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THE STABILITY OF ORDINAL MEASURES OF ASSOCIATION IN CONTINGENCY TABLES

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ABSTRACT

The stability of several ordinal measures of association are compared when a grid placed on five non-normal bivariate distributions, is varied. The ability of the measures to reproduce their counterpart in the continuous bivariate distribution depends on the proportion of tied pairs of observations and the distribution. Kendall's tau b is more stable than tau c in the tau-type measures and rho c is the most stable of Spearman's rho-type measures. Goodman and Kruskal's gamma overestimates the association in all distributions studied. A new measure which corrects for this is given.

CONTENTS

1. INTRODUCTION

Ordinal measures of association are frequently used to express the relationship between two variables in a contingency table with ordered categories. The measures are adaptations of the ordinal correlation coefficients, Kendall's τ and Spearman's ρ_s (see Goodman and Kruskal [4], Kruskal [6]). The two way contingency table is considered to be a sample from a bivariate continuous population with a large number of tied observations and the adaptations of the measures correct for ties. The aim of the correction is to bring the measure of association in the contingency table as close as possible to the appropriate ordinal correlation coefficient when no ties are present.

For the measure to be satisfactory it should be stable both with respect to the size of the table and the choice of marginals, so that different choices of categorisations of the two variables lead to the same conclusions. Agresti [1] investigated the behaviour of some of the most frequently used measures of association when a grid placed on three bivariate normal distributions with ρ = .2, .5, .8 was varied by changing the marginal probabilities and the number of rows and columns.

In this study we investigate the behaviour of these measures when similar grids are placed on other types of bivariate distributions.

2. BIVARIATE DISTRIBUTIONS USED

Five bivariate distributions were used in the study. They were chosen from simply structured one parameter families of distributions, whose marginals could be fixed in advance. The single parameter represented the association and usually had a simple interpretation. The distributions were the contingency-type distributions of Plackett [8]; the bivariate family of Morgenstern [7] and Farlie [3] and the bivariate logistic distribution of Gumbel [5].

Suppose that X and Y are random variables with distribution functions $F(x)$ and $G(y)$ respectively and let $H(x,y)$ be their joint distribution function. Plackett [8] defined a family of bivariate distributions by considering a bivariate distribution dichotomised at the point (x_0,y_0). If $P_1 = H(x_0,y_0)$, $P_2 = F(x_0) - H(x_0,y_0)$, $P_3 = G(y_0) - H(x_0,y_0)$ and

$P_4 = 1-F(x_0) - G(y_0) + H(x_0,y_0)$ then the coefficient of contingency is defined as

$$(P_1P_4)/(P_2P_3) = \psi$$

For given marginal distributions F and G, the solution H of the quadratic equation

$$\frac{H(1-F-G+H)}{(F-H)(G-H)} = \psi \quad 0 < \psi < \infty$$

defines a bivariate distribution function. For any choice of marginals F and G we may construct a bivariate distribution H and find the correlation coefficient ρ as well as ρ_S and τ. If we perform a monotonic transformation on each of the marginals ρ will change but ρ_S and τ, being invariant under ordinal transformations, remain the same. This means that for our purposes we need only consider a canonical form of H obtained after monotonic probability integral transformations have converted the marginals into uniform distributions over (0,1). The distribution function then reduces to

$$H(x,y,\psi) = (S-(S^2+4\psi(\psi-1)xy)^{\frac{1}{2}})/(2(\psi-1))$$

where

$$S = 1 + (x+y)(\psi-1) \quad .$$

Two distributions with uniform marginals and $\psi = 3$ and $\psi = 15$ were used and we refer to them as Plackett 3 and Plackett 15 respectively. Closed-form expressions for the correlation coefficient and the rank correlation coefficient do not exist, so these were calculated numerically.

The distribution function of the Morgenstern [7] and Farlie [3] distributions has the form

$$H(x,y) = F(x)F(y)[1+\alpha A(F(x)B(G(y))]$$

where α is a parameter $(-1 < \alpha < 1)$ and $A(\cdot)$ and $B(\cdot)$ are bounded functions with bounded first derivatives. We again chose uniformly distributed marginals for the reason given above. Setting $A(x) = B(x) = 1-x$ we obtain the Morgenstern [7] distributions for which $\rho = \alpha/3$, $\rho_s = \alpha/3$ and $\tau = 2\alpha/9$. For these distributions the range of the correlation is restricted to $-1/3 < \rho < 1/3$ and the regression of Y on X, $E(Y|X)$, is linear. We took $\alpha = 0.9$ and refer to this distribution as Morgenstern .9. Secondly we took $A(x) = B(x) = 1-x^2$. In this case $\rho = \rho_s = 3\alpha/4$ and $\tau = \alpha/2$. The regression of Y on X is nonlinear with $E(Y X) = \frac{1}{2} - \alpha(1-3x^2)/4$. In this case also the range of the correlation is restricted to $-3/16 < \rho < 3/8$. We took $\alpha = 0.4$ and refer to this distribution as Farlie 0.4. Finally we used the bivariate logistic distribution Gumbel [5] with distribution function $H(x,y) = (1+e^{-x}+e^{-y})^{-1}$.

The regression of Y on X is nonlinear, $\rho = \frac{1}{2}$, $\tau = 1/3$ and $\rho_s = 0.48$ which was calculated numerically.

The values of the correlation coefficients for these continuous distributions are given in Table 1, (see Appendix I).

3. ORDINAL MEASURES AND GRIDS STUDIED

The six coefficients most often used for measuring association in ordered contingency tables were used in our study. These were also used by Agrestri [1] with an underlying bivariate normal distribution. To allow further comparisons with the normal distribution the table sizes and marginal probabilities adopted by Agresti were used.

Consider a two way contingency table with r ordered row categories and c ordered column categories. Let p_{ij} be the probability that an observation falls in the cell in row i and column j,

$m = \min(r,c)$

$$p_{i.} = \sum_{j=1}^{c} p_{ij} , \quad p_{.j} = \sum_{i=1}^{r} p_{ij} ,$$

$$F_{k.} = \sum_{i=1}^{k-1} p_{i.} + p_{k.}/2 , \quad F_{.k} = \sum_{j=1}^{k-1} p_{.j} + p_{.k}/2 ,$$

$$\mu_1 = \sum_{i=1}^{r} i p_{i.} , \quad \mu_2 = \sum_{j=1}^{c} j p_{.j} ,$$

$$P_c = 2 \sum_{i=1}^{r} \sum_{j=1}^{c} p_{ij} \Big(\sum_{i'>i} \sum_{j'>j} p_{i'j'} \Big) ,$$

$$P_d = 2 \sum_{i=1}^{r} \sum_{j=1}^{c} p_{ij} \Big(\sum_{i'>i} \sum_{j'<j} p_{i'j'} \Big) ,$$

$$P_t = \sum_{i=1}^{r} p_{i.}^2 + \sum_{j=1}^{c} p_{.j}^2 - \sum_{i=1}^{r} \sum_{j=1}^{c} p_{ij}^2 .$$

Then the ordinal measures of association are

$$\gamma = (P_c - P_d)/(1 - P_t) ,$$

$$\tau_b = (P_c - P_d)/\Big(\big(1 - \sum_{i=1}^{r} p_{i.}^2\big)\big(1 - \sum_{j=1}^{c} p_{.j}^2\big)\Big)^{\frac{1}{2}} ,$$

$$\tau_c = (P_c - P_d)/[(m-1)/m],$$

$$R = \sum_{i=1}^{r} \sum_{j=1}^{c} (i-\mu_1)(j-\mu_2) p_{ij} \Big/ \Big(\big(\sum_{i=1}^{r} (i-\mu_1)^2 p_{i.}\big)\big(\sum_{j=1}^{c} (j-\mu_2)^2 p_{.j}\big)\Big)^{\frac{1}{2}}$$

$$\rho_b = \sum_{i=1}^{r} \sum_{j=1}^{c} (F_{i.} - .5) p_{ij} /$$

$$((\sum_{i=1}^{r}(F_{i.}-.5)^2 p_{i.})(\sum_{j=1}^{c}(F_{.j}-.5)^2 p_{.j}))^{\frac{1}{2}} .$$

$$\rho_c = 1 - 6m^2 \sum_{i=1}^{r} \sum_{j=1}^{c} p_{ij}(F_{i.} - F_{.j})^2/(m^2-1) .$$

The three measures based on the proportions of concordant and discordant pairs, P_c and P_d, are extensions to cross-classification tables of Kendall's τ [4] which is defined as $\tau = P_c - P_d$ for a continuous bivariate distribution. $P_t = 1 - P_c - P_d$ is the proportion of pairs of observations that are tied on at least one of the two rankings when the data are grouped. P_t measures the departure from continuity and is virtually unaffected by the choice of bivariate distribution. We shall be particularly interested in how well γ, τ_b and τ_c reproduce τ uncorrected since it is known that for an underlying normal population τ deflates seriously when the data are grouped [1, page 50]. The measures R, ρ_b and ρ_c are based on correlation type measures of association. R is the Pearson product moment correlation using integer row and column scores [10] and ρ_b and ρ_c are extensions of Spearman's rank order correlation coefficient ρ_s [12].

Table sizes used were 2×2, 2×3, 2×4, 2×5, 2×10, 3×3, 3×4, 3×5, 3×10, 4×4, 4×5, 4×10, 5×5, 5×10 and 10×10. It was unnecessary to consider tables with $r > c$ since all the measures are symmetric in this sense. The marginal probabilities were selected from the following:

Categories	
2	(.5,.5), (.4,.6), (.3,.7), (.2,.8), (.1,.9)
3	(.333,.333,.333), (.1,.3,.6), (.25,.25,.50)
4	(.25,.25,.25,.25), (.1,.1,.4,.4)
5	.2 in each category
10	.1 in each category.

For tables of a given size a pair of marginal probabilities was selectec and a set of tables obtained by taking all permutations of the marginal probabilities which gave different values for at least one of the six measures. For example, in the 2×3 table with marginal distributions (.3,.7) and (.5,.25,.25) each measure has the same value as for the table with marginals of (.7,.3) and (.5,.25,.25) so one of these tables was omitted. There were 226 distinct grids used in all and these were placed over each of the five bivariate distributions.

4. GENERAL RESULTS

Summary statistics for all measures averaged over the 226 grids used are given in Table 2. In all distributions the mean value of τ_b was closest to τ. (See Table 2 Appendix I.)

The mean values of the other measures were not as close to their continuous counterparts; γ grossly overestimates τ; R, ρ_b and ρ_c underestimate ρ_s. The standard deviations are very similar with those of τ_b being lowest. A trend worth noting is the difference between the Plackett 3 and Plackett 15 distributions. The τ values of these distributions are .240 and .546 respectively. There is an increase in standard deviation which corresponds fairly closely to the increase in τ. This occurs for all measures except γ and ρ_c. The root mean squared error of the measure about its continuous counterpart is given in Table 2b and shows that τ_b tends to be closest to the parent value for ungrouped data for every distribution, τ_c is next closest and R_c third.

A more informative measure of closeness to the continuous counterpart is given by the slope of the least squares regression line of each measure against

P_t, the proportion of tied pairs. The intercept of the line , at $P_t = 0$, is constrained to equal the continuous value. This measure of closeness was suggested by Agresti [1] and is most informative since it gives both the magnitude and the direction of the deviation of the measure. P_t itself is an indicator of grid size and pattern. It measures departure from continuity and is virtually unaffected by the bivariate distribution. From the slopes of the regression lines given in Table 2d it can be seen that all the measures underestimate the continuous value except γ, which overestimates it considerably. The inflationary behaviour of γ has been noted by many authors, [1], [10], [11]. (See Table 2d Appendix I.)

Agresti [1] noted that $|\tau_b - \tau| \leq \tau/10$ for nearly all the 226 grids placed on a bivariate normal distribution, provided $P_t \leq .75$. Excluding tables with $P_t > .75$ and all 2×2 tables, which will be discussed separately later, we calculated P_e, the percentage of tables for which $|M - \mu|/\mu \leq .1$ where M is a measure and μ is its continuous counterpart. The results are given in Table 3 and show that γ behaves very poorly. Not even the 10×10 table came with 10 percent of the continuous value in any of the five distributions.

(See Table 3 Appendix I.)

τ_b is best among the τ estimators, and ρ_c among the ρ_s estimators. Agresti [1] reported that $|\tau_b - \tau| \leq .1$ for nearly all grids with $P_t \leq .75$ and an underlying normal population. It is clear that τ_b does not behave as well with other underlying distributions, for example in the Morgenstern .9 distribution only 30 per cent of the τ_b lay with this limit, whereas 89 per cent of the τ_b from the Plackett 15 distribution were within the limit.

5. PLOTS OF THE MEASURES VERSUS P_t

For each of the five distributions, the six measures were plotted against P_t, the proportion of tied pairs. Plots of the estimators of τ for the Plackett 3, Plackett 15, Morgenstern .9 and Logistic are given in figures A, B, C and D. Plots of the estimators of ρ_s for the Morgenstern .9 and Logistic distributions are given in figures E and F. These plots are most informative, since it is seen immediately that convergence to the appropriate continuous value is curvilinear rather than linear, and also that all the measures, apart from γ, can be both above and below the continuous value. γ lies wholly above, confirming the inflationary nature of γ.

In general, from the plots, γ grossly overestimates τ in all cases and converges in a convex fashion as $P_t \to 0$. It can be seen that the convergence patterns of γ possess a distinct upper bound and this suggests that a correction factor could be found to improve the convergence to τ. This will be pursued in a later section.

τ_b underestimates τ if P_t is large but has the attractive property that for tables of any dimensions, the values of τ_b are close to τ if the marginals of the tables are uniform. Overall τ_c behaves more extremely than τ_b showing larger deviations both above and below the τ value. Moreover the values of τ_c follow definite branches in all distributions studied. This is due to the strong effect of the discontinuous term, $m = \min (r,c)$ in the definition of τ_c. This effect is especially prominent if m is small.

The patterns are not entirely homogenous over the distributions. With an underlying logistic distribution the values are far more scattered, but in the Morgenstern distribution the measures show well defined trends. None of the distributions produce patterns as smooth as those observed by Agresti [1] in his study of the bivariate normal. Thus one should be aware that these measures are not "distribution free".

R and ρ_b behave very similarly in all cases and tend to increase steadily to ρ_s as $P_t \to 0$. Agrestri [1] observed that ρ_c was inflated for small m. We observe the reverse, especially for the Morgenstern .9 distribution. In addition ρ_c and τ_c show the same branching tendencies.

6. 2×2 TABLES

The 2×2 contingency tables are a special case. Although in these tables the concept of order falls away, ordinal measures are widely used to express the association between two variables. γ is constant for 2×2 tables of the Plackett distributions, since

$$\gamma = (\psi - 1)/(\psi + 1) \qquad 6.1$$

where

$$\psi = (p_{11}p_{22})/(p_{12}p_{21})$$

$\gamma = .5$ for the Plackett 3 and .875 for the Plackett 15 distribution. It can also be shown for 2×2 tables that

$$\tau_b = \rho_b = R = (p_{11}\, p_{22} - p_{12}\, p_{21})/(p_{1.}\, p_{2.}\, p_{.1}\, p_{.2})^{\frac{1}{2}}$$

The values of all the measures fluctuate greatly for 2×2 tables and are generally unstable. To demonstrate this instability, figure G shows the 25 values of τ_b

for all the 2×2 tables from the logistic distribution. τ_b was chosen since it is the most stable of all the measures and lies closest to τ in all respects, so instabilities demonstrated for τ_b will be worse for the other measures. The logistic distribution was chosen since the spread of the measures was larger for this distribution than the others. The τ value for the logistic distribution is .33. The value of τ_b fluctuated between .1 and .5 as P_t went from .7 to .1. Figure G also shows interesting branches in the pattern of τ_b values. They are linked by dotted lines and all cross the true τ line at some point. Each branch is labelled by one of the marginals of the 2×2 table, for example (.1,.9). Each of the tables in this branch has as one of its marginals the pair (.1, .9). The circled element has as its other marginal (.5, .5) and points further from this element have more skewed marginals. The curve formed by the circled elements matches the characteristic convergence curve of τ_b. Similar plots were made for the Plackett 15 and Plackett 3 distributions. Although these showed less variability, the same patterns were discernible and more marked for larger τ_b values. Moreover, these patterns show that τ_b can estimate the τ value fairly well in a 2×2 contingency table provided both marginals are close to uniform. Once they become more skewed than (.4, .6) τ_b becomes extremely unreliable and particularly so if both marginals are skew. Similar patterns were observed in 3×3 tables, but the branching was not as distinct because of the more complex marginal structures of these tables. In a practical situation, if it is possible to choose the marginal categorisation, we would recommend making them as close to uniform as possible.

7. TABLES WITH UNIFORM MARGINALS

A useful way of examining the effect of the number of categories is to keep both the row and column marginals uniform. So for a r×c table the marginal probabilities will be 1/r and 1/c for the row and column categories respectively. Under these conditions $R = \rho_b$ and

$$1/\min(r,c) \leq P_t \leq 1/r + 1/c - 1/rc \; . \quad (7.1)$$

So if r and c tend to infinity and the grids placed on the bivariate distribution satisfy the marginal constraints, $P_t \to 0$, $P_c \to \pi_c$, $P_c \to \pi_d$, where π_c and π_d are the true probabilities of concordance and discordance respectively, then γ, τ_b and τ_c all converge to τ. Under these circumstances $\rho_b = R$ converges to the Pearson correlation of the two marginal distribution functions, F(x) and G(y), which is ρ_s. Also ρ_c converges to $1 - 6E(F(x) - G(y))^2 = \rho_s$.

Thus with uniform marginals all the measures converge under these conditions. Twenty-five tables had uniform marginals and the values for each measure were plotted against P_t. The patterns obtained were very similar for all distributions and the τ estimators for the Plackett 3 and ρ_s estimators for the Morgenstern .9 are shown in Figures H and I, respectively. γ is consistently too large, but τ_b stays close to τ for all r and c. The elements of τ_c that are circled are from 2×c tables and it is interesting to note that as c increases, the τ_c moves further from τ. This happens for 3×c tables also. With ρ_c, as the number of columns, c, increases the estimates for the 2×c tables fall steadily away from ρ_s. The branching phenomenon noticed with τ_b in the 2×2 tables also appears with ρ_c. This is due to the discontinuous term, m, in the definitions of τ_c and ρ_c. The fact that the value deteriorates as c increases for each fixed value of r, leaves us with little confidence in either of these measures. R and ρ_b lie very close to ρ_s much in the same way as τ_b does.

In general under the conditions of uniform marginals all the measures except ρ_c and τ_c behave consistently and converge to the parent values. γ, however, only becomes a good estimator if P_t is small, but it remains consistently above the τ value.

8. A NEW MEASURE OF ASSOCIATION

Although γ fared worse than all the measures as regards closeness to τ, it was always consistently above the τ value whereas the other measures fluctuated around τ. γ also had an upper bound in all distributions studied. When plotted against P_t the form of the convergence curve appeared to be exponential and it seemed that a correction could be applied to improve the convergence. The definition of γ is

$$\gamma = (P_c - P_d)/(1 - P_t) \qquad (8.1)$$

Various corrections were applied but the one found to give the best results was

$$\gamma' = (P_c - P_d)/(1 - P_t^2) \qquad (8.2)$$

Plots of γ' against P_t are given in Figures J and L for the Plackett3 and the Logistic distributions. Values of $(MSE)^{\frac{1}{2}}$ and the slope of the least squares line are given in Table 4. For low vlaues of P_t, γ' lies on the τ line and for high values it is fairly symmetrically distributed around this line. This holds for all distributions except the Plackett 15 where there is a tendency to drop below τ for large values of P_t.

(See Table 4 Appendix I.)

Eq.8.2 can be written as

$$\gamma' = (P_c - P_d)/(P_c + P_d)(2 - P_c - P_d)$$

When $P_d = 0$, $\gamma' = 1$ and $\gamma' = 1/(2 - P_c)$. When $P_t = 0$ and $P_d = 0$, γ' will be 1, otherwise it will be less than 1. For larger values of P_t the limits of γ' lie inside (-1, 1). This is clearly evident in the plot of γ' versus P_t for the Plackett 15 distribution. Thus γ' tends to be conservative in the case of high association and high P_t. However, this type of deviation is more acceptable than those of the other measures considered under the same circumstances. Based on this, we recommend γ' as a reliable measure of association.

APPENDIX I TABLES

1. Correlation coefficients

Distribution	ρ	τ	r_s
Plackett 3	.35	.24	.35
Plackett 15	.73	.55	.73
Morgenstern .9	.30	.20	.30
Farlie .4	.30	.20	.30
Logistic	.50	.33	.48

2. Summary Statistics of Measures and their Relationship to P_t, for 226 Grids Placed on Five Distributions

Distribution	τ	γ	τ_b	τ_c	ρ_s	R	ρ_b	ρ_c
				a. Mean				
Plackett 3	.240	.434	.241	.227	.352	.261	.264	.326
Plackett 15	.546	.820	.506	.477	.728	.540	.547	.589
Morgenstern .9	.200	.374	.204	.192	.300	.222	.224	.290
Farlie .4	.200	.341	.188	.179	.300	.203	.208	.276
Logistic	.333	.621	.353	.329	.478	.389	.385	.434
				b. Standard deviation				
Plackett 3		.042	.041	.064		.052	.053	.067
Plackett 15		.040	.100	.141		.116	.118	.073
Morgenstern .9		.047	.034	.053		.044	.044	.074
Farlie .4		.055	.045	.059		.054	.056	.076
Logistic		.105	.049	.075		.066	.059	.065
				c. $(M.S.E.)^{\frac{1}{2}}$ about underlying measure				
Plackett 3		.198	.041	.065		.105	.103	.071
Plackett 15		.277	.107	.157		.221	.216	.158
Morgenstern .9		.180	.034	.054		.089	.088	.074
Farlie .4		.151	.046	.062		.111	.108	.079
Logistic		.306	.053	.075		.111	.111	.079
d. Slope of l.s. line with intercept equal to underlying measure								
Plackett 3		.287	-.009	-.033		-.145	-.141	-.031
Plackett 15		.418	-.089	-.142		-.315	-.305	-.225
Morgenstern .9		.259	-.002	-.023		-.123	-.119	-.005
Farlie .4		.204	-.027	-.044		-.151	-.146	-.027
Logistic		.445	-.022	-.022		-.142	-.148	-.061

3. The percentage of tables with correlation within 10 percent of the continuous value

Distribution	p^a	γ	τ_b	τ_c	R	ρ_b	ρ_c
	Measure						
Plackett 3	73	0	52	53	11	14	60
Plackett 15	76	0	89	58	12	15	23
Morgenstern .9	71	0	30	47	12	15	55
Farlie .4	71	0	50	50	11	12	49
Logistic	73	0	50	46	29	21	50

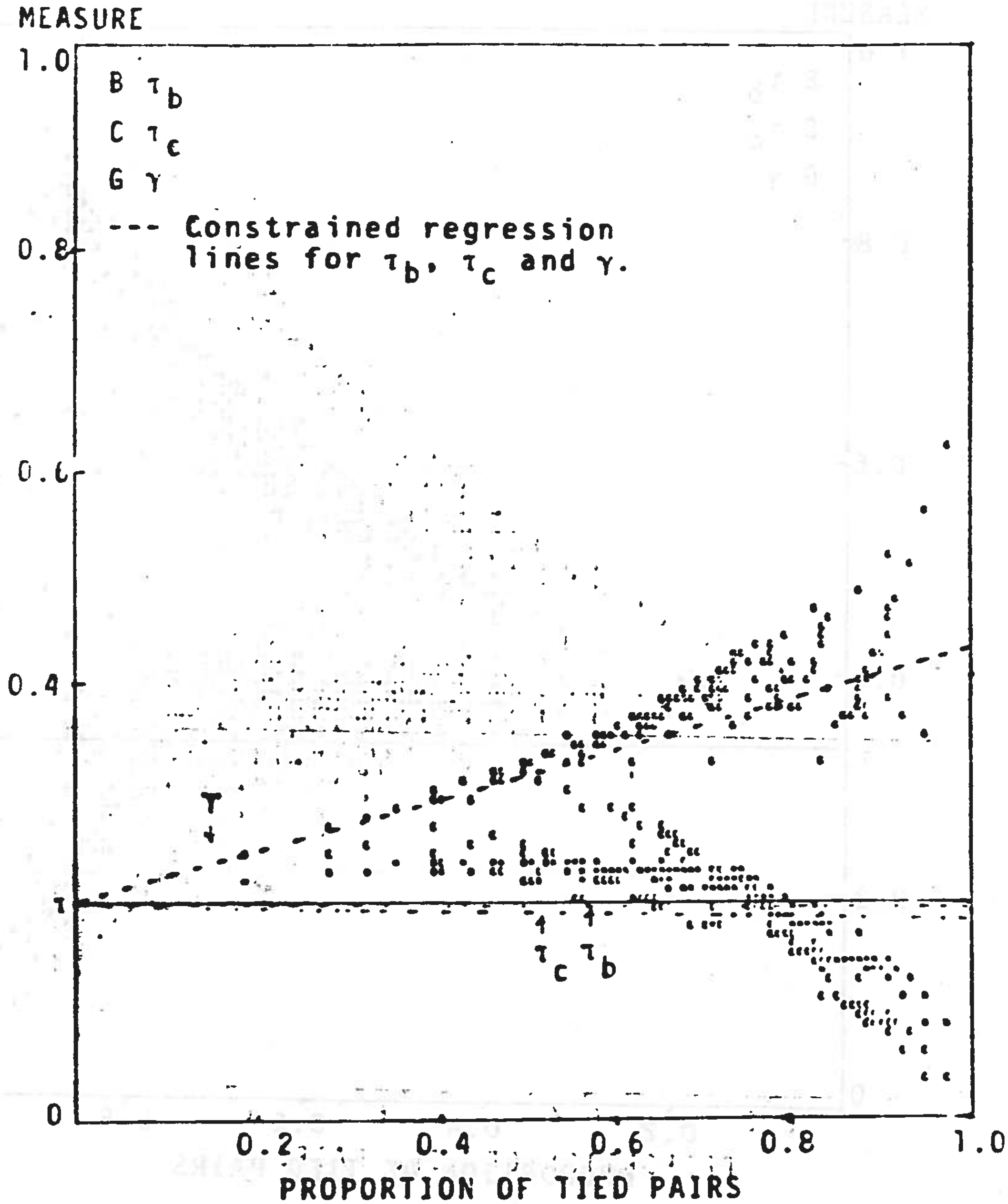
[a] p is the percentage of tables with $P_t \leq .75$.

4. Comparison of γ, γ' and τ_b

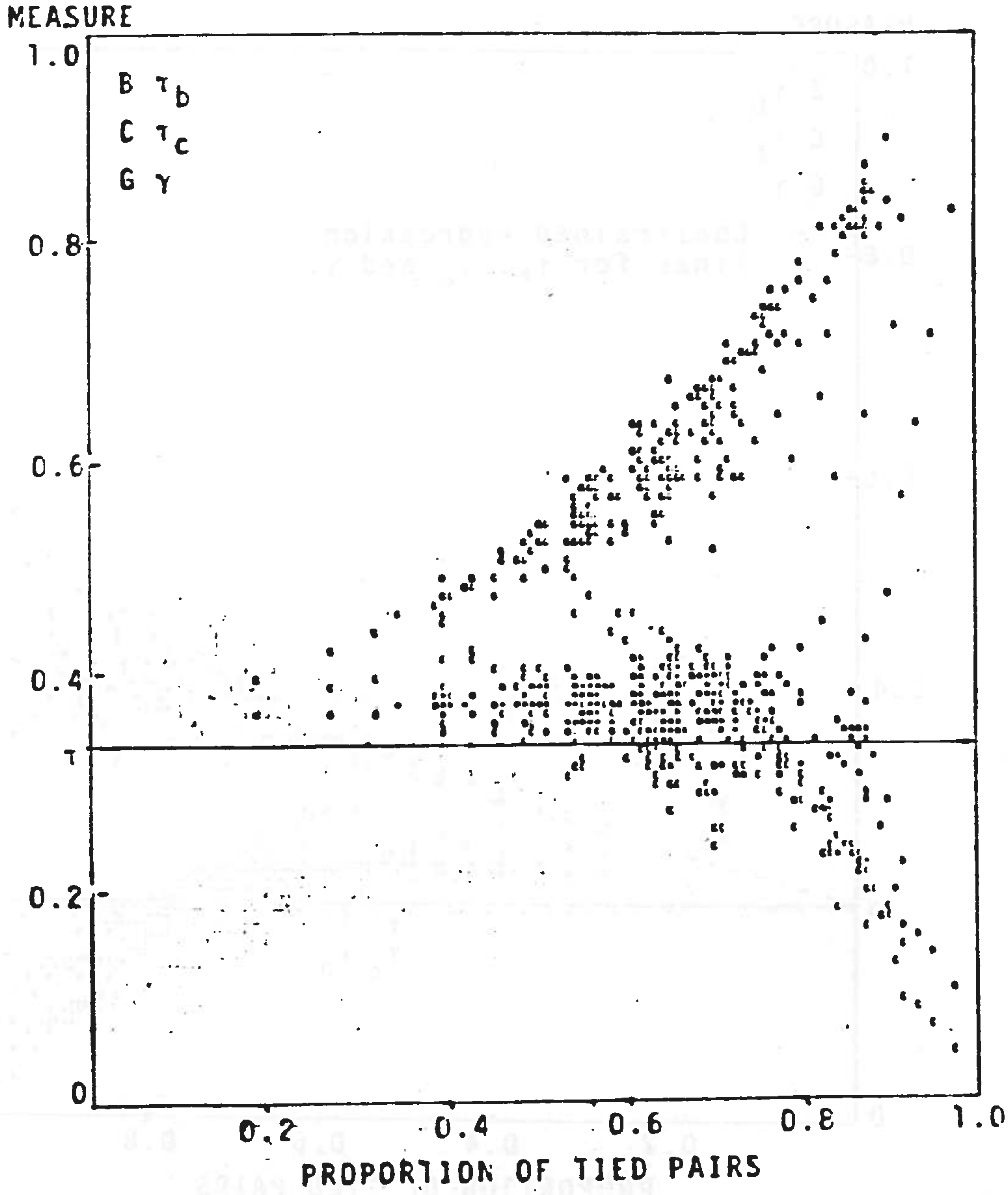
Distribution	Measure γ	γ'	τ_b
	a.	$(MSE)^{\frac{1}{2}}$	
Plackett 3	.198	.022	.041
Logistic	.306	.052	.053
	b.	Slope of least squares regression line	
Plackett 3	.287	.028	-.009
Logistic	.445	.063	.022

APPENDIX II FIGURES

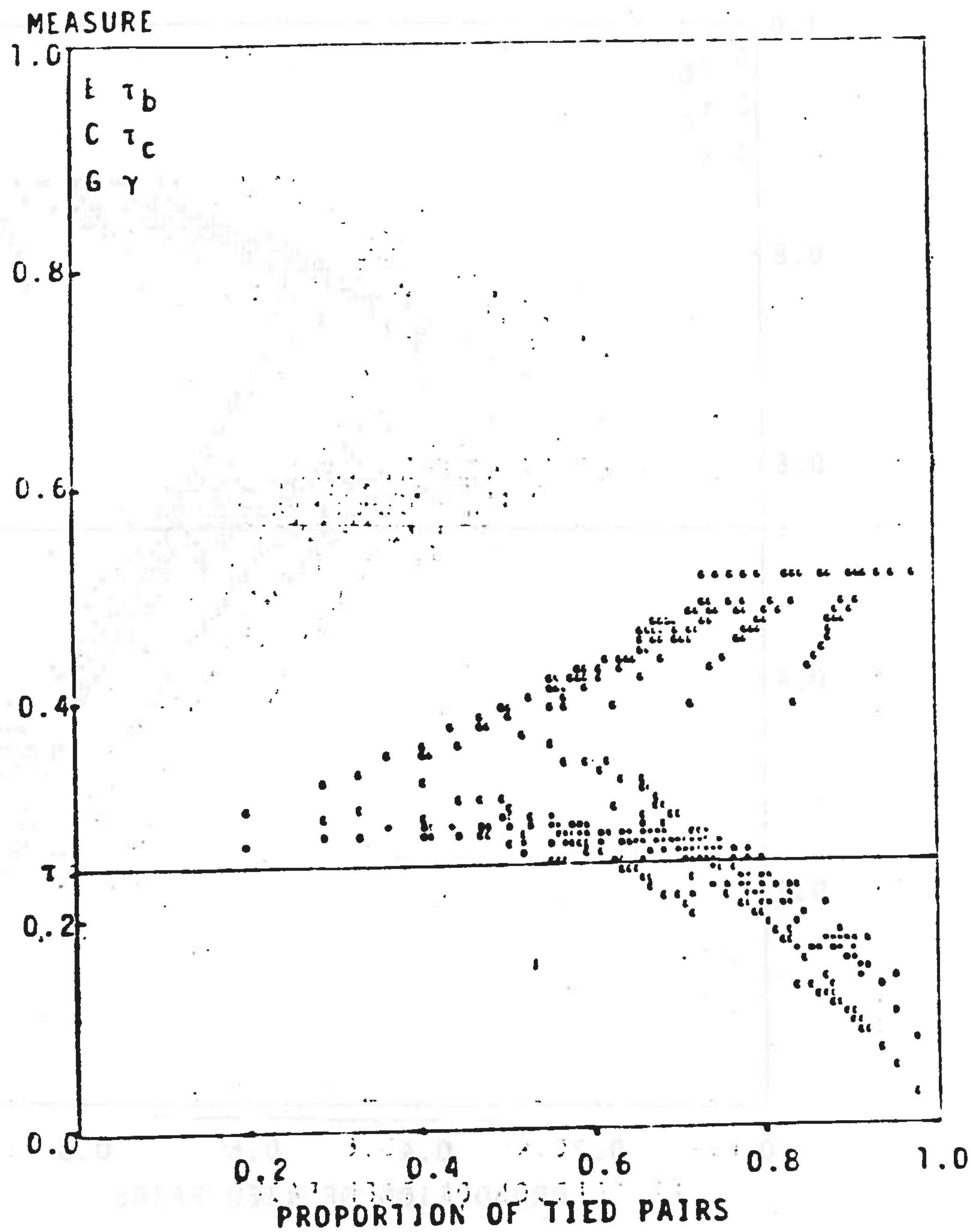
A. τ_b, τ_c and γ against P_t for tables from the Morgenstern .9 distribution.



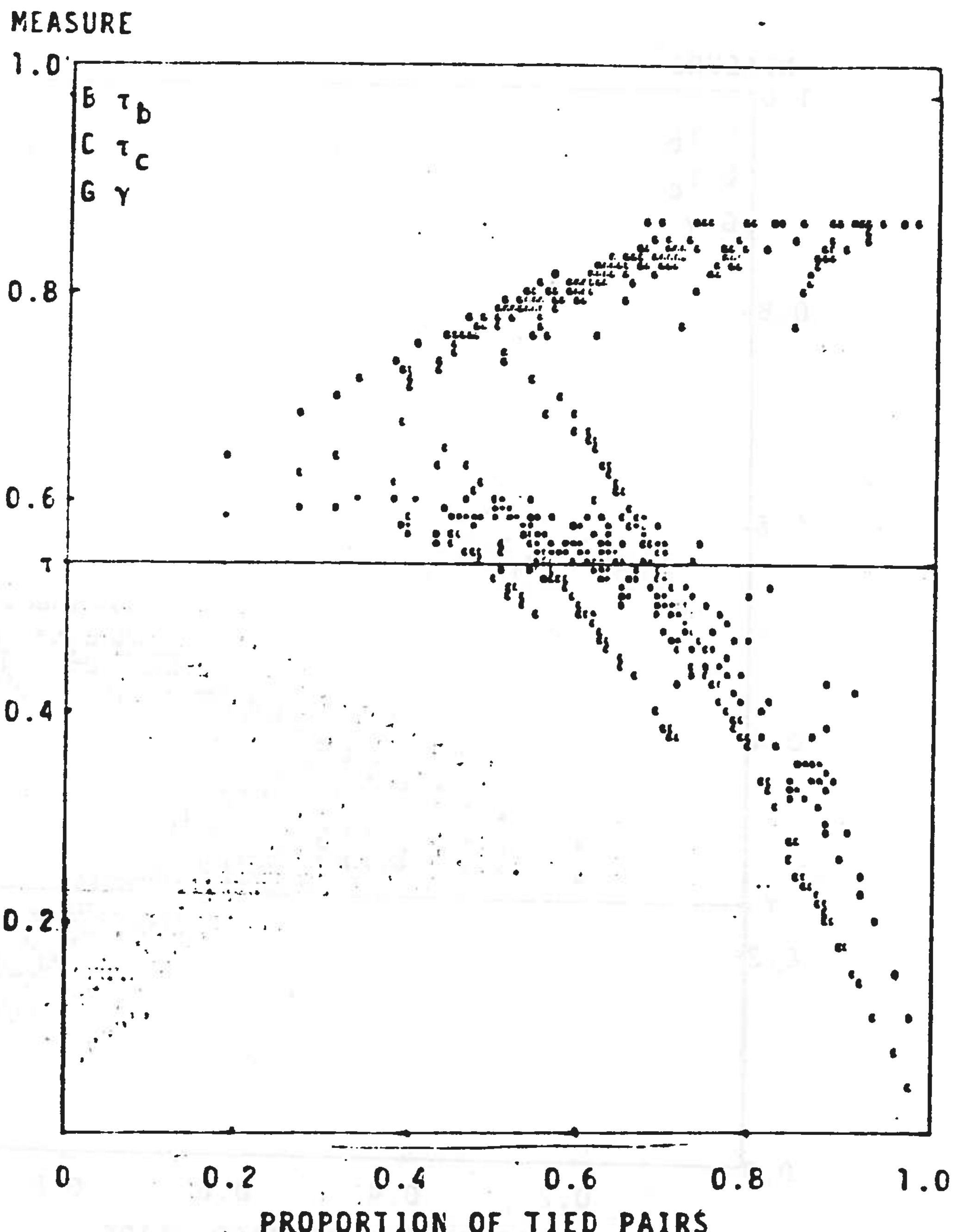
B. τ_b, τ_c and γ against P_t for tables from the Logistic distribution.



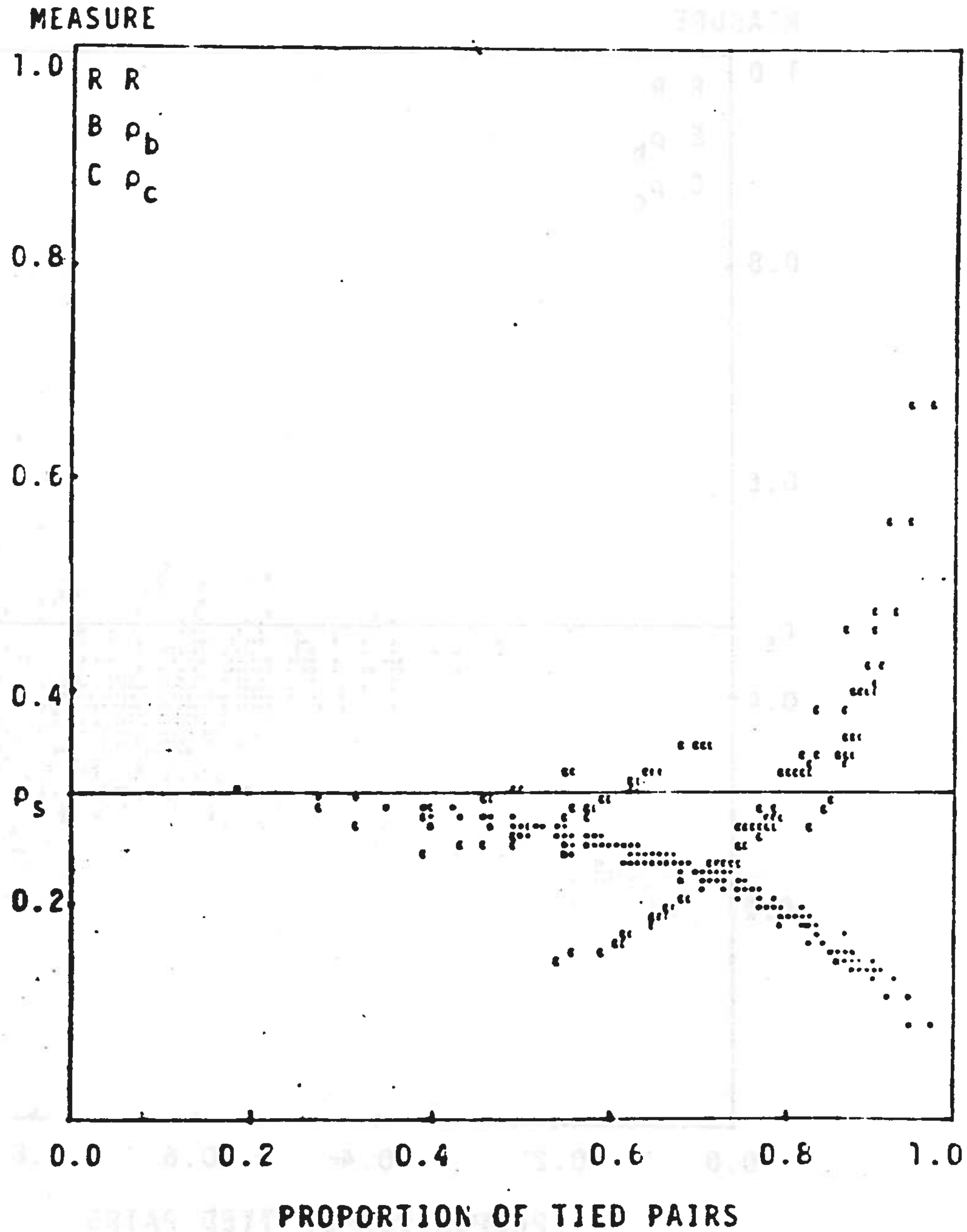
C. τ_b, τ_c and γ against P_t for tables from the Plackett 3 distribution.



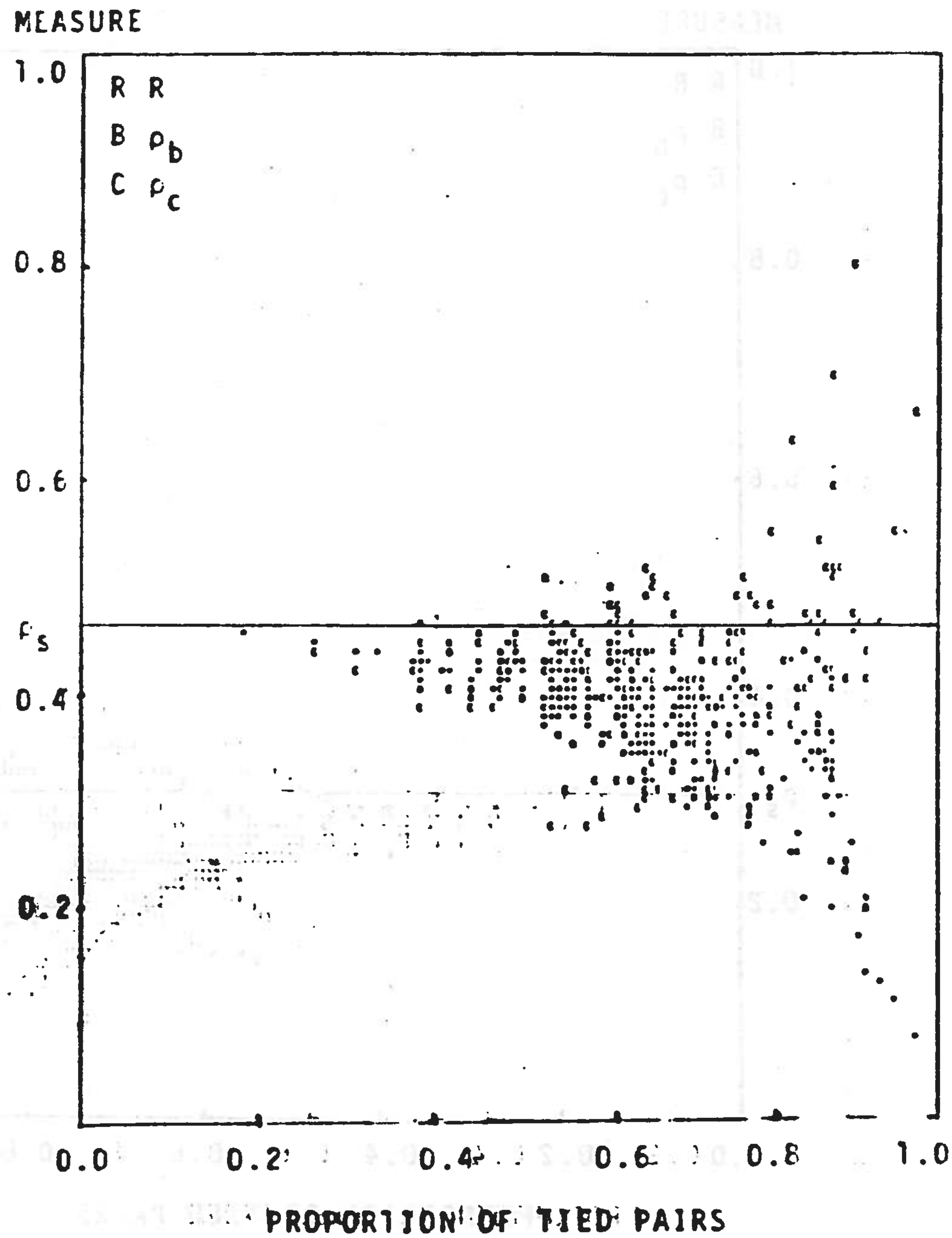
D. τ_b, τ_c and γ against P_t for tables from the Plackett 15 distribution.



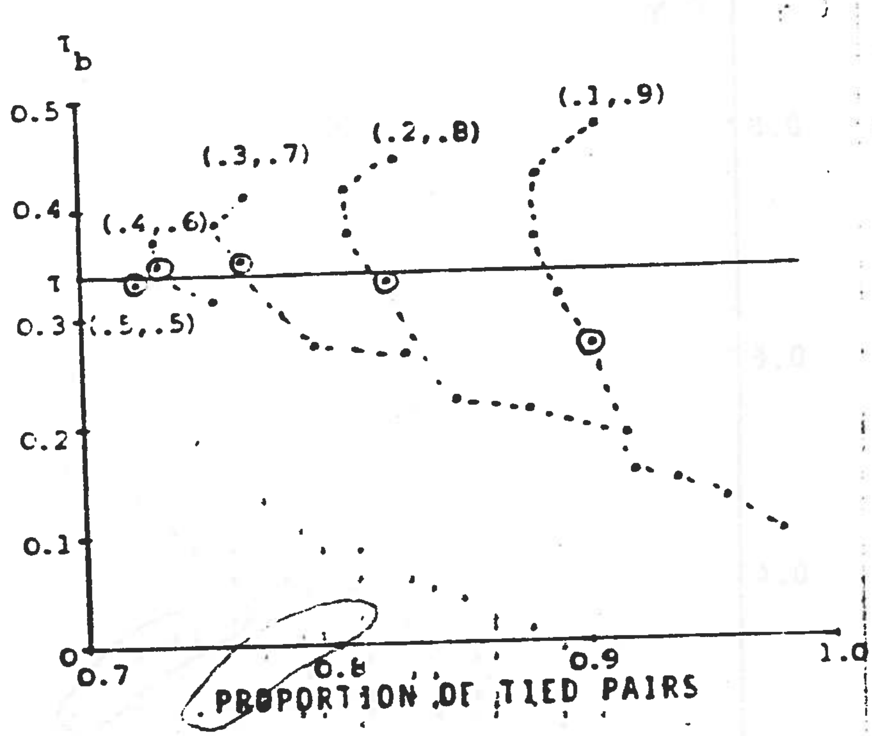
E. R, ρ_b and ρ_c against P_t for tables from the Morgenstern .9 distribution.



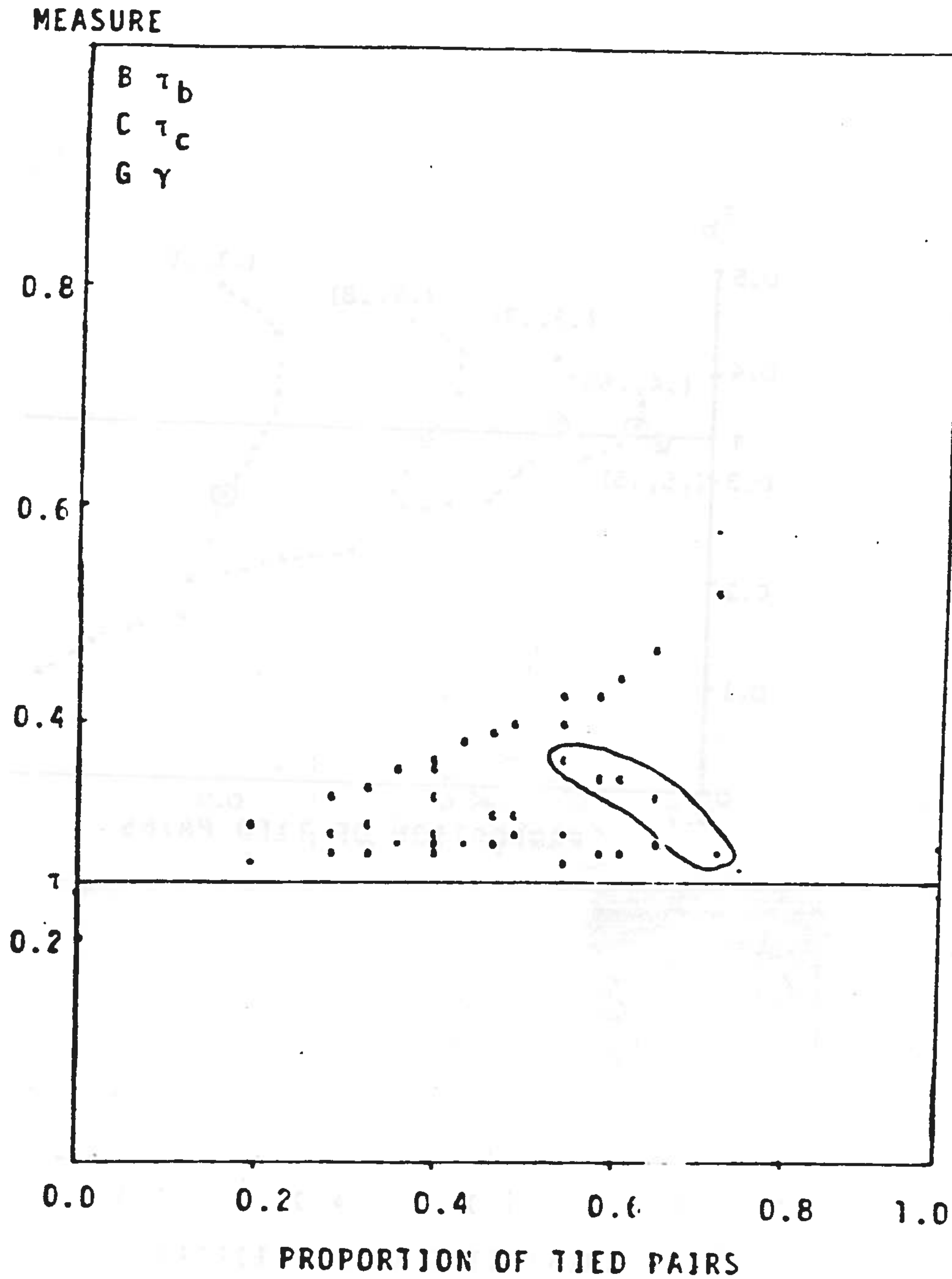
F. R, ρ_b and ρ_c against P_t for tables from the Logistic distribution.



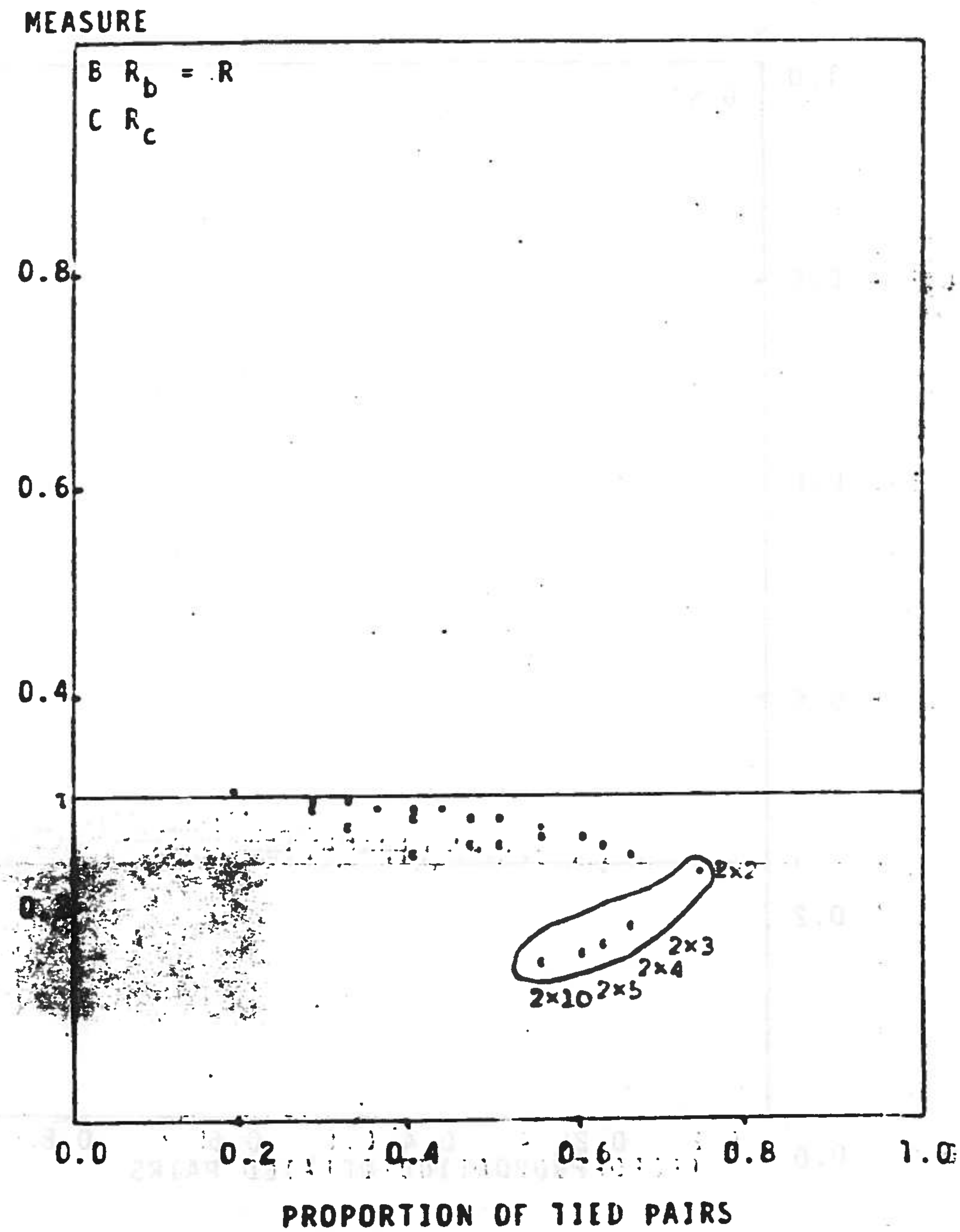
6. τ_b in 2×2 tables formed from the Logistic distribution versus P_t



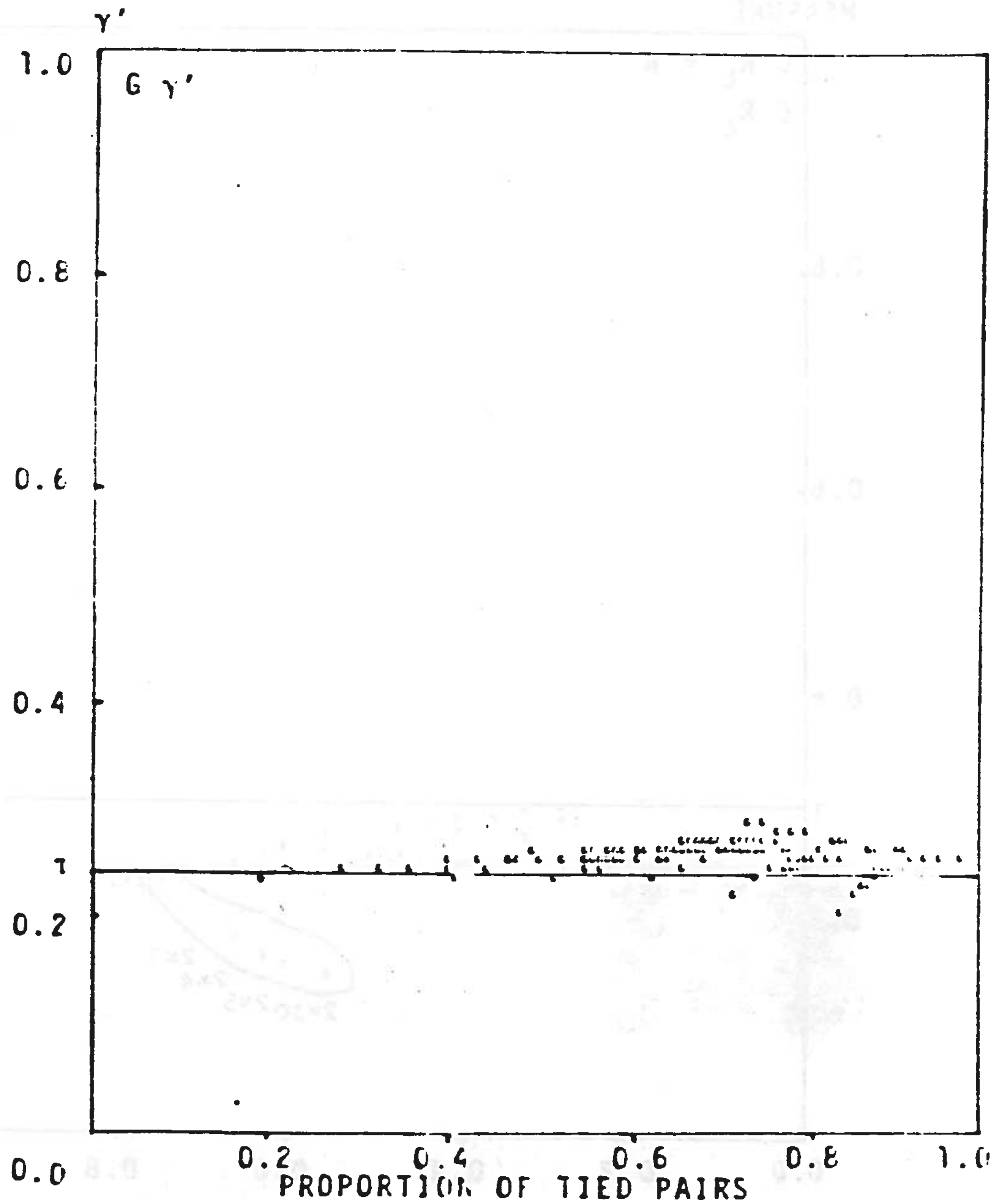
H. γ, τ_b and τ_c versus P_t in tables with marginals 1/r and 1/c formed from the Plackett 3 distribution.



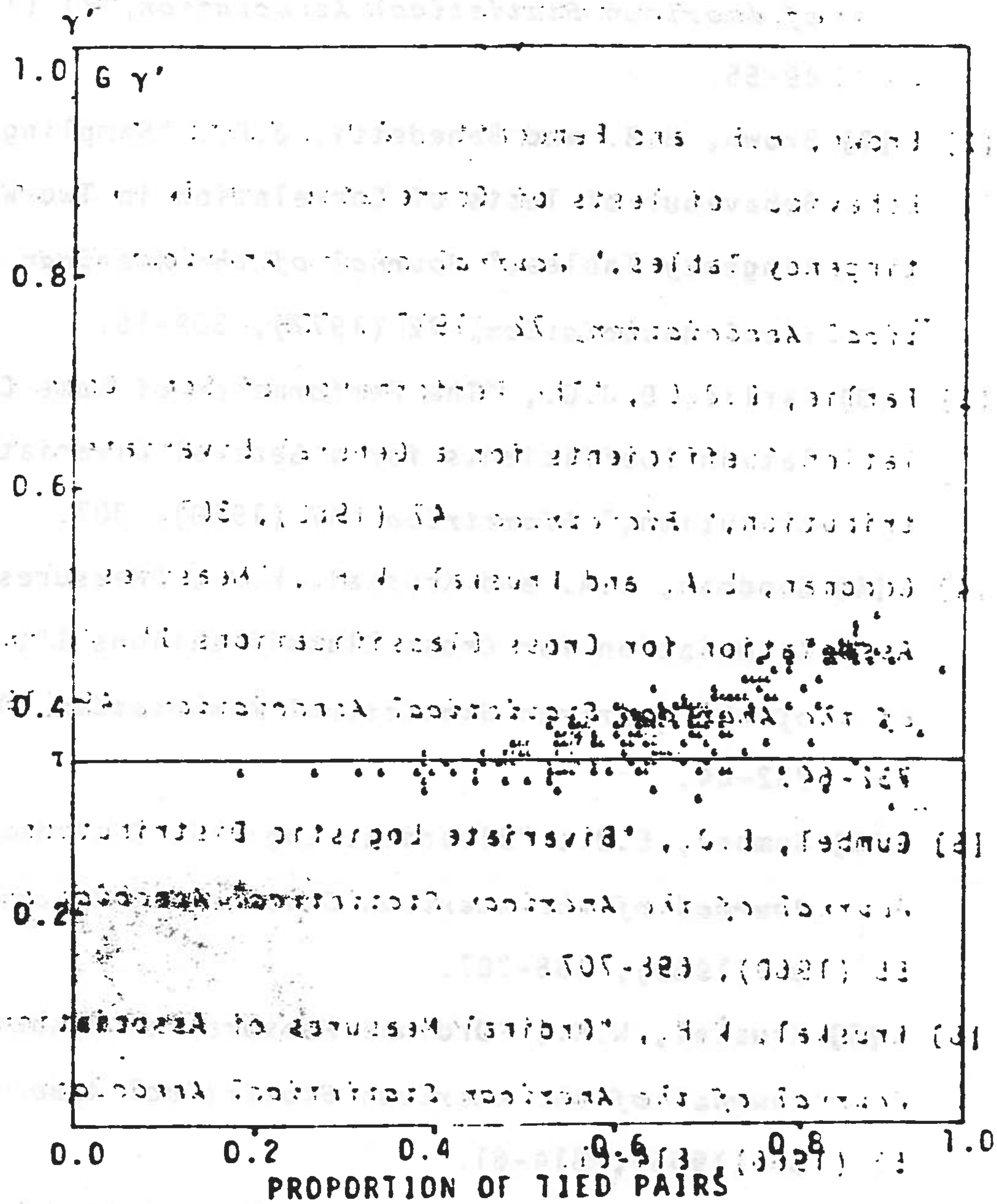
I. ρ_b and ρ_c against P_t in tables with marginals 1/r and 1/c formed from the Morgenstern .9 distribution.



j. γ' versus P_t for tables from the Plackett 3 distribution.



K. γ' versus P_t for tables from the Logistic distribution.



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