	0.2
e_{Ck} (own-price elasticity)	-1.0	-0.5	-0.25	-0.5
t_L (labor tax rate)	0.4	0.4	0.4	0.05
Formula / Result				
1. Partial Equilibrium				
$-\frac{t_{Ck}^2}{2p_{Ck}^2}e_{Ck}s_kY$	$0.125s_kY$	$0.010s_kY$	$0.00125s_kY$	$0.0100s_kY$
2. General Equilibrium without Income Effects				
$-\frac{t_{Ck}^2}{2p_{Ck}^2}e_{Ck}s_kY+\frac{t_{Ck}t_L}{p_{Ck}}e_Ls_kY$	$0.175s_kY$	$0.030s_kY$	$0.01125s_kY$	$0.0125s_kY$
-- relative difference from partial equilibrium	*0.40*	*2.00*	*8.00*	*0.25*
3. Full General Equilibrium				
$\left[-\frac{t_{Ck}^2}{2p_{Ck}^2}e_{Ck}s_kY+\frac{t_{Ck}t_L}{p_{Ck}}e_Ls_kY\right]/(1-t_Le_{LY})$	$0.162s_kY$	$0.028s_kY$	$0.01042s_kY$	$0.0124s_kY$
-- relative difference from partial equilibrium	*0.30*	*1.78*	*7.33*	*0.24*

Notes:

s_k ≡ share of income spent on good k

Y ≡ income

t_{Ck} ≡ tax rate on consumer good k

p_{Ck} ≡ price of consumer good k

All cases assume a compensated labor supply elasticity (e_L) of 0.25 and an income elasticity of labor supply (e_{LY}) of -0.2

Table 2

Comparison of Approximations from Excess-Burden Formulas

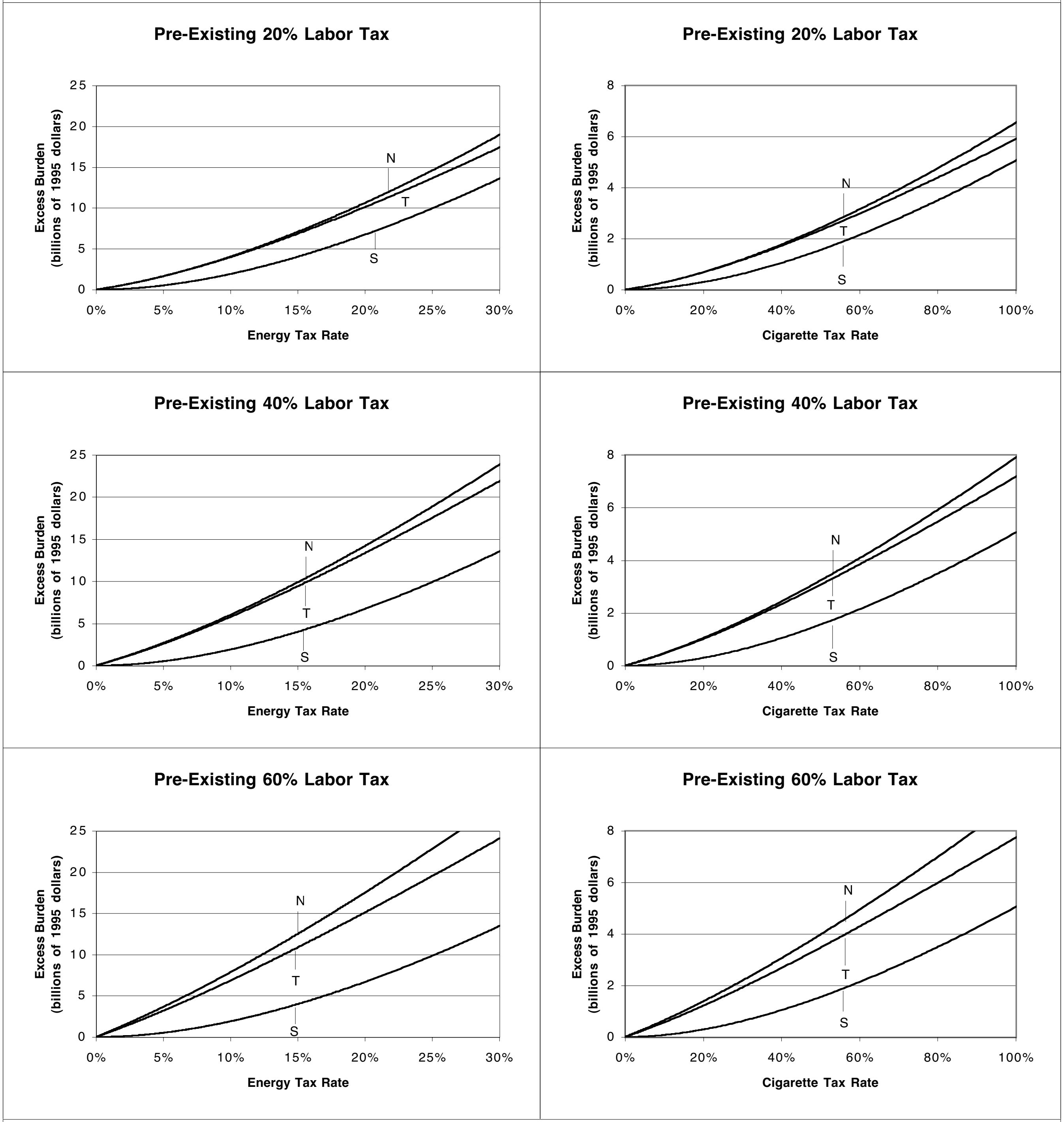
(Excess Burden in Billions of 1995 dollars)

		"True" Excess Burden	Simple Formula: Estimated Excess Burden	Simple Formula: Error	New Formula: Estimated Excess Burden	New Formula: Error
Cigarette Tax						
Elasticity of Demand	Tax/pack					
0.2	14.1%	0.613	0.074	-88.0%	0.640	4.4%
	31.3%	1.486	0.335	-77.5%	1.548	4.2%
	65.3%	3.456	1.264	-63.4%	3.648	5.6%
0.4	14.1%	0.670	0.149	-77.7%	0.696	3.9%
	31.3%	1.728	0.669	-61.3%	1.796	3.9%
	65.3%	4.259	2.473	-41.9%	4.544	6.7%
0.8	14.1%	0.788	0.307	-61.0%	0.813	3.2%
	31.3%	2.195	1.339	-39.0%	2.291	4.4%
	65.3%	5.647	4.735	-16.2%	6.219	10.1%
Energy Tax						
Elasticity of Demand	Tax Rate					
0.6	2.5%	1.168	0.092	-92.1%	1.220	4.5%
	5%	2.460	0.355	-85.6%	2.567	4.3%
	10%	5.345	1.325	-75.2%	5.588	4.6%
0.9	2.5%	1.204	0.139	-88.5%	1.254	4.2%
	5%	2.596	0.533	-79.5%	2.698	3.9%
	10%	5.827	1.982	-66.0%	6.075	4.2%
1.35	2.5%	1.259	0.210	-83.3%	1.307	3.8%
	5%	2.795	0.800	-71.4%	2.896	3.6%
	10%	6.489	2.922	-55.0%	6.771	4.3%

Notes: "Simple Formula" refers to the first formula on page one of text. "New Formula" is equation (25) of text. "True Excess Burden" is the burden resulting from the general equilibrium model of Section IV.A. Cigarette tax rates are equivalent to $0.265, $0.59, and $1.23 per pack.

Figure 1
Comparison of Excess Burden Formulas

Energy Tax	Cigarette Tax



T—"True" Excess Burden (from general equilbrium simulation model)
S—Simple Formula (see page 1)
N—New Formula (see equation 25)