Anti-Aliasing Filters for Coupled
Reynolds-Averaged/Large-Eddy
Simulations

J. U. Schlüter and H. Pitsch

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Center for Turbulence Research
Stanford University
Stanford, CA 94305-3030

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Abstract

The increasing complexity of engineering problems makes the coupling of multiple simulation codes attractive. In fluid mechanical applications, the physical range of flow phenomena that can be modeled can be extended significantly by coupling flow solvers based on the Reynolds-averaged Navier Stokes (RANS) approach and on Large-Eddy Simulations (LES). These separate flow solvers run simultaneously and exchange information at the interface. However, since the LES flow solver operates usually with a much smaller time-step, the LES data has to be sampled in order to provide data for the RANS flow solver. In the sampling process aliasing errors can occur. This study investigates possibilities in order to suppress aliasing errors while preserving the amplitude and phase of the long wave spectrum.
Nomenclature

Flow parameters:

- \( D \): Diameter of the pipe
- \( f \): Frequency
- \( Re \): Reynolds number
- \( Sr \): Strouhal number
- \( S \): Swirl number
- \( t \): Time on the LES time scale
- \( \tau \): Time on the RANS time scale
- \( u_c \): Convective velocity
- \( x, r, \phi \): Coordinates in axial, radial and azimuthal direction
- \( u_x, u_r, u_\phi \): Velocity components in axial, radial and azimuthal direction
- \( x_0 \): Location of the interface point

Filter parameters:

- \( a_m, b_n \): Filter constants
- \( H(\lambda) \): Desired filter response
- \( \lambda \): Non-dimensional frequency
- \( N, M \): Order of the filter
- \( r(t_k) \): Filter response on the time grid \( t_k \)
- \( s(t_k) \): Original signal on the time grid \( t_k \)
1 Motivation

Currently, a wide variety of flow phenomena are addressed with numerical simulations. Many flow solvers are optimized to simulate a limited spectrum of flow effects effectively, such as single parts of a flow system, but are either inadequate or too expensive to be applied to a very complex problem.

As an example, the flow through a gas turbine can be considered. In the compressor and the turbine section, the flow solver has to be able to handle the moving blades, model the wall turbulence, and predict the pressure and density distribution properly. This can be done by a flow solver based on the Reynolds-Averaged Navier-Stokes (RANS) approach[1] . On the other hand, the flow in the combustion chamber is governed by large scale turbulence, chemical reactions, and the presence of fuel spray. Experience shows that these phenomena require an unsteady approach[2] . Hence, for the combustor the use of a Large-Eddy Simulation (LES) flow solver is desirable[3] .

While many design problems of a single flow passage can be addressed by separate computations, only the simultaneous computation of all parts can guarantee the proper prediction of multi-component phenomena, such as compressor/combustor instability and combustor/turbine hot-streak migration. Therefore, a promising strategy to perform full aero-thermal simulations of gas-turbine engines is the use of a RANS flow solver for the compressor section, an LES flow solver for the combustor, and again a RANS flow solver for the turbine section (Fig. 1).
The approach to couple simulation codes has been done already in other areas of application, most notably in global climate simulations [4], and found recently more attention in the engineering community[5]. However, the idea to couple RANS and LES flow solvers is a very recent approach. It is a unique method to construct an LES-RANS hybrid. While other LES-RANS hybrid approaches, such as Detached-Eddy Simulations (DES) [6] and Limited-Numerical Scales (LNS) [7] combine LES and RANS in a single flow solver, the approach to couple two existing flow solvers has the distinct advantage to build upon the experience and validation that has been put into the two codes during their development.

A demonstration for an applied coupled RANS-LES computation is the simulation of the flow development downstream of a compressor stage [8] (Fig. 2). The interactions between the compressor and the prediffuser of the combustor can be studied. Here, the compressor stage is computed with a RANS flow solver, since the wall bounded flows around the stator and the moving rotor can be efficiently predicted with this approach. The diffuser on the other hand is computed with an LES flow solver, since only this approach can assure the prediction of detachments in this portion of the flow. The flow solvers are two-way coupled, which means that both flow solvers communicate the flow information at the interface to the peer flow solver.

In order to ensure the information transfer of flow properties in such complex applications, validation studies have to be performed. Recent work was focusing on boundary conditions. The present work
will focus on the dynamic properties of the data exchange. It will
demonstrate the presence of aliasing and will present strategies to
attenuate the aliasing error.

## 2 Interface Conditions

The simultaneous computation of the flow in all parts of a gas tur-
bine different flow solvers requires an exchange of information at the
interfaces of the computational domains of each part. Previous work
has established algorithms, which ensure, that two or more simulta-
neously running flow solvers are able to exchange the information at
the interfaces efficiently\cite{9, 10}.

The necessity of information exchange in the flow direction from
the upstream to the downstream flow solver is obvious: the flow in a
passage is strongly dependent on mass flux, velocity vectors, and tem-
perature at the inlet of the domain. However, since the Navier-Stokes
equations are elliptic in subsonic flows, the downstream flow conditions
can have a substantial influence on the upstream flow development.
This can easily be imagined by considering that, for instance, a flow
blockage in the turbine section of the gas turbine can determine and
even stop the mass flow rate through the entire engine. This means
that the information exchange at each interface has to go in both,
downstream \textit{and} upstream, directions.

Considering an LES flow solver computing the flow in the com-
bustor, information on the flow field has to be provided to the RANS
flow solver computing the turbine as well as to the RANS flow solver computing the compressor, while at the same time, the LES solver has to obtain flow information from both RANS flow solvers (Fig. 3).

The coupling can be done using overlapping computational domains for the LES and RANS simulations. For the example of the compressor/combustor interface this would imply that inflow conditions for LES will be determined from the RANS solution at the beginning of the overlap region, and correspondingly the outflow conditions for RANS are determined from the LES solution at the end of the overlap region [8].

However, the different mathematical approaches of the different flow solvers make the coupling of the flow solvers challenging. Since LES resolves large-scale turbulence in space and time, the time step between two iterations is relatively small. RANS flow solvers average all turbulent motions over time and predict ensemble averages of the flow. Even when a so-called unsteady RANS approach is used, the time step between two ensemble-averages of the RANS flow solver is usually larger by several orders of magnitude than that for an LES flow solver.

The smaller time-step of the LES flow solver results in the necessity of the LES flow solver to filter its own LES data over the RANS time-step in order to provide data for the RANS flow solver at the requested times. However, the sampling process may introduce errors in the frequency spectra.

As an analogy, the digitization of an continuous signal during ex-
periments can be seen. Here, the highest frequency recorded without error is the Nyquist frequency, defined as half of the sampling frequency. Experimentalists use low pass filter in order to remove high frequency disturbances prior to the sampling process. Omitting the filtering would result in aliasing errors, that means, under-resolved frequencies \( f > f_{\text{NYQUIST}} \) would be found in the long wave spectrum. Hence, the low-pass filtering prior to the digitization is necessary.

For the communication between LES and RANS flow solver a similar procedure has to be developed in order to avoid aliasing of frequencies in the sampling of the LES data. The current study investigates the use of filters to ensure the communication of dynamic properties, such as the frequency, the amplitude and the phase of a given perturbation.

## 3 Test-Case and Flow Solver

For this investigation, the transfer of dynamic properties between two flow solvers has been studied. The idea is to excite periodically the flow in the upstream domain and observe the information transfer to the downstream domain.

Fig. 4 shows the considered test case: a pipe is split into an upstream domain computed by one flow solver and a downstream domain computed by another flow solver. Both pipe segments are 3 diameter \( D \) long with an overlap of 1\( D \).

For demonstration purposes laminar flow conditions have been cho-
The Reynolds-number is \( Re = 1000 \) based on the bulk velocity and the diameter of the pipe. Laminar conditions have been chosen for clarity, since in this case turbulence is not interfering with the periodic excitation. A turbulent test case is examined in the last chapter.

In the present study, the downstream RANS flow solver is replaced by an LES flow solver for simplicity reasons. However, for the scope of this study it is not important to use a downstream flow solver at all, since the data acquisition on the upstream LES flow solver is examined. The use of a second flow solver was done solely for practical purposes. In order to simulate a RANS flow solver, the two simultaneously running LES flow solvers exchange information at a chosen RANS time step \( \Delta \tau \), which is larger by more than one order of magnitude than the LES time step \( \Delta t \).

The inflow of the upstream pipe was defined as a laminar parabolic inflow in axial direction and a parabolic profile in azimuthal direction, thus simulating a laminar swirling pipe flow. The swirl number of this pipe flow was \( S = 0.15 \) with:

\[
S = \frac{1}{D} \int_0^D r^2 u_x \bar{u}_\phi \, dr = \frac{1}{D} \int_0^D r u^2 \, dr,
\]

with \( u_x \) the axial velocity component, \( u_\phi \) the azimuthal velocity component, and \( D \) the diameter of the pipe. In order to simulate a convective wave, the azimuthal velocity component has been modulated by:

\[
\bar{u}_{\phi,\text{forcing}}(t) = \bar{u}_{\phi,\text{mean}} \cdot [1.0 + 0.3 \cdot \sin(2\pi \cdot Sr \cdot t)]
\]

with \( \bar{u}_{\phi,\text{mean}} \) the mean azimuthal velocity for \( S = 0.15 \), and \( Sr \) the Strouhal number defined as \( Sr = fD/U_{\text{bulk}} \).
The choice was made to modulate the swirl velocity rather than the axial velocity, since this simulates a convective wave without changing temporarily the bulk velocity.

The LES flow solver developed at the Center for Turbulence Research[11] has been used. The flow solver solves the filtered momentum equations with a low-Mach number assumption on an axi-symmetric structured mesh. A second-order finite-volume scheme on a staggered grid is used [12]. The sub-grid stresses are approximated by a dynamic procedure [13, 14].

For the real-time exchange of flow variables during the simultaneous computation of both domains, an interface has been used [9, 10] developed at the Stanford University. The interface establishes a communication between the two flow solvers and lets the flow solvers exchange flow variables after a given time-step $\Delta\tau$. Each of the flow solvers obtains a data set of flow variables for each point at the boundary. Then, each flow solver defines its own boundary conditions on the basis of the obtained data.

LES inflow boundary conditions for turbulent calculations can be defined using the mean velocity profile from RANS and adding turbulent fluctuations from a turbulence database [15]. For the laminar case this was not necessary, since it is a laminar flow and the obtained mean flow field specifies directly the inflow.

The LES outflow boundary condition at the outlet of the upstream LES flow solver can be defined using a body force to drive the LES solution near the outlet to the desired Reynolds-averaged flow field.
However, for the course of this investigation, no such body force was employed, which means, that the feedback from the downstream flow solver to the upstream flow solver was suppressed. This was done to ensure that aliased frequencies in the downstream domain are not transferred back to the upstream domain, where they would be able to compromise the original signal. While initial tests with a true two-way coupling did not show an such an effect, the feedback has been suppressed in order to demonstrate clearly, that downstream aliased frequencies have not been present in the upstream domain. Since there is no feedback from the downstream flow solver, the time-evolving LES solution of the upstream flow solver is identical for all cases reported here.

4 Aliasing Problem

In order to demonstrate the presence of aliasing, the upstream flow was periodically excited with two frequencies: one at a Strouhal number $Sr = 1.0$ and another at $Sr = 7.5$. The interface frequency defined by the chosen RANS time step ($f_{\text{interface}} = 1/\Delta t$) is set to $Sr = 10.0$ which leads to a Nyquist frequency of $Sr = 5.0$. Hence, the long wave frequency at $Sr = 1.0$ is well resolved and can be transferred to the downstream domain. However, the second frequency is under-resolved and will lead to aliasing.

In order to quantify the transfer of the dynamic properties, the transient data for several points was recorded over 50 periods of the
lower frequency \((Sr = 1.0)\) and analyzed. The points were located on the \(x = 2D\) plane. For the upstream flow solver the points are just in the same location, where the data is acquired for the downstream flow solver. For the downstream flow solver the points are located right in the inflow plane, and hence, the downstream flow solver has not yet affected the data in this plane. This justifies the approach to use an LES flow solver instead of a RANS flow solver downstream, since in this plane, both solutions are identical. In the following, data for the point \(x = 2D, r = 0.5R; \phi = 0\) will be presented.

The transient data was analyzed by a Fourier transform of the kinetic energy in order to assess its spectral characteristics. Fig. 5 shows the energy spectrum in the upstream domain. Since this is a laminar flow, the spectrum is very smooth and shows mainly the two distinct peaks resulting from the forcing of the flow. There are some additional smaller peaks from the folding of the two frequencies (such as \(Sr = 8.5, Sr = 6.5\)) and subharmonic responses of the flow (such as \(Sr = 2.0\)). The goal of a successful signal processing is to transfer the long-wave frequency \((Sr = 1.0)\) with no energy loss, while suppressing the high frequency disturbance \((Sr = 7.5)\).

Fig. 6 shows the energy spectrum for the same physical point, but in the downstream domain. Since the flow solver computing the upstream domain has transferred the signal without any treatment, the high frequency perturbation in the upstream domain has been aliased and can be found now in the long wave spectrum at \(Sr = 2.5\).

This may cause considerable problems, since this frequency is re-
solved by any unsteady RANS flow solver operating at a time-step correspondent to the interface frequency. Since this peak in the long wave spectrum is not present in the upstream domain, this error has been introduced entirely by the sampling process. Hence, the upstream flow solver has to treat the signal during the sampling process in order to suppress the high frequency perturbation, while preserving the low-frequency oscillation.

5 Temporal Filters

A common procedure in order to avoid aliasing errors in experiments is to use a low-pass filter prior to the sampling process. The low-pass filter suppresses all frequencies above the Nyquist frequency while letting all lower frequencies pass. This filtering process has to be done prior to the sampling process, since otherwise the aliasing error has already taken effect and is indistinguishable from the rest of the long wave spectrum.

Using the same strategy for the sampling of LES data leads to the need of a digital filter. A digital filter can be defined as:

$$ r(t_k) = \sum_{n=0}^{N} b_n s(t_{k-n}) + \sum_{m=1}^{M} a_m r(t_{k-m}) $$

with $r$ the filter response, $s$ the original signal, $b_n$ and $a_m$ the filter constants, $N$ and $M$ define the order of the filter, and $t_k$ the time, where $t_k - t_{k-1}$ is the LES time-step $\Delta t$. Since in LES computations the time-step is usually not constant, but varies in order to maintain the highest possible time-step that satisfies the CFL condition, a pre-
sampling process has to be made. This pre-sampling averages the data with a higher frequency than the actual sampling frequency. In order to avoid aliasing in the pre-sampling process, the frequency of the pre-sampling has to be chosen well within the energy decay, so that the energy of frequencies higher than the Nyquist frequency are considerably smaller than the energy of the lower frequencies. A filter such as Eq. 3 can then be applied.

While a filter in the form of Eq. 3, which uses the history of the signal and the history of prior filter responses in order to define the filter, a so called infinite impulse response filter (IIR filter), can be used, a simplified filter, which uses only the history of the signal, a so called finite impulse response filter (FIR filter) may have advantages:

\[ r(t_k) = \sum_{n=0}^{N} b_n s(t_{k-n}) \]  

(4)

First, FIR filters are always stable. Due to the absence of the filter response, no feedback is possible and hence, this kind of filters will never be able to amplify errors. Second, FIR filters have a linear phase response. The advantage resulting out of that will be made clear later.

Due to the high number of points at the interface the filter has to be applied to, the order \( N \) of the filter is sought to be small, since \( N \) determines the number of time-steps that have to be recorded.

For the current investigation, two different filter have been used. The detailed description of the determination of the filter constants can be found in the appendix.

One filter is based on the Fourier Series Method (FSM), which
is the exact solution for an infinite number of filter coefficients. The filter response of this filter is shown in Fig. 7. The dashed line denotes the ideal filter response: below the cutoff frequency it is unity, above zero. Since only a limited number of filter coefficients are available, the actual filter response differs from the ideal filter.

The order of the filter was chosen to $N = 21$. This order is the lowest order ensuring a filter response of unity at the 0Hz, and thus ensuring mean momentum conservation. The low order of the filter results in an overshoot right next to the cutoff frequency, which is known as the Gibb’s phenomenon.

The second filter employed attempts to minimize the Gibb’s phenomenon. In order to this, so called window functions can be used to smoothen the ripple effects. Here, the usage of a Kaiser window is proposed resulting a smoothed filter response (Fig. 8)

5.1 Temporal Filter: Amplitude Response

In the next step, the filters were applied to the LES computation of the upstream domain. Each filter was applied separately to the LES data in two LES computations. The signal response of both computations using the two different filters show the desired results (Fig. 9 and 10). The long wave perturbation at $Sr = 1.0$, the frequency which is desired to be transmitted, can be found in the downstream domain without a loss of energy. Since the filters have filtered out the high frequency perturbation prior to the actual sampling process, no aliasing can be observed. A comparison with the unfiltered spectrum shows that
the energy of aliased frequency was reduced by 99.6% using the FSM method and by 99.7% for using the Kaiser window.

While for the current test case the results of both filters are nearly identical, more complex test-cases may require to choose a filter based on the filter response. These results show, that digital filters are able to improve the quality of the signal taken from LES data in order to correct the amplitude response and attenuate aliasing.

5.2 Temporal Filter: Phase Response

The application of these filters have a major drawback. Fig. 11 shows the phase response of both filters. The phase response is linear in the passing frequency range. This translates to a constant time delay of:

$$\Delta t = \frac{N - 1}{2 \cdot f_{\text{sample}}}$$  \hspace{1cm} (5)

This means, that the signal coming from the upstream flow solver arrives in the downstream flow solver delayed. The time-delay can be minimized by decreasing the order of the filter. However, in the author’s opinion, the here presented filters with an order of $N = 21$ are already the minimum order for a filter with an acceptable quality of amplitude response.

If unsteady coupling effects are investigated, this time delay introduced by the filter is usually not acceptable, especially since previous investigation went at great lengths to avoid the smaller time delay by the explicit coupling of the two flow solvers[17].

Since the phase delay is unavoidable using these temporal filters,
the application of these filters is limited to the following. Most unsteady RANS flow solvers for turbomachinery applications do not claim to compute a truly unsteady flow, but an ensemble-average or a phase-average. In phase-averaged flows a number of averages of the flow are taken in relation to the phase of a base frequency $f_{\text{base}}$, which is tied to the rotational speed of the turbomachinery. Assuming that the LES delivers data to a RANS flow solver computing phase-averages, the LES flow solver can compute phase-averages on the basis of the LES data at the interface. Then, a filter is designed which creates the time-delay for one full period of the base frequency. Here, the advantage of a linear phase response of a FIR filter is apparent: the linear phase response translates to a constant time delay, which can be controlled by the order of the filter. The order of the filter is then determined by:

$$N = 2 \cdot \frac{f_{\text{sample}}}{f_{\text{base}}} + 1$$

(6)

If the time delay created by the explicit coupling of the flow solvers[17] can be also corrected in this filter delay, if the order is reduced by 1.

While this procedure might be working for a number of applications, most unsteady LES-RANS computations will neither tolerate the time-delay nor the usage of phase-averages at the interface.

6 Spatial Filters

The major reason why temporal filters are creating a time-delay is the lack of information of the signal in the future. A relationship known
as the Taylor-hypothesis may help in fluid mechanic applications:

\[
\frac{\partial}{\partial t} = -u_c \frac{\partial}{\partial x} \tag{7}
\]

with \( u_c \) the local convection velocity. The Taylor-hypothesis is valid in the absence of diffusion and \( u_c \) as the only convection. This relationship translates a temporal signal into a spatial signal. The temporal filter then becomes the spatial filter:

\[
r(t_k) = \sum_{n=0}^{N} b_n s(x_{k-n}) \tag{8}
\]

with \( x_n - x_{n-1} = \frac{u_c}{f_{\text{sample}}} \tag{9} \)

Instead of using the time history of the signal, the downstream development is sampled. Unlike the case of the temporal filters, where the time history of the interface points have to be stored, no additional memory is necessary for the spatial form of the filter.

So far, the phase delay is still present, unless the origin of the filter is shifted upstream putting the filter centrally around the desired interface point:

\[
x_{0, \text{new}} = x_{0, \text{old}} - \frac{N - 1}{2} \frac{u_c}{f_{\text{sample}}} \tag{10}
\]

Here, the location of the sampling points is defined by the sampling frequency. In many flow solvers, especially when using structured meshes, it may be of advantage to define the sampling frequency on the mesh spacing. The location of the sampling points are then defined as points on the mesh and the sampling frequency by the distance of the points:

\[
f_{\text{sample}} = \frac{u_c}{\Delta x} \tag{11}
\]
The advantage of this definition is first of all practical nature, since it is easier to retrieve data from these points. Furthermore, no error due to aliasing in the pre-sampling process is introduced, since the sampling points resolve the entire spectrum on the given mesh.

The disadvantage of this definition of the sampling points is the independence of the sampling frequency from the interface frequency. A variation of the RANS time-step (and hence, a variation of the interface frequency) requires a new definition of the filter, since the desired cutoff frequency has changed, while the sampling frequency remained constant.

For the current study, the spacing of mesh points in axial direction was $\Delta x = 3D/128$. With $u_c = U_{bulk} = 1$ this results in a Strouhal number of $Sr_{sample} = 42.67$. The cutoff frequency of $Sr = 5.0$ results in a normalized cutoff frequency $f_{cutoff} = 0.117$.

The number of sampling point was limited to $N = 17$. A small number of sampling points is desirable since the Taylor-hypothesis looses validity with increasing distance to the interface point. Furthermore, the extend of the spatial filter is sought to be small for several reasons. First, in geometries more complex than the current pipe flow, the spatial filter has to be put into an area, where the flow is nearly parallel and has a nearly constant convection velocity over the spatial extend of the filter, which may not be the case over a large portion of the flow. Second, in parallel computations the extend of a spatial filter may be larger than the extend of the flow field computed on a single processor, so that interactions between parallel processors
may be necessary.

Since it is rather difficult to design a filter with a low cutoff frequency such as \( f_{\text{cutoff}} = 0.117 \) on the basis of few sampling points, a running average filter was employed \( (b_n = 1/N) \), since it was the only filter which could ensure an amplitude response of unity at the mean flow. The resulting filter response can be seen in Fig. 12. This filter is still in need of improvement, but shall be sufficient for the current investigation.

### 6.1 Spatial Filter: Amplitude Response

The spatial filter was implemented to the upstream LES flow solver computing the pipe flow. The integrated LES-LES computation was performed and the received signal at the inlet of the downstream flow solver examined (Fig. 13). It can be seen, that the low frequency perturbation has passed the interface, although, due to the filtering, has lost some energy \( (\approx 3\%) \). The high frequency perturbation has been filtered out sufficiently so that aliasing is successfully suppressed. In comparison to the unfiltered spectrum, the aliased frequency was attenuated by 99.1%.

### 6.2 Spatial Filter: Phase Response

The phase delay of this filter can be expressed as a constant time delay \( \Delta t \), which is a sum of the filter time-delay and the time correction by
the shift of the origin Eq. (10):

$$\Delta t_{\text{total}} = \Delta t_{\text{filter}} + \Delta t_{\text{origin shift}}$$

(12)

The filter time-delay $\Delta t_{\text{filter}}$ is defined corresponding to Eq. (5). The time correction due to the shift of the origin upstream is given by:

$$\Delta t_{\text{origin shift}} = \frac{\Delta x}{u_c} = - \frac{N - 1}{2} \frac{u_c}{f_{\text{sample}}} \frac{1}{u_c}$$

(13)

Equation (12) then becomes:

$$\Delta t_{\text{total}} = \frac{N - 1}{2} \frac{N - 1}{f_{\text{sample}}} = 0$$

(14)

The zero time delay ensures a true phase transfer of a given perturbation.

This result shows, that anti-aliasing with spatial filters is possible, allowing the proper transfer of amplitude and phase of a given long wave perturbation, and hence, allowing a true unsteady coupling between LES and RANS flow solvers.

7 Filtering: Application to Turbulent Flows

In order to demonstrate the the filtering procedures on turbulent flows, the computations were repeated using a turbulent pipe flow with a Reynolds-number of $Re = 15,000$. Both LES computations are now truly turbulent.

Since the inflow of the upstream pipe has to turbulent, the inflow of the was generated by a separate LES computation of a periodic pipe flow with $Re = 15,000$ and $S = 0.15$. The outflow plane of this
computation was recorded into a data base which in turn is then fed into the inflow of the actual LES computations [18].

As in the laminar case, the swirl velocity is periodically excited. Here, the mean swirl velocity is modulated while the turbulent fluctuations are left untouched. Since the propagation of a convective wave is disturbed by the turbulence, the upstream pipe was shortened by 1.5\(D\) so that the inlet plane is 0.5\(D\) upstream of the interface plane. This allows to obtain a clearer signal from the forcing at the interface plane, since time is decreased, where the forcing wave and stochastic turbulence can interact.

The upstream LES flow solver filters the its solution using a RANS time step \(\Delta\tau\) and transfers it to the downstream LES. The downstream LES flow solver regenerates its inflow boundary conditions by using the mean flow field delivered by the upstream flow solver and adding turbulent fluctuations from a data base in order to regenerate the resolved turbulence [15]. The data base was generated by an LES computation of a periodic pipe at a higher Reynolds-number of \(Re = 30,000\) in order to point out differences in the high frequency spectrum in the upstream and downstream domain.

Fig. 14 shows the resulting energy spectrum in the upstream domain. The two forcing frequencies can be easily identified. The high frequency forcing is disturbed by the turbulence and is less distinct. Yet, in the unfiltered downstream spectrum aliasing can be observed (Fig. 15).

Applying a temporal filter to the upstream LES computation sup-

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presses the aliasing successfully (Fig. 16). Please note that the differences in the high frequency spectrum between upstream and downstream domain are due to the reconstruction of high frequency turbulence using a data base in the downstream domain.

The use of a spatial filter (Fig. 17) delivers essentially the same result as a temporal filter. The aliasing is successfully suppressed.

8 Conclusions

In integrated LES-RANS computations the higher temporal resolution of the flow in the LES domain may lead to aliasing errors when sampling the data for the RANS flow solver using a larger time-step and hence, operating at a lower frequency. This problem is similar to the aliasing problem in experiments, where a continuous signal is digitally sampled, and where anti-aliasing is achieved with low-pass filtering prior to the sampling.

The current study was able to demonstrate the presence of aliasing and proposed low-pass filtering prior to the sampling process as a possible solution. The effects of two different digital filters were shown and both filters were able to suppress the aliasing for the chosen test-case of a periodically perturbed pipe flow. However, the large phase delay introduced by the filter limits the application of temporal filters to phase-averaged solutions.

With the usage of the Taylor-hypothesis a temporal filter can be transformed into a spatial filter. The spatial formulation allows to
correct the phase response of the filter. Despite some drawbacks, the
spatial filter was able to suppress aliasing successfully, while enabling
a true unsteady coupling of flow solvers without phase delay.

The identification of the aliasing problem in integrated LES-RANS
computations and its solution using spatial filters is another important
step towards truly unsteady flow predictions using coupled LES-RANS
flow solvers.

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A Digital Filter Design

Digital filter design, especially those of low order, is currently more an
art than an exact science. The minimum specifications for a digital
filter vary from application to application. For coupled RANS-LES
computations, the temporal digital filters were designed using a mini-
imum number of coefficients, while having a filter response of unity at
the 0Hz in order to ensure the mean momentum conservation. Here,
two different filters are presented. One is derived mathematically from
the desired filter response and is the basis for all filters. The other
filter improves the filter response by employing a window function.
A.1 Fourier Series Method

The first filter is designed using the Fourier Series Method [19]. The coefficients can be derived from:

\[ b_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(\lambda)[\cos(m\lambda) + j\sin(m\lambda)]d\lambda \quad (15) \]

with \( m = n - \frac{N - 1}{2} \) \quad (16)

with \( H(\lambda) \) the desired filter response and \( \lambda \) the normalized frequency, here normalized to the pre-sampled frequency. The optimal filter response would have a cutoff frequency of \( \frac{1}{2 \cdot f_{\text{interface}}} \). The pre-sampling frequency was chosen here twice the interface frequency, which results in a cutoff frequency \( \lambda_{\text{cutoff}} = 1/4 \). Eq. 15 then becomes:

\[ b_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos(m\lambda)d\lambda + j \frac{1}{2\pi} \int_{-\pi}^{\pi} \sin(m\lambda)d\lambda \quad (17) \]

The second integrand is zero, since the integrand is an odd function and the limits of the integration are symmetric. Eq. 17 then becomes:

\[ b_n = \frac{\sin(m\pi)}{2m\pi} \bigg|_{\lambda=-\frac{\pi}{4}}^{\lambda=\frac{\pi}{4}} = \frac{\sin(m\frac{\pi}{4})}{m\pi} \quad (18) \]

Please note, that the definition of the filters does not include the knowledge of the actual sampling frequency, but only the cutoff frequency relative to the sampling frequency. This means, if the interface frequency is changed, the filters will adapt automatically. The filter coefficients for \( N = 21 \) can be found in the following table:
\begin{align*}
b_0 &= b_{20} = 0.0311536 \\
b_1 &= b_{19} = 0.0244766 \\
b_2 &= b_{18} = -0.0000006 \\
b_3 &= b_{17} = -0.0314696 \\
b_4 &= b_{16} = -0.0519226 \\
b_5 &= b_{15} = -0.0440576 \\
b_6 &= b_{14} = 0.0000006 \\
b_7 &= b_{13} = 0.0734296 \\
b_8 &= b_{12} = 0.1557666 \\
b_9 &= b_{11} = 0.2202866 \\
b_{10} &= 0.2446776
\end{align*}

The filter response of this filter is shown in Fig. 7. The deviations of the actual filter response from the ideal filter response are due to the low number of filter coefficients. Most notably, an amplification close to the cutoff frequency can be observed. This amplification is called the Gibb’s effect.

### A.2 Window Method

The second filter used in this study uses a window method in order to smoothen the filter response. One of the major shortcomings of the Fourier Series Method is the assumption, that the signal is periodic. This creates some problems due to an in-continuity at the end and at the beginning of the recorded signal. One possibility to dampen this
effect is to use window functions:

\[ r(t_k) = \sum_{n=0}^{N} b_n w_n s(t_k-n) \]  \hspace{1cm} (19)

with \( w_n \) the window function. Some of the most common window functions are the Hann, Hamming, Parzen of Kaiser windows[20]. The choice of the appropriate window method is subjective and depends on the preferences of the designer. Currently, a number of filter design tools are available, which allow to investigate the effect of a given window to the filter response. Here, the commercial package Matlab has been used.

Applying several windows to the filter \((N = 21)\) results in varying filter responses. Some filters are not able to attenuate the Gibb’s phenomenon completely (such as the Hann and Hamming window). Others disturb the low frequency spectrum significantly (such as the Parzen window) for the given order of the filter. Here, as a compromise the Kaiser window was chosen (Fig. 18).

The window function is usually combined with the filter coefficients leading to a new set of coefficients:
\begin{tabular}{c c}
\hline
$b_0$ & $b_{20}$ = 0.0004736 \\
$b_1$ & $b_{19}$ = 0.0013386 \\
$b_2$ & $b_{18}$ = -0.0000006 \\
$b_3$ & $b_{17}$ = -0.0069236 \\
$b_4$ & $b_{16}$ = -0.0179856 \\
$b_5$ & $b_{15}$ = -0.0217416 \\
$b_6$ & $b_{14}$ = 0.0000006 \\
$b_7$ & $b_{13}$ = 0.0583416 \\
$b_8$ & $b_{12}$ = 0.1425086 \\
$b_9$ & $b_{11}$ = 0.2189896 \\
$b_{10}$ & 0.2500006 \\
\hline
\end{tabular}

This filter has been designed to work under the current circumstances, but may have to be adapted for other application, most notably, when a larger number of filter coefficients can be used.

References

[1] R. Davis, J. Yao, J. P. Clark, G. Stetson, J. J. Alonso, A. Jame-
son, C. Haldeman, and M. Dunn. Unsteady interaction between 
a transsonic turbine stage and downstream components. *ASME 

chapter 5 Reynolds averaged and large eddy simulation modeling


Figure 1: Decomposition of gas turbine engine. (RANS/LES of compressor/diffuser[8], LES of combustor[3], RANS of turbine section[1])

Figure 2: Example of a two-way coupled RANS-LES: compressor stage and a pre-diffuser in a gas turbine [8].
LES to RANS
Provide time averaged data

RANS to LES
Create turbulent fluctuations

LES to RANS
Provide time averaged data

RANS to LES
Upstream influence of pressure very important

Figure 3: Gas turbine combustor with interfaces.

periodically excited

overlap

LES upstream

Re=1000
S = 0.15

feedback suppressed

Figure 4: Geometry of the test-case.
Figure 5: Energy spectrum at a point \((x = 2D, r = 0.5R; \phi = 0)\) in the interface plane of the upstream domain.

Figure 6: Energy spectrum at the interface plane of the downstream domain. Physically identical point as Fig. 5 \((x = 2D, r = 0.5R; \phi = 0)\). No filter used.
Figure 7: Filter response of filter designed with Fourier Series Method (solid line). $N = 21$. Cutoff frequency: 0.25. Dashed line: ideal filter response.
Figure 8: Filter response of filter designed with Window Method (solid line) using a Kaiser window. $N = 21$. Cutoff frequency: 0.25. Dashed line: ideal filter response.
Figure 9: Energy spectrum at the interface plane of the downstream domain. Filter designed with Fourier Series Method (Fig. 7) used.

Figure 10: Energy spectrum at the interface plane of the downstream domain. Filter designed with Window method (Fig. 8) used.
Figure 11: Phase response of filter designed with Window Method (solid line) using a Kaiser window. $N = 21$. 

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Figure 12: Amplitude response of spatial filter based on a running average (solid line). $N = 17$. Dashed line: ideal filter response.

Figure 13: Energy spectrum at the interface plane of the downstream domain. Spatial running average filter (Fig. 12) used.
Figure 14: Energy spectrum at the interface: original signal in the upstream domain.

Figure 15: Energy spectrum at the interface: downstream solution without filtering.
Figure 16: Energy spectrum at the interface: downstream solution using temporal filter (Kaiser window).

Figure 17: Energy spectrum at the interface: downstream solution using spatial filter.
Figure 18: Kaiser window used to improve filter response, $N = 21$, $\beta = 6$